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DAVID HUME'S CRITIQUE OF INFINITY

BY

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With finite but inexhaustible love
for Tina

'Twere certainly to be wish'd, that some expedient were fallen upon to reconcile philosophy and common sense, which with regard to the question of infinite divisibility have wag'd most cruel wars with each other.

— David Hume, *A Treatise of Human Nature* ('Abstract')

CONTENTS

PREFACE

The problem of infinity is of central but often unappreciated importance in Hume's philosophy. Although the question of whether extension is infinitely or only finitely divisible raises some of the most challenging philosophical paradoxes for Hume's empiricism, there have been few detailed and no fully comprehensive systematic discussions of Hume's objections to infinity. In this book, I offer a detailed exposition and critical evaluation of Hume's refutation of infinite divisibility, placing Hume's arguments in the historical context of Enlightenment philosophy of mathematics and metaphysics of space and time, and assessing the prospects of his strict finitism in light of contemporary mathematics, science, and philosophy.

Hume's timeless relevance is partly a result of his preoccupation with universal philosophical themes. He is particularly concerned with the limitations of relying on sense experience for knowledge. He acknowledges the conflict between what can be known from experience and the deeply entrenched 'rationalistic' beliefs and conceptual commitments for which there is no adequate perceptual basis. The collision of experience and wayward philosophical enthusiasms is nowhere more poignant for Hume than in the case of infinite divisibility. Here philosophy confronts a concept that seems to be indispensable for the exact sciences, but which in obvious ways also appears to be humanly incomprehensible. Hume attacks the question from both directions. He lays siege to the concept of infinity on many fronts with many different arguments, and he outlines an experientially defensible theory of sensible extensionless indivisibles as an alternative empiricist theory for the finite divisibility of space and time. He argues that we do not and cannot have an adequate idea of infinity or infinite divisibility in the first place, and that we can get along perfectly well without it, thereby avoiding the methodological confusions that the concept entails.

Hume is fascinating for reasons of style as well as substance. He is one of the few technically rigorous philosophers whose personality, despite the intervention of several centuries, emerges as a living presence from the printed page. One feels an immediate empathy with Hume's honest doubts as he struggles in his philosophical writings to strike a measured balance between natural reason and skeptical philosophical inquiry. Hume's investigations of the origin of ideas and the limits of knowledge evoke the leisured atmosphere of eighteenth-century letters, of sherry and walking sticks and animated conversations around the fireplace. Beyond this, Hume is subtle in thought, occasionally confusing in exposition, and hence a challenge to interpret correctly. Yet every paragraph promises the resolution of longstanding conceptual difficulties in a system of epistemology and moral theory founded on a recognition of the scope of human experience, and a firm commitment to keeping philosophy within its bounds.

The plan of these chapters is to provide a critical discussion of Hume's arguments against infinite divisibility and in support of the concept of sensible extensionless indivisibles. The account of Hume's critique of infinity fits these diverse proofs together into a coherent picture of Hume's metaphysics of extension in space and time, explaining their interrelationships and assessing their philosophical importance. For these purposes, it has proven useful to give reconstructions of each of Hume's arguments by identifying assumptions and conclusions in the style of an informal deductive inference. Such reconstructions have the advantage of exhibiting the implicit structure and propositional content of plausible interpretations of an author's thought in a perspicuous way that facilitates its philosophical evaluation.

The sense of 'reconstruction' in these restatements of Hume's arguments is not that in which an expert repairs or restores a damaged building or artwork. Hume's arguments are sometimes hard to understand, but they are generally complete and intact. Nor is this the paleontologist's detective work, who, from a single fossil rib or fin bone can reconstruct for the astonished layman an entire ichthyosaurus. The informal reconstructions of Hume's arguments about infinite divisibility and the theory of sensible extensionless indivisibles try instead to reassemble the elements of Hume's proofs into more easily discernible structures, to interpolate implicit assumptions and conclusions, and to sharpen their outlines and make them more readily understandable from a contemporary perspective. The Procrustean bed of misinterpretation to be avoided in the history of philosophy is not the sequential formatting of reconstructed inferences, but the naive imposition of anachronistic concepts, categories, and terminologies on classical writings, warping original meanings by violently prying ideas out of cultural chronological context, and transposing them into misleading conceptual frameworks. There may be philosophical texts that do not lend themselves to reconstruction as argument chains. But that is no reason to avoid the method in explaining the thoughts of philosophers like Hume who explicitly claim to be offering proofs for their conclusions that must accordingly stand the test of adequacy as sound arguments.

The Introduction on the 'Two-Fold Task of Hume's Critique' gives an overview with historical background of Hume's refutation of infinite divisibility and theory of sensible extensionless indivisibles. Part One, 'The Inkspot Experiment', and Part Two, 'Refutations of Infinite Divisibility', offer a detailed study of Hume's most extensive treatment of infinite divisibility and analysis of spatial extension and geometrical magnitude in the *Treatise* and the *Enquiry Concerning Human Understanding*. An evaluation of the implications of Hume's project for present day mathematics, science, and philosophy is presented in the Conclusion, 'Hume Against the Mathematicians'. The Afterword, 'Hume's Aesthetic Psychology of Distance, Greatness, and the

Sublime', considers Hume's strict finitism in application to predominantly infinitary eighteenth-century preoccupations with the aesthetic experience of the sublime, paying special attention to the problem as it reflects on Hume's theory of the relation between reason and the passions.

I hope that this investigation will contribute to renewed interest in Hume's arguments against infinity. Hume's empiricism and the implications of his humanized doctrine of natural belief for the concept of extension in my opinion deserve more serious consideration than they have so far received. Hume's critique of infinite divisibility is not only historically interesting, although it is certainly that, but represents a profound and carefully considered defiance of mainstream infinitism in pure and applied mathematics, in our day as in his. The purpose of this study of Hume's objections to infinite divisibility will be satisfied if it stimulates interest in the philosophical problems concerning the nature of extension that induced Hume to reject the very idea of infinity, and to advance in its place an uncompromising strict finitism in mathematics and metaphysics.

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Bergen, Norway
16 August 2000

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INTRODUCTION

TWO-FOLD TASK OF HUME'S CRITIQUE

Our system concerning space and time consists of two parts, which are intimately connected together... The capacity of the mind is not infinite; consequently no idea of extension or duration consists of an infinite number of parts or inferior ideas, but of a finite number, and these simple and indivisible... The other part of our system is a consequence of this. The parts, into which the ideas of space and time resolve themselves, become at last indivisible; and these indivisible parts, being nothing in themselves, are inconceivable when not fill'd with something real and existent.

— Hume, *Treatise*, Book I, Part II, Section IV

INTRODUCTION

TWO-FOLD TASK OF HUME'S CRITIQUE

Hume's Strict Finitism

The concept of infinity is a cornerstone of classical mathematics. It is shielded from the objection that it transcends imagination and entails intuitive paradoxes by mathematics' reputation as a paradigm of impeccably exact reasoning in the service of absolutely certain knowledge. The theoretical grandeur and pragmatic success of infinitary mathematics in arithmetic, geometry, and the calculus, with its impressive applications in physics and engineering, further testify not only to the utility but the intelligibility and reality of infinite quantity, magnitude, and relation. To question infinity or the possibility of infinite divisibility is to declare oneself mathematically inept, lacking in essential insight for the higher reaches of the formal sciences.

All this is challenged by David Hume's revolutionary program to reconstitute philosophy by 'the experimental method of reasoning' in fashioning a new theory of human nature. In the 'Introduction' to *A Treatise of Human Nature*, Hume places the methodology of his humanized empiricism in judgment even over the most respected findings of mathematics, natural philosophy (science), and natural religion (theology considered independently of revelation or faith). From the outset, Hume holds out the two-fold task of criticizing the received presuppositions and implications of these disciplines, and the potential for advancing their development in revisionary ways. The fact that infinite divisibility is endorsed by the authority of mathematics and may be indispensable to mathematical theory and practice by itself is not decisive for Hume in evaluating its philosophical legitimacy. Hume writes:

> 'Tis evident, that all the sciences have a relation, greater or less, to human nature; and that however wide any of them may seem to run from it, they still return back by one passage or another. Even *Mathematics, Natural Philosophy, and Natural Religion*, are in some measure dependent on the science of Man; since they lie under the cognizance of men, and are judged of by their powers and faculties. 'Tis impossible to tell what changes and improvements we might make in these sciences were we thoroughly acquainted with the extent and force of human understanding, and cou'd explain the nature of the ideas we employ, and of the operations we perform in our reasonings.[1]

This statement of Hume's project offers a revealing contrast with Descartes's rethinking of the foundations of knowledge in the *Meditations on First Philosophy*. Descartes reports that the inspiration for his philosophy was the desire to make all knowledge as secure as that found in arithmetic and geometry.[2] Hume by comparison has no such predisposition toward the sanctity of classical mathematics.[3]

Descartes, in applying methodological doubt to raze an inherited edifice of knowledge, refuses to admit any belief, no matter how extensively accepted in received philosophy or commonsense opinion, unless it stands scrutiny against the malignant demon hypothesis as the strongest sanely imaginable

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[1] Hume, *Treatise*, 'Introduction', p. xv.

[2] Descartes, *Discourse*, *Works*, Vol. I, p. 85: "Most of all was I delighted with Mathematics because of the certainty of its demonstrations and the evidence of its reasoning; but I did not yet understand its true use, and, believing that it was of service only in the mechanical arts, I was astronished that, seeing how firm and solid was its basis, no loftier edifice had been reared thereupon."

[3] *Treatise*, pp. 166, 198. Hume maintains that mathematical necessity depends on acts of human understanding, and that as a result mathematical proofs like judgments of fact, especially if they are long or detailed, are at most only probably true. See Atkinson, "Hume on Mathematics", for an account of Hume's concept of the qualifiedly 'synthetic' necessity of mathematical truths.

basis for skepticism. Thus, Descartes sets aside as dubitable the testimony of the senses, the Scholastic tradition of Aristotelian metaphysics and science, and generally all propositions less certain than his own existence, regardless of their popularity or usefulness. Hume stands Descartes's method on its head, adopting an experiential criterion of acceptability in place of pure reason or the 'light of nature' (*lumen naturale*), while preserving Descartes's radical spirit of discarding any disqualifying beliefs or ideas.[4] Far from agreeing with Descartes that all knowledge should finally be made as certain as classical mathematics, Hume maintains that mathematics too provides knowledge only to the extent that its concepts first pass muster according to the principles of a properly humanized empirical epistemology and theory of mind.[5]

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[4] It is instructive to compare Hume's objections to the possibility of having clear and distinct ideas of infinity or infinite divisibility with his challenge to Descartes's *cogito* discovery of the better knowability of the ego or self. In a famous passage of the *Treatise*, Hume writes: "For my part, when I enter most intimately into what I call *myself*, I always stumble on some particular perception or other, of heat or cold, light or shade, love or hatred, pain or pleasure. I never can catch *myself* at any time without a perception, and never can observe any thing but the perception." Hume in both cases claims to have tried Descartes's method, but concludes that upon careful inspection he is unable phenomenologically to locate certain ideas cherished by rationalist metaphysics. In Meditation I, Descartes questions even the reliability of his previous beliefs in mathematical truths; but this does not seem to vitiate mathematics as a paradigm for the completion of his epistemic program after his emergence from the skeptical attitude upon establishing rationalist foundations for knowledge. In his August 26-31, 1737 letter to Michael Ramsay, Hume specifically mentions among other sources Descartes's *Meditations* as essential background reading to understanding the *Treatise*. The letter appears in Popkin, "So, Hume Did Read Berkeley", pp. 774-775. The letter was first published by Kozanecki in "Dawida Hume'a Nieznane Listy w Zbiorach Muzeum Czartoryskich (Polska)", pp. 133-134.

[5] The distinction between rationalism and empiricism is oversimplified, especially when rationalism is arbitrarily confined to the seventeenth century, and empiricism to the eighteenth, or when it is supposed that any particular thinker representing one of these movements, Descartes, Leibniz, or Spinoza, or Locke, Berkeley, or Hume, is respectively a pure rationalist

When Hume looks into the ideas of infinity and infinite divisibility, he finds them inadequate, excluded by the standards of what the mind can know according to the principles of human nature. Hume discovers two categories of objections to infinitary mathematics as in effect he applies Protagoras's dictum to make man (humankind) the measure of all things to the received propositions of mathematics, science, metaphysics and religion.

First, Hume precisely defines the contents of thought. Thinking involves perceptions, consisting only of impressions of sensation and reflection, and ideas as faded faint impressions, all of ultimately experiential origin. Hume argues that human beings as finite creatures with finitely limited minds are incapable of entertaining adequate ideas of the infinite. Infinity and infinite divisibility are necessarily beyond the limits of human perception. For Hume, this means that no definite

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or pure empiricist, whatever these concepts are taken to signify. There is no pure rationalism that does not recognize experience as contributing to knowledge among the so-called rationalist systems, and there is no pure empiricism in the historical canon that tries to do entirely without reason. What makes a philosophy rationalist or empiricist in the proper sense of the word, as with due caution I shall continue to use these terms under this definition, is rather a matter of emphasis and of the predominance of reason over experience or the reverse. Descartes is a radical though not a pure rationalist in this sense because he officially suspends acceptance of the world of experience until he establishes foundations in reason for the veridicality of perception in the proof that God exists and is no deceiver. Berkeley and Hume are radical though again not pure empiricists in the opposite sense because they are skeptical about the ability of reason to decide substantive metaphysical questions, and, while not rejecting let alone refusing to exercise reason, understand experience as more important than mere logical inference and armchair definition and analysis of meaning in their theoretical-methodological prioritization of epistemic principles. With these kinds of qualifications, I see no good objection to reinterpreting rather than simply abandoning the rationalism-empiricism terminology, nor, within the limits of legitimate usefulness of the concept, the study of the complementarity and dynamic dialectical opposition and interplay between impure rationalism and impure empiricism in the history of modern philosophy.

impressions or ideas can positively correspond to the use of words or mathematical symbols for such concepts. Hume's conclusions are not as straightforward as they might first appear, since from the fact that finite minds cannot fully contain or encompass infinite ideas, ideas of infinite magnitude or infinitely fine structure, it does not obviously follow that finite minds cannot contain ideas *of* or *about* the infinite.[6] The objection requires more careful argument, invoking Hume's distinction between impressions and ideas, and his generic natural history of their empirical ancestry in sensation. It is significant that, having articulated a humanized epistemology and theory of mind, Hume proceeds immediately in Part II to apply its principles concerning the finite experiential origin of all perceptions to the metaphysics of space and time. Hume's argument in the *Treatise* Book I, Part II, 'Of the ideas of space and time', systematically demolishes the classical mathematical concept of the infinite divisibility of extension. Then, more devastatingly, Hume maintains in even stronger terms by a series of *reductio* refutations that no mind could possibly have an adequate idea of infinity or infinite divisibility, because any such idea would be self-contradictory.

This is the negative part of Hume's two-fold critique of infinity. Like Descartes, again, however, Hume's purpose is not purely destructive. He engages in skepticism not for its own sake, but as a preliminary to offering a more acceptable alternative. Amid the ruins of the ideas of infinity and infinite

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[6] Locke, *Essay*, pp. 213-214: "... we are carefully to distinguish between the *Idea* of the Infinity of Space, and the *Idea* of a Space infinite: The first is nothing but a supposed endless Progression of the Mind, over what repeated *Ideas* of space it pleases; but to have actually in the Mind the *Idea* of a Space infinite, is to suppose the Mind already passed over, and actually to have a view of all those repeated *Ideas* of Space, which an endless repetition can never totally represent to it, which carries in it a plain contradiction." Locke, as will be seen, allows only a negative idea of infinity or infinite divisibility. Hume's critique, in its rejection of negative ideas, can be understood in part as disallowing even this possibility in Locke, conflating the distinction between having an idea of or about infinity and having an idea that itself exhibits infinite extent or divisibility.

divisibility, Hume offers to rebuild something more secure in their place. The positive part of Hume's critique substitutes a humanized empiricist theory of the finite divisibility of extension into sensible extensionless indivisibles or extensionless *minima sensibilia* as the atomic constituents of physical space. These spatial points are supposed to have color and tactile properties, but are so tiny that although they are experienced in sensation and imagination they cannot be further divided without altogether vanishing. Hume, in opposition to the classical abstract rationalist conception of geometrical magnitude as infinitely divisible, thinks of space and spatial extension as a finite mosaic of finitely many juxtaposed sensible extensionless indivisibles, into which extension as a result is only finitely divisible.

Thus, Hume's critique of infinity attempts to satisfy a twofold, negative and positive, purpose. In the negative part, Hume refutes the concepts of infinity and infinite divisibility in two ways. He argues that there can be no adequate idea of infinite divisibility according to his empiricist epistemology and theory of mind, and challenges as logically inconsistent such concepts of infinite divisibility as are found in classical mathematics and rationalist metaphysics. More positively, Hume then proceeds to replace the inadequate idea of infinitely divisible spatial extension with a nonclassical theory of sensible extensionless indivisibles as atomic constituents of finitely divisible extension in space.

Dialectical Structure of Hume's Critique

In the *Treatise*, Book I, Part II, Section IV, '*Objections answer'd*', Hume divides his project into two negative and positive parts:

> Our system concerning space and time consists of two parts, which are intimately connected together... The capacity of the mind is not infinite; consequently no idea of extension or duration consists of an infinite number of parts or inferior ideas, but of a finite number, and these simple and indivisible... The other part of our system is a consequence of this. The parts, into which the ideas of space and time resolve themselves, become at last indivisible; and these indivisible parts, being nothing in

> themselves, are inconceivable when not fill'd with something real and existent.[7]

The exposition of Hume's critique of infinity in the following chapters is guided by Hume's description of his two-fold dialectical task. The refutation of infinite divisibility, as the negative part, and the theory of space as a dense distribution of sensible extensionless indivisibles, as the positive part, jointly constitute Hume's empiricist theory of space. The two complementary objectives of Hume's theory, together with an appreciation of the alternating refrains of skepticism and naturalism that pervade his philosophy, offer a framework in which Hume's critique of infinity can be understood.

Hume's *Treatise* contains six proofs against the infinite divisibility of extension. The centerpiece is an argument that satisfies both parts of Hume's two-fold task by simultaneously refuting infinite divisibility and justifying the finite division of space into sensible extensionless indivisibles. The proof, which I shall call the inkspot argument, is based on Hume's famous description of a visual experiment involving the perception of a distant inkspot. The inkspot argument is distinctively Humean, in that it relies on the humanized epistemic principles Hume endorses in his attempt to apply what he calls the experimental method of reasoning to moral subjects. The inkspot argument is reinforced by four *reductio* proofs, and a dilemma about the application of ideas of equality and quantity in geometry. These do not make essential use of Hume's positive doctrines, but attempt to turn the assumptions of proponents of infinite divisibility against themselves. They are primarily dialectical barbs aimed at classical mathematically-minded metaphysicians, intended to undercut infinitist objections to the theory of sensible extensionless indivisibles. By eliminating infinitism as a viable alternative, they contribute to Hume's proof that there are extensionless indivisibles, which the inkspot argument shows to be sensible.

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[7] *Treatise*, p. 39.

The motivations for Hume's *reductio* objections to infinite divisibility are complex. The lack of an explicit plan for this part of Hume's analysis has led to confusion about his purposes, which have remained obscure to many of his commentators. Hume summarizes the object of his *reductio* arguments in the anonymous 'Abstract of a Book Lately Published...', when he explains: "Having denied the infinite divisibility of extension, our author finds himself obliged to refute those mathematical arguments, which have been adduced for it; and these indeed are the only ones of any weight."[8] Yet the *reductio* proofs have been criticized more often than Hume's inkspot experiment, as though Hume meant the case against infinite divisibility to rest primarily on these disproofs, rather than on his main argument from the limits of perception as the origin of all ideas. The *reductio* arguments have drawn fire especially in connection with Hume's apparent misunderstanding of the theory of limits in mathematical analysis, and the *Treatise* note about the difference between *proportional* and *aliquot* parts.[9] The *reductio* arguments are in some sense external to but ultimately an essential part of Hume's theory of extension. They point up inconsistencies in the traditional mathematical approach to the metaphysics of space, and by their rejection of empyrean rationalist assumptions that Hume for the most part does not share, leave his empiricism standing alone as unchallenged master of the field.

The *Enquiry Concerning Human Understanding* exhibits the same two-part division of positive and negative arguments as the *Treatise.* The later work by contrast includes only two proofs, offered almost as asides to a discussion of skepticism in notes to sections 124 and 125. The argument in *Enquiry* 124 is a *reductio ad absurdum* that is different from but related to and in some ways a hybrid of two arguments, one positive and the other negative, found in the *Treatise.* The argument of 125 further implicitly supports Hume's theory of spatial extension as

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[8] Ibid., 'Abstract', p. 658.

[9] Ibid., p. 30, n. 1.

composed of sensible extensionless indivisibles, and does duty in the later work as a substitute for the *Treatise* inkspot argument. The two-fold task of refuting the infinite divisibility of extension and upholding the alternative theory of sensible extensionless indivisibles, and the strategy of achieving these aims both by positive phenomenal argument and negative *reductio* proofs is (unequally) reflected in both the *Treatise* and first *Enquiry*. The positive arguments of both texts, unlike the *reductio* proofs, are more complete, in the sense that they combine Hume's two-fold task of refuting the infinite divisibility of extension and defending the theory of space as a composition of sensible extensionless indivisibles.

All eight of Hume's proofs in the *Treatise* and *Enquiry* are crucial for understanding his metaphysics of space. But they have seldom been discussed systematically, and never previously in their entirety and interrelation. The arguments against infinite divisibility collectively address one of the most difficult test cases for Hume's thesis of the experiential origin of ideas, and the two sets of proofs from these writings bear significantly on the question of whether Hume's thought underwent a major ideological or methodological shift in the transition between the early and later periods.

Hume's refutation of the infinite divisibility of space, and his doctrine of the sensible extensionless indivisibles that constitute extension, are perhaps the least loved, and until recently, least examined aspects of his philosophy. Hume's arguments are so abstruse, and his conclusions so extraordinary, so far removed from prevailing mathematical and scientific thinking, that his critique of infinity has often been ignored or summarily dismissed. The fact that Hume reduces his lengthy forty-two page treatment of the problem in the *Treatise* to just two pages of the first *Enquiry*, together with his disavowal of the first work in the 'Advertisement' to the *Enquiry*, has further encouraged the tendency to disregard his arguments.[10] It is

[10] Hume, *Essays and Treatises on Several Subjects*, Vol. II [1777], facsimile page for the 'Advertisement', reprinted in *Enquiries Concerning Human*

tempting to think that Hume in the later work finally sees insuperable defects in his previous views about the mathematics and metaphysics of space, and, accordingly, abandons them unceremoniously for more promising topics.

Among those who have seriously considered Hume's theory of extension, opinions of its merits are widely if not infinitely divided. To give a sense of the disparity, Donald L.M. Baxter, in "Hume on Infinite Divisibility", maintains: "Far from begging the question, Hume has available a respectable argument against the infinite divisibility of finite spatial intervals."[11] C.D. Broad, on the contrary, in his Dawes Hicks Lecture on Philosophy to the British Academy, "Hume's Doctrine of Space", concludes: "I think, then, that Hume's whole account of spatial divisibility can be fairly safely dismissed as rubbish."[12] And later: "... there seems to me to be nothing whatever in Hume's doctrine of Space except a great deal of ingenuity wasted in recommending and defending palpable nonsense."[13]

Understanding and Concerning the Principles of Morals, p. 2. See Franklin, "Achievements and Fallacies in Hume's Account of Infinite Divisibility", p. 86: "But in omitting his treatment of space and time almost entirely from the later *Enquiry*, Hume seems to admit tacitly that it was not a success with its intended audience."

[11] Baxter, "Hume on Infinite Divisibility", p. 140.

[12] Broad, "Hume's Doctrine of Space", p. 171. See Broiles, *The Moral Philosophy of David Hume*, p. 3: "Think of all the sections which are seldom discussed in Book I of the *Treatise.* Hardly anyone is familiar with Hume's work on space and time." Flew begins his essay "Infinite Divisibility in Hume's *Treatise*", with these words, and then adds, p. 257: "And furthermore, it might easily be argued that there is no very good reason why anyone should struggle to gain such familiarity — except, of course, simply in order better to understand Hume and to appreciate his weaknesses as well as his strength." On p. 269, he states: "One wishes Hume had thus been led to challenge the questionable fundamentals on which depend both the atomistic ontology which he only suggests and the bizarre first account of geometry which he presents so proudly."

[13] Broad, "Hume's Doctrine of Space", p. 176. I do not share Broad's dissatisfaction with Hume's theory, but I agree with his emphasis on the importance of phenomenological evidence in Hume's inkspot experiment, as against commentators who focus exclusively on the *reductiones ad absurdum.*

Although Hume's critique of infinity and theory of sensible extensionless indivisibles is not universally condemned, the tide of opinion has been overwhelmingly opposed to Hume's theory, even when the objections brought against it are based on misinterpretations.

Whether or not Hume's arguments against infinity are sound, they are crucial to understanding his philosophical system. Hume is the architect of a penetrating, historically important, and still vital approach to empiricism. But the problems raised by an epistemology limited to what can be received by or constituted out of sense experience go beyond the principles of Hume's particular brand of empiricism in rejecting the concept of infinity. The significance of Hume's opposition to infinity includes enduring problems in the metaphysics of space, time, and extension, and the philosophy of mathematics and scientific method. When a complete picture of his theory is given, Hume's critique of infinity is more interesting than his detractors and even the later Hume acknowledge.

Historical-Philosophical Context

The historical background for Hume's arguments against infinity begins with the ancient Greeks. The concept of infinity and infinite divisibility of extension predates Euclidean geometry. Infinity makes its first recorded philosophical appearance in the paradoxes of Zeno of Elea in defense of Parmenides' presocratic doctrine of the One. Zeno's disproofs of real motion and extension in the world of sensory experience depend essentially on the infinite divisibility of space and time.[14]

It is common to define infinity in this tradition as limitlessness, boundlessness, or endlessness. By this conception, there is always continuation beyond any chosen point in an infinite set, series, or expanse. Infinite divisibility obtains just in case there always exists another point between any two chosen points.

[14] Kline, *Mathematical Thought from Ancient to Modern Times*. Kretzman, ed., *Infinity and Continuity in Ancient and Medieval Thought*. Sorabji, *Time, Creation, and the Continuum: Theories in Antiquity and the Early Middle Ages*.

These accounts are intended both as definitions and aids to assist an overtaxed mathematical imagination. It is in this sense that Lucretius in *De Rerum Natura* repeats an Epicurean argument in favor of the infinity of space. He considers a thought experiment in which we suppose that the universe has an invisible limit, an end or boundary of space, against which a javelin is thrown. The javelin either penetrates or is blocked by the limit. If the javelin pierces through, then, contrary to the hypothesis, it cannot have encountered the real spatial limit of the universe, no matter where the limit is set. If the javelin is blocked, then it must be stopped by something beyond the invisible limit in space. In either case, there is something more, something beyond any postulated spatial limit to the universe. If the boundary is thought to retreat from the spear, then repeating the operation again and again for any proposed limit will show that there is no true limit, but always another space beyond.[15]

Do these definitions and thought experiments offer a reasonable way of thinking about infinity? Or do they at most capture the concept of indefinite, indeterminate, or inexhaustible but finite quantity and partition? The difficulty threatens when we do not just nod our heads in agreement with some of these venerable formulas as fully expressing the idea of the infinite, but stop and ask whether 'always' implies infinite time or something less. If time itself is infinite, then, true enough, there will always be another number, another corridor of space to receive Lucretius' javelin, another real number between any two real numbers on the number line, another point between two points on a geometrical line or expanse of space. If time is not infinite, however, then the claim that there is always something more or always something between, will not take us from the finite experience of these relations to full-blooded infinity.

The trouble is that these definitions give us no independent way to understand the concept of infinite time except by saying that there is always another moment of time, or that moments

[15] Lucretius, *De Rerum Natura*, Book I, lines 969-983.

of time can be mapped onto an infinite numerical series. If we do not stipulate or presuppose that time is infinite, *always* is not long enough to model the infinity or infinite divisibility of other sets, series, and relations. If we are struggling to understand the concept of infinity, it is unenlightening to be told that there is always more space, always another number or element of a sequence, always another number, point, or place between any two numbers, points, or places, if 'always' quantifies over what is supposed to be an infinite extent of time. If 'always' just means for every moment of time, whether finite or infinite in extent, then the concept is evidently too weak to define an infinite set, series, or relation. Finally, if 'always' is meant metaphorically for a more ethereal properly mathematical notion, then we should be able to do without time altogether in the definition of infinity, and cash in the temporal analogy for something more definite.

But what? A satisfactory explanation has yet to be proposed. Timeless definitions of infinity and infinite divisibility are obviously possible. But they are also uninformative. Consider this atemporal formulation of the concept of infinite divisibility:

> For any line segment (LS) x, x is *infinitely divisible* (ID) $=_{\mathrm{df}}$ for any two points on x, P_1^x and P_n^x, there exists on x a distinct point, P_i^x, between (B) P_1^x and P_n^x: $(\forall x)(\mathrm{LS}(x) \supset (\mathrm{ID}(x) \equiv (\forall P_1^x)(\forall P_n^x)(\exists P_i^x)\mathrm{B}(P_i^x, P_1^x, P_n^x)))$ $(1 \neq i, n;\ i \neq n)$

The definition tries to make universal generalization over a dense linear array of distinct points do the work of the temporal infinity or 'always' of traditional definitions. If a midpoint lies between any two arbitrary points, then between that midpoint and one of the original points there is yet another midpoint, and so on. *And so on*. . . to infinity?

That depends on whether or not in the first place we are quantifying over a domain of infinitely many points. If we are, well and good. But then we are presupposing infinity rather than defining it out of whole cloth. We are no better off than if we were to say that there is *always* a point between two points, meaning over infinite time or in the infinite mind of God. The dilemma is that a definition of infinity seems either to be viciously circular by virtue (or rather, by vice) of presupposing or explicitly stipulating a domain of infinitely many points, or else inevitably lacks adequate means to express an idea of infinite divisibility. At some finite stage of subdivision we will run out of distinct points, just as we eventually run out of time in the successive subdivision of a line or expanse of space, unless we assume infinite or infinitely divisible time. The definition works to explain infinite divisibility only if we need no explanation but already understand the concept, and only if we are prepared to quantify over a domain of infinitely many points. As a way of conveying the idea of infinity in the guise of divisibility into infinitely many points, the atemporal definition is no improvement over the blatant circularity of temporal '*always*' definitions that quantify over infinitely many moments.[16]

This is the problem of understanding the infinite, of whether there really is a *concept* of infinity. By itself, the above dilemma should already be enough to cast serious doubt on the philosophical respectability of the idea of infinity. For it seems to be a higher order rather than foundational concept for which we can only make provision in a conceptual scheme by assuming it as primitive. Some thinkers are untroubled by this issue, and are willing to develop and make use of elaborate mathematical symbolisms and philosophical theories about the infinite, while claiming to have an adequate intuitive grasp of infinity or maintaining that theory makes no such requirement. Their views have mostly been in the ascendency, and they

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[16] The same criticism applies to other atemporal definitions of infinite divisibility. See Moore, *The Infinite*, p. 42: "... let us exploit the infinitude of the natural numbers. Then 'This body is infinitely divisible' can be glossed as either: (1) For every natural number *n*, there is a possible situation *s*, such that this body is divisible into more than *n* parts in *s* [;] or (2) There is a possible situation *s*, such that for every natural number *n*, this body is divided into more than *n* parts in *s*." Here reliance on a prior understanding of the concept of infinity and availability of an infinite set or series to set in correspondence with the subdivisions of an extension is explicit.

throw up their hands in frustration or smile in condescension at the mathematical incompetence of skeptics who balk at the idea of infinity and regard infinitists as merely playing with empty words and meaningless mathematical symbols. Perhaps there is a kind of color blindness for the concept of infinity. There seems to be nothing those who claim to understand infinity can do to help share their insight with those who do not, and nothing those who claim not to understand infinity can say to shake the convinction of those who do.

It is in this spirit that John Locke maintains, in *An Essay Concerning Human Understanding*, Chapter XVII 'Of Infinity':

> But yet if after all this, there be men who persuade themselves that they have clear positive comprehensive ideas of infinity, it is fit they enjoy their privilege: and I should be very glad (with some others that I know, who acknowledge they have none such) to be better informed by their communication. For I have been hitherto apt to think that the great and inextricable difficulties which perpetually involve all discourses concerning infinity, — whether of space, duration, or divisibility, have been the certain marks of a defect in our ideas of infinity, and the disproportion the nature thereof has to the comprehension of our narrow capacities. For, whilst men talk and dispute of infinite space or duration ... it is no wonder if the incomprehensible nature of the thing they discourse of, or reason about, leads them into perplexities and contradictions, and their minds be overlaid by an object too large and mighty to be surveyed and managed by them.[17]

The conflict appears earlier in classical times, in disagreements about the interpretation of Zeno's paradoxes. As a sample of these puzzles, consider the paradox of Achilles and the Tortoise. The infinite divisibility of extension implies that no runner regardless of effort can catch up to or pass another who continues to move at any speed whatever after beginning with however slight a head start. The second runner faces an infinite succession of midpoints to be transversed along a

[17] Locke, *Essay*, pp. 222-223. Locke's thesis that there is no positive, but at most a negative idea of infinity is discussed in Part One, Chapter 2.

continuum between his starting place and the continuously advancing position of the first runner. When any midpoint is gained, infinitely more remain to be passed by the second runner before the first runner can be reached. The conclusion of the paradox, blatantly at odds with everyday experience, might be thought flatly to discredit the concept of infinite divisibility. But in Zeno's application, accepting the infinite divisibility of extension supports the Parmenidean distinction between the way of seeming and the way of being, by which phenomenal motion and spatial extension are judged unreal, and the empirical evidence of the senses is scorned as illusory.[18]

Aristotle discusses Zeno's paradoxes in the *Physics*. He mediates there as in other problems between an idealist-formalist desire to preserve a realm of potential infinities grasped by the intellect, while maintaining the nonoccurrence of actual infinite divisibility.[19] Aristotle is more of an empiricist than Zeno or Parmenides, or, for that matter, Plato. But his distinction between potential and actual infinity as a solution to Zeno's paradox does not question the intelligibility of the concept of potential infinity, only the application of the concept of actual infinity to the world of experience. If infinity is not found in nature, the concept itself is unchallenged by Aristotle. The unreality of infinite divisibility is enough to avoid Zeno's paradoxes, at least insofar as they pertain to motion, time, and extension in applied mathematics. Aristotle's discussion of the idea of potential infinity shares the material inadequacy or circularity of the Greek conception by which infinity numbers

[18] Salmon, ed., *Zeno's Paradoxes*. Grünbaum, *Modern Science and Zeno's Paradoxes*. Ferber, *Zenons Paradoxien der Bewegung und die Struktur von und Zeit*. Hasse, Scholz, and Zeuthen, *Zeno and the Discovery of Incommensurables in Greek Mathematics*. Sweeney, *Infinity in the Presocratics: A Bibliographical and Philosophical Study*. Owen, "Zeno and the Mathematicians". Jacquette, "A Dialogue on Zeno's Paradox of Achilles and the Tortoise".

[19] Aristotle, *Physics* 233^a23-263^a28. See Russell, "The Problem of Infinity Considered Historically", *Our Knowledge of the External World*, Lecture 6, pp. 182-198.

the steps or stages of an endlessly successive operation. Thus, Aristotle declares in *Physics* 206^a27-29 and 207^a6-8:

> For generally the infinite has this mode of existence: one thing is *always* being taken after another, and each thing that is taken is *always* finite, but *always* different.

> Thus, something is infinite if, taking it quantity by quantity, we can *always* take something outside.[20]

We are thereby introduced to the idea of the potentially infinite, but only if *always* presupposes the potentially infinite duration or succession of events in time. We are given no further help in understanding this idea, however, nor reason for thinking infinity so defined might exist even potentially. Aristotle's distinction between actual and potential infinity as a solution to Zeno's paradoxes held sway through the medieval and modern periods until recent times. While most mathematicians continued to speak without qualification of infinity and infinite divisibility, it was generally assumed that philosophical difficulties like Zeno's paradoxes could be avoided where necessary by invoking Aristotle's concept of potential infinity and denying the actual infinity or infinite divisibility of space and time.[21]

In the rise of rationalism that marked the intellectual climate of the seventeenth and early eighteenth centuries, infinity and infinite divisibility became the common currency of the idealized mechanics of physical phenomena in the mathematics and natural philosophy of Descartes, Newton, Leibniz, and their many followers.[22] Infinity was exemplified most notably in Leibniz's calculus of infinitesimals and Newton's the-

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[20] *The Complete Works of Aristotle* (Revised Oxford Edition), Barnes, ed., Vol. I (emphases added). Aristotle's word is 'αει', which can mean 'forever' as well as 'always'. I am indebted to Henry W. Johnstone, Jr. for etymological advice.

[21] An excellent critical examination of these topics is given by Benardete, *Infinity: An Essay in Metaphysics*, pp. 1-71. See also Moore, *The Infinite*, esp. pp. 17-44.

[22] Moore, *The Infinite*, pp. 57-95.

ory of fluxions. The calculus extended the formal techniques of Descartes's procedure for finding the tangents and 'normals' (line segments perpendicular at the tangent point) to a curve, and John Wallis's method of exhaustion by quadratures, in which the area bounded by a curve is determined by approximation to any desired degree of accuracy by fitting beneath the curve a succession of appropriately sized rectangles, the sum of whose respective areas is easy to calculate.[23] The theory of infinitesimals and fluxions presupposes but goes yet another step beyond Euclidean infinite divisibility. Leibniz's integral calculus was an improvement over Wallis's theory, in which sums of infinitesimals measure the bases of infinitely small isosceles triangles in analyzing the composition of the area enclosed by any curve. Newton's theory of fluxions accomplishes the same for a kinematic geometry, where fluxions are 'first ratios of nascent arguments' involving the coordinates of a point (fluent) in continuous motion dynamically tracing a curve in space over infinitely small moments of time.[24]

The infinite divisibility of space is presupposed by the theory of infinitesimals and fluxions. The Euclidean mathematical points into which continuous extension was thought to be infinitely divisible were seldom confused with infinitesimals.[25] The Newtonian John Keill, in his 1702 *Introductio ad verum physicam*, articulates the concept of infinite divisibility of extension into extended parts rather than points.[26] This was yet another more cogent form of infinite divisibility against which Hume reacted,

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[23] Wallis, *A Treatise of Algebra Both Historical and Practical.*

[24] Newton, *The Method of Fluxions and Infinite Series.* Newton's method is extensively used in *Sir Isaac Newton's Mathematical Principles of Natural Philosophy and his System of the World.* See Cajori, *A History of the Conceptions of Limits and Fluxions in Great Britain from Newton to Woodhouse.*

[25] An exception is Hayes, *A Treatise of Fluxions*, p. 1.

[26] For background, see, *inter alia*, Boyer, *The History of the Calculus and its Conceptual Development*; Baron, *The Origins of the Infinitesimal Calculus*; Hall, *Philosophers at War: The Quarrel Between Newton and Leibniz.* McGuire, "Space, Geometrical Objects and Infinity: Newton and Descartes on Extension", in Shea, ed., *Nature Mathematicized*, pp. 69-112.

occurring in a source with which Hume might have been familiar from his student days at the University of Edinburgh, especially after its translation into English in 1720 as *An Introduction to Natural Philosophy*. Keill writes:

> ... every magnitude is not compounded of points, but parts, that is, other magnitudes of the same kind, whereof every one is constituted of other parts, and each of these is still made up of others, and so on *in infinitum*.[27]

Hume's critique of the concept of infinity can be understood in part as an empiricist reaction to well-entrenched rationalist commitments to infinity in all three categories. Infinite number, infinite quantity or distance in space or time, infinite geometrical magnitude, infinite divisibility of extension, and infinitesimals or fluxions, in Hume's view, all partake of the same fatal philosophical error.

Infinitist theories for Hume contain mere words and symbols that may function with apparent theoretical and pragmatic success, but that fail to represent genuine ideas grounded in experience. Infinity is a mathematical idealization that extrapolates beyond human perception and imagination; it is not comprehended by finite thought, but is supposed to be more abstractly understood by an act of transcendent reason. Infinity and infinite divisibility thus provide a sharp focus for dispute about the metaphysics, epistemology and scientific methodology of Descartes, Leibniz, and Newton versus Locke, Berkeley, and Hume.

Hume's disavowal of infinity goes beyond Aristotle's distinction between actual and potential infinity. Hume's theory of knowledge, in which ideas derive only from immediate impressions of finitely experienced sensation or reflection, precludes even the possibility of an adequate idea of infinity or infinite divisibility, actual or potential. The impact of Hume's rejection of infinite divisibility on his philosophy as a whole is important and far-reaching. It has obvious and immediate implications for

[27] Keill, *Introduction to Natural Philosophy, or Philosophical Lectures*, Lecture 3, p. 21.

his philosophy of mathematics and metaphysics of space and time, but also for his aesthetic psychology of distance, greatness, and the sublime, and even for his religious skepticism. An infinitely knowing, infinitely powerful, and infinitely benevolent divine creator of the universe is as unintelligible for Hume as infinite number, an infinite expanse of space or time, or infinitely divisible extension. The critique of infinity in mathematics and metaphysics supports Hume's doubts about conventional religious belief, exemplified by Newton's striking image of infinite space as God's visual field.[28]

Bayle's Trilemma for the Divisiblity of Extension

To understand the context of philosophical controversy to which Hume's theory belongs, we now look to two of its most important historical precedents in the writings of Pierre Bayle and George Berkeley.

Bayle discusses infinite divisibility in the article on 'Zeno of Elea' in his *Dictionary Historical and Critical.*[29] Bayle's treatment

[28] Newton, *Opticks*, Book III, Query 28, p. 370: "Does it not appear from phenomena that there is a Being incorporeal, living, intelligent, omnipresent, who in infinite Space, as it were in his Sensory, sees the things themselves intimately, and thoroughly perceives them and comprehends them wholly by their immediate presence to himself..." See Hurlbutt, *Hume, Newton, and the Design Argument*, p. 10: "Infinite space is, 'as it were' God's visual field, and the things in this space are known by him in their complete inner nature and complexity."

[29] Kemp Smith, *Philosophy of David Hume*, pp. 284-290, 325-338. See Flew, "Infinite Divisibility in Hume's *Treatise*", pp. 257-269. Fogelin, "Hume and Berkeley on the Proofs of Infinite Divisibility", pp. 47-69. Bayle's trilemma is not entirely original, but recapitulates a medieval argument. See Thijssen, "David Hume and John Keill and the Structure of Continua", pp. 271-286. But Thijssen writes, p. 285: "Although Hume may have used Bayle as a source his discussion of the three possible views regarding continuity (i.e., divisibility *in infinitum*, composition out of mathematical points, and composition out of physical points) reflects a much older scholastic heritage. The same is true for almost all of the arguments that Hume (and Bayle) employ in their discussion of continuity." In his letter to Ramsay, Hume also recommends Bayle's entries on 'Zeno' and 'Spinoza' as background reading for understanding the *Treatise*. Thijssen, p. 285, refers to Maier, *Die Vorläufer*

includes objections to infinite divisibility that reappear among Hume's *reductio* disproofs. More importantly, Bayle rehearses a medieval problem about the conceivability and divisibility of extension which Hume attempts to solve by elaborating his theory of sensible extensionless indivisibles.[30]

Bayle poses a trilemma intended to expose the pretensions of reason, as a counterpoise to Descartes's efforts to establish indubitable foundations for knowledge.[31] The trilemma is that there are just three concepts of the divisibility of extension, each of which is impossible because of internal inconsistencies. Extension as essentially divisible paradoxically therefore does not exist, and human reason is accordingly chastened, ultimately in the service of ecclesiastical purposes. The problem or elements of the problem can be traced back through Antoine Arnauld's *Port Royal Logic*, a slate of medieval thinkers, and Book VI of Aristotle's *Physics*.[32] The three concepts considered in Bayle's trilemma are that extension is: (i) infinitely divisible; (ii) a finitely divisible fabric of extended physical points; or (iii) a finitely divisible system of extensionless ideal mathematical points.

The argument against the infinite divisibility of extension rests on several criticisms, including a proof later adopted by Hume as a *reductio* from the addition of infinite parts. The objection is that if a tiny object like a barley corn or the space it occupies is infinitely divisible, then it must contain

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Galileis im 14. Jahrhundert, pp. 159-161, as documenting the medieval origins of the trilemma in the continuum debate. Hume indicates his awareness of the trilemma's ancestry, when he writes in considering the argument in the *Treatise*, p. 40: 'It has often been maintain'd *in the schools*...'.

[30] Fogelin and Mijuskovic suggest that besides *The Port Royal Logic* an additional source for the infinity arguments may have been Barrow, *The Usefulness of Mathematical Learning Explained and Demonstrated*, p. 76, to which Hume refers in the *Treatise*, p. 46. This is the English translation Hume knew; see below note 41.

[31] Bayle, 'Zeno of Elea', *The Dictionary Historical and Critical of Mr. Peter Bayle*, p. 611.

[32] Ibid., p. 610.

infinitely many distinct extended parts, each of which in turn is infinitely divisible. The idea is absurd, because the juxtaposition or adding together of infinitely many extended parts must be infinitely extended. Bayle concludes:

> An infinite number of parts of extension, each of which is extended, and distinct from all others, as well with respect to it's entity, as with respect to the space it fills, cannot be contained in a space one hundred thousand millions of times less than the hundred thousandth part of a barley corn.[33]

The two remaining concepts of extension as a finite system of extended physical or extensionless mathematical points are similarly rejected. The (non-Humean) mathematical theory of infinite divisibility into (nonsensible) ideal or abstract extensionless indivisibles from a synthetic point of view, Bayle argues, cannot explain how extension is constituted. Extensionless mathematical points are just so many 'nothingnesses', according to Bayle, and as such they cannot possibly comprise extension in the aggregate. Extended physical points, on the other hand, cannot be the ultimate constituents of extension, since if extended they are divisible into right and left halves, and as such are divisible rather than indivisible.

> A few words shall suffice as to Mathematical points; for ... several nothingnesses of extension joined together will never make an extension... Nor is it less impossible or inconceivable that it should be composed of the Epicurean atoms, that is, of extended and indivisible corpuscles; for every extension, how small soever, hath a right and left side ... a body cannot be in two places at once; and consequently every extension which fills several parts of space contains several bodies.[34]

Bayle maintains that none of these alternatives is a real possibility, but that each leads to contradiction. He confirms what he takes to be the purpose of the skeptical attack on reason implicit in Zeno's arguments against the existence

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[33] Ibid., p. 614.
[34] Ibid.

of extension. "Nay, I am persuaded," he writes, "that the proposing of these arguments may be of great use with respect to religion. . ."[35]

Hume agrees in condemning all three conceptions. He advances his own doctrine of sensible extensionless indivisibles as a fourth alternative that Bayle does not consider, but that Hume believes escapes all three prongs of Bayle's trilemma. The proof of the trilemma contains the inspiration for several of the arguments Hume deploys against the logical coherence of the idea of infinite divisibility. More importantly, the trilemma motivates his theory by providing a problem to be solved by a methodologically satisfactory metaphysics of space.

Hume rejects the first possibility in Bayle's trilemma by all six *Treatise* arguments, the inkspot argument together with the four *reductio* proofs and geometry dilemma. The *reductio* arguments reinforce Bayle's rejection of the first horn of the trilemma. Hume approves Bayle's objections to the second horn. He claims that extension can never exist without parts, so that the concept of a physical point must combine the incompatible features of being physical and hence divisible with being an indivisible point. The third conception goes by the board because no assemblage of (nonsensible) extensionless ideal or abstract mathematical points ('nothingnesses') can constitute real extension. Hume argues:

> It has often been maintain'd in the schools, that extension must be divisible, *in infinitum*, because the system of mathematical points is absurd; and that system is absurd, because a mathematical point is a non-entity, and consequently can never by its conjunction with others form a real existence. . . The system of *physical* points . . . is too absurd to need a refutation. A real extension, such as a physical point is suppos'd to be, can never exist without parts, different from each other; and wherever objects are different, they are distinguishable and separable by the imagination.[36]

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[35] Ibid.

[36] *Treatise*, p. 40.

After discarding all three conceptions, Hume does not embrace the skeptical religious conclusion dangled by Bayle, but instead advances an unanticipated fourth choice. If successful, Hume's theory of extensionless colored and tactile indivisibles as the constituents of extension avoids all three prongs of Bayle's trilemma.

First, Hume rejects infinite divisibility partly for the same reasons Bayle gives. Hume's theory of space as constituted by extensionless indivisibles, moreover, implies that extension is only finitely divisible. Second, Hume sees no absurdity in the concept of his indivisibles as in the idea of an extended 'Epicurean' physical point. The reason is that Hume's indivisibles, despite being colored and tactile, are extensionless. Hume's indivisibles, by virtue of being extensionless, are not divisible into left- and right-hand parts, and as such are not subject to the contradictions Bayle refutes in the second prong of his trilemma. Nor, third and finally, are Hume's sensible extensionless indivisibles subject to the limitations of ideal Euclidean mathematical points, which Hume agrees with Bayle in dismissing as fictions. Hume's extensionless indivisibles, by virtue of their sensible properties, can be described as constituting extension in combination. The difference is that, although Hume's least atomic units of extension, like ideal Euclidean points (not segments), are extensionless and indivisible, they are also not abstract, but sensible, and although individually extensionless, constitute extension in finite aggregates. Hume explains:

> This wou'd be perfectly decisive, were there no medium betwixt the infinite divisibility of matter, and the non-entity of mathematical points. But there is evidently a medium, *viz.* the bestowing a colour or solidity on these points; and the absurdity of both the extremes is a demonstration of the truth and reality of this medium.[37]

Far from acceding to Bayle's skepticism about reason's inability to understand divisibility in the metaphysics of extension, Hume reverses the effect of the trilemma by using it to show

[37] Ibid.

that there can be no adequate alternative to his positive theory of sensible extensionless indivisibles. The force of the trilemma is thereby rechanneled into proving, not that reason is out of its depths in trying to grasp the subtle principles of the divisibility of extension, but that reason is compelled to accept the constitution of finite extension by and its finite divisibility into sensible extensionless indivisibles as the only possible solution to Bayle's problem.

Bayle dismisses the possibility that extension may be constituted by extensionless indivisibles, which he refers to as 'Mathematical points'. Bayle's argument in the third prong of the trilemma appears at first to contradict Hume's theory of extensionless indivisibles as well. There is an important difference between what Bayle calls these 'nothingnesses of extension', no number of which 'joined together will [ever] make an extension', and Hume's doctrine of sensible extensionless indivisibles.

Hume largely agrees with Bayle's criticisms of infinite divisibility. Bayle, satisfied with a skeptical conclusion about the limitations of human versus divine reason, concludes that the trilemma demonstrates human imprudence in trying to penetrate the mysteries of the universe. He continues with a long quotation of approval explaining what are supposed to be the religious implications of Zeno's paradox concerning the divisibility of matter, which he also signals by the marginal note, 'What use ought to be made of the foregoing dispute':

> ... and I say here with regard to the difficulties of motion, what Mr. Nicolle said of those of the divisibility *in infinitum*. 'The advantage which may be drawn from these speculations is not meerly [*sic*] to acquire this sort of knowledge ... but to learn to know the limits of our understanding, and to force it however unwilling to own that some things exist, though it is not capable of comprehending them: for which reason it is proper to fatigue the intellect with these subtilties, in order to subdue its presumption, and deprive it of the assurance of ever opposing its faint light to the truths which the church proposes, under pretext that it cannot comprehend them: for since all the force of human understanding cannot comprehend

> the smallest atom of matter, and is obliged to own that it clearly sees that such an atom is infinitely divisible, without being able to conceive how that can be: is it not plain that the man acts against reason who refuses to believe the wonderful effects of God's omnipotence, which is of it self incomprehensible, because our minds cannot comprehend these effects...'[38]

Hume, no less than his rationalist opponents, sees himself as a champion of reason, properly understood, and is by no means receptive to Bayle's religious cabals debasing reason. Bayle's trilemma thus confronts Hume's ingenuity primarily as a challenge to be overcome. The pretensions of reason must be delimited, and its nature understood, particularly in the operations of which it is capable and incapable, and its subordination to the passions. But not every conceptual puzzle uncovers a true defect of reason, and philosophy must proceed with caution here as elsewhere in charting the mind's limitations. Hume believes that Bayle's trilemma is avoidable, and in the *Treatise* and again in the first *Enquiry*, he advances the theory of sensible extensionless indivisibles as a triumph of reason over self-defeating 'Pyrrhonian' skepticism that tries to turn reason against itself.[39]

Legacy and Influence of Berkeley on Hume's Metaphysics of Space and Philosophy of Mathematics

After Bayle, the second most important influence on Hume's critique of infinity is Berkeley. The two-fold task of Hume's critique of infinity is fully anticipated by Berkeley's philosophy of mathematics. Berkeley offers an empiricist-conceptualist argument against the possibility of infinity, infinite divisibility, and infinitesimals and fluxions in the calculus. His all-purpose refutation of the infinite, like Hume's, is reinforced by *reductio* objections to the concepts of infinitesimals and fluxions. Finally, but equally significantly, Berkeley like Hume defends a theory of *minima sensibilia*. Although a version of Berkeley's idea of

[38] Bayle, 'Zeno of Elea', p. 614.

[39] *Enquiry*, pp. 149-158.

minima sensibilia appears already in Aristotle and other classical authors, it is the more likely immediate source for Hume's positive doctrine of sensible extensionless indivisibles.[40]

Infinite quantity and relation are not experienced in nature, but are supposed by their adherents to be abstract general ideas. Berkeley rejects these as unthinkable. He holds that only ideas of possibly existent objects can be conceived, and notes that abstract general ideas are ideas of things that cannot possibly exist. He reinforces this conclusion with a phenomenological challenge for those who claim to have such ideas to imagine or hold before the mind an idea of a triangle that is neither isosceles, scalene, nor equilateral, neither right-angled nor not-right-angled, with lines that are neither red nor blue nor white nor any other particular color. Since no objects so incomplete and inconsistent in their properties can possibly exist, the abstract general idea of any such object is inconceivable. If there are no abstract general ideas, then there are no abstract general ideas of infinity, infinite divisibility, infinitesimals, or fluxions. In *A Treatise Concerning the Principles of Human Knowledge*, Berkeley states:

> If any man has the faculty of framing in his mind such an idea of a triangle as is here described, it is in vain to pretend to dispute him out of it, nor would I go about it. All I desire is, that the reader would fully and certainly inform himself whether he has such an idea or no. And this, methinks, can be no hard task for any one to perform. What more easy than for any one to look a little into his own thoughts, and there try whether he has, or can attain to have, an idea that shall correspond with the description that is here given of the general idea of a triangle, which is, *neither oblique, nor rectangle, equilateral, equicrural, nor scalenon, but all and none of these at once?*[41]

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[40] An historical-philosophical discussion of indivisibles in the metaphysics of space in Hume's predecessors and successors without explicit reference to Hume is given by Zimmerman, "Could Extended Objects Be Made Out of Simple Parts?" and "Indivisible Parts and Extended Objects: Some Philosophical Episodes from Topology's History".

[41] Berkeley, *Treatise, Works*, Vol II, p. 33.

Berkeley's refutation of abstract general ideas discounts the dominant abstractionist philosophy of mathematics originating with Aristotle and transmitted through the Scholastic period to the seventeenth and eighteenth centuries in such works as Isaac Barrow's *Lectiones Mathematicae* and Wallis's *Mathesis Universalis.* Mathematical entities in this tradition are conceived as abstracted from experience in a kind of imaginative thinking away of certain properties and focusing on those that remain.[42] The simplest relevant case is that in which we regard extension as the abstraction of breadth and depth, say, from the idea of a road, leaving only its distance or length. Abstraction in mental operations of this kind are supposed in principle to explain the ontology and conceptual basis of all mathematical objects. But if Berkeley's arguments against abstract general ideas are correct, then abstractionism fails as a cogent theory of mathematical entities. The application to abstract general ideas of the infinite divisibility of extension is offered by Berkeley in these terms in the *Principles*:

> Every particular finite extension which may possibly be the object of our thought is an *idea* existing only in the mind, and consequently each part thereof must be perceived. If, therefore, I cannot perceive innumerable parts in any finite extension that I consider, it is certain they are not contained in it.[43]

Berkeley soon arrives at an alternative conception of general ideas that has come to be known as the theory of representative generality. The model is one in which the mind entertains only particular ideas that by proxy represent others belonging to the same category. If in reasoning a conclusion is drawn that pertains only to the particular representative chosen and not generally to all others of its type, Berkeley imagines that counterexamples from within the category will rush in to declare

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[42] Barrow, *Lectiones Mathematicae XXIII; in quibus principia matheseôs generalia exponuntur*, pp. 29-33. Wallis, *Johannis Wallis S.T.D... Opera Mathematica*, Vol. 1, p. 21.

[43] Berkeley, *Principles*, p. 98.

themselves exceptions to the inference.[44] Prior to discovering the theory of representative generality, in his private notebooks later published as *Philosophical Commentaries*, Berkeley briefly considers the prospects of a phenomenal atomistic mathematics of extension based on *minima sensibilia*, which would have overturned virtually every theorem of classical Euclidean infinitary geometry. Having found a suitable substitute for abstract general ideas in the theory of representative generality, Berkeley in the *Principles* and thereafter claims to be able to accommodate the theorems of traditional geometry, despite rejecting the concept of infinite divisibility.[45] He explains:

> To make this plain by an example, suppose a geometrician is demonstrating the method, of cutting a line in two equal parts. He draws, for instance, a black line of an inch in length, this which in it itself is a particular line is nevertheless with regard to its signification general, since as it is there used, it represents all particular lines whatsoever; for that what is demonstrated of it, is demonstrated of all lines, or, in other words, of a line in general.[46]

The suggestion is reminiscent of Aristotle's distinction between actual and potential infinity. Berkeley holds that geometry requires only unlimited rather than infinite divisibility, an idea available to the mind through representative generality. We can imagine a succession of particular lines, each representing

[44] Ibid., pp. 29-40; 45. Berkeley, *Three Dialogues*, *Works*, Vol. II, pp. 192-194. Peter Browne, one of Berkeley's professors at Trinity College, Dublin, has been identified as a likely inspiration for Berkeley's refutation of abstract general ideas. Browne, *The Procedure, Extent, and Limits of Human Understanding*, esp. pp. 186-187. See Atherton, "Berkeley's Anti-Abstractionism".

[45] Jesseph, *Berkeley's Philosophy of Mathematics*, pp. 69-78. See p. 73: "On Berkeley's analysis, then, the thesis of infinite divisibility must be read as the claim that every geometric magnitude actually contains an infinite number of parts. But this claim, he thinks, can be rejected without requiring a full-scale overhaul of traditional geometry, because lines in geometric proofs serve as representatives of other, larger lines." I am greatly indebted throughout this section to Jesseph's excellent study.

[46] Berkeley, *Principles*, p. 32.

half the extension of the previous one, and each visually or imaginatively experienced as divided in two. No single line is infinitely divided in thought, but the representative generality of an unlimited succession of divided lines beyond the limits of immediate perception or imagination is sufficient for classical geometry.

As Douglas M. Jesseph argues in *Berkeley's Philosophy of Mathematics*, Berkeley embraces a complicated three-part ontology of the objects of three distinct kinds of traditional mathematics.[47] Geometry, as a formal descriptive theory of sensible magnitude, requires definite ideas originating in the perception of spatial extension with representatively general application to all similar figures. Arithmetic and algebra, on the contrary, Berkeley treats nominalistically as formal symbol games, while his theory of applied mathematics in physical science is instrumentalist. Berkeley makes peace with classical geometry by way of his theory of representative generality. But he is unable to extend the same courtesy to infinitesimals and fluxions because of what he perceives as conceptual inconsistencies. Nor is he willing to accept the calculus merely as a useful formal game. He cannot overlook its apparent paradoxes on overriding pragmatic grounds, as he does in the case of imaginary numbers in algebra, such as the square root of -1. The difference is that Berkeley regards the calculus as a geometrical theory, and, as such, a descriptive mathematical theory based on ideas and concepts originating in sense perception.[48]

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[47] Jesseph, *Berkeley's Philosophy of Mathematics*, pp. 222-226. Jesseph regards Berkeley as the first formalist, anticipating David Hilbert's characterization of mathematics as formal token manipulation.

[48] Jesseph, *Berkeley's Philosophy of Mathematics*, p. 116: "This is further evidence of the important distinction between geometry and arithmetic in Berkeley's philosophy of mathematics. For Berkeley, the calculus is fundamentally a geometric theory, whose proper object is perceivable extension. Thus, the key terms in the calculus must be interpretable in terms of perceivable extension, i.e., we must be able to frame ideas corresponding to these terms." See also pp. 219-223.

Berkeley, therefore, in addition to his blanket objections to abstract general ideas of infinitesimals, finding no acceptable alternative conception in terms of representative general ideas, raises pointed internal criticisms against Leibniz's infinitesimals and Newton's fluxions in his books *The Analyst; or, A Discourse Addressed to an Infidel Mathematician*, and *A Defence of Free-Thinking in Mathematics*.[49] Berkeley's criticism of Newton's theory is that fluxions presuppose infinite divisibility in nascent ratios measuring a fluent's movement at instantaneous speeds. Leibniz's infinitesimals in turn are unacceptable because they entail the contradictory properties of being greater than zero but less than any finite quantity, by which they seem both to obey and not to obey the ordinary laws of arithmetic. Infinitesimals must be nonzero in quantity when added together in calculating nonzero curve lengths, tangents to curves, and areas swept by curves, but they must be of negligible, in effect zero, quantity, when added to or multiplied by any finite magnitude. Ivor Grattan-Guinness writes, in "Berkeley's Criticism of the Calculus as a Study in the Theory of Limits":

> The use of infinitesimals in the early decades of the calculus seems to have been promoted to a large extent by this kind of problem [demonstrating the value of the derivative when the limit of *hx* goes to 0]. Since they obeyed the law of addition
>
> $$a + h = a$$
>
> to ordinary numbers a, they were so small as to allow the limiting valued effectively to be achieved; on the other hand, being non-zero, they avoided the difficulty of 0/0. But such a view is obviously inconsistent, and it led to the mathematical inconsistencies from which the foundations of the calculus was then suffering.[50]

[49] Berkeley, *The Analyst*, *Works*, Vol. IV, pp. 55-102; 103-156. Berkeley appears to endorse infinite divisibility in his early public lecture, "Of Infinities", ibid., pp. 233-238. See Meyer, *Humes und Berkeleys Philosophie der Mathematik, vergleichend und kritisch dargestellt.* Also Jesseph, pp. 185-186.

[50] Grattan-Guinness, "Berkeley's Criticism of the Calculus as a Study in the Theory of Limits", p. 219.

Without disputing the truth of its theorems, Berkeley explains the apparent success of the calculus despite the incoherence of infinitesimals by identifying two compensating errors required by infinitesimals and fluxions at two different stages of the theory's deductions.[51]

Many historians of mathematics agree that Berkeley's criticisms of the calculus were for their time a well-founded indictment of significant difficulties in the concepts of infinitesimals and fluxions. Later mathematics has redeemed a refined version of the calculus by reinterpreting its methods in terms of the modern theory of limits, with values approaching zero as a limit, while infinitesimals have been resurrected by set theoretical devices in so-called nonstandard analysis. But these efforts only confirm the validity of Berkeley's attack on the conceptual foundations of the original systems, in criticisms that were taken very seriously by Berkeley's contemporaries.[52]

Hume like Berkeley never accepted the idea of infinity or infinite divisibility. But it is surprising that Hume in his critique of infinity nowhere discusses the problem of infinitesimals or fluxions. Even though Hume shares Berkeley's skepticism about infinity and infinite divisibility, he does not continue Berkeley's battle against the calculus or include infinitesimals along with the Euclidean points of infinitely divisible extension as among the disputed concepts of infinity. In the absence of definitive historical documentation to explain Hume's omission, I shall venture two hypotheses, one biographical and the other philosophical.

The first suggestion raises the question of Hume's real interest and competence in mathematics. Little is known of Hume's mathematical training. He left no original proofs or

[51] Berkeley, *The Analyst*, pp. 78: "Now I observe in the first place, that the Conclusion comes out right, not because the rejected Square of *dy* was infinitely small; but because this error was compensated by another contrary and equal error." See Jesseph, *Berkeley's Philosophy of Mathematics*, pp. 199-215.

[52] Jesseph, *Berkeley's Philosophy of Mathematics*, pp. 226-230. Grattan-Guinness, "Berkeley's Criticism of the Calculus as a Study in the Theory of Limits", p. 227.

related formal results, and we do not ordinarily think of Hume as having exceptional mathematical skills or devoting as much time to investigations in geometry or analysis as an adept well-versed amateur like Berkeley. Ernest Campbell Mossner in *The Life of David Hume* offers these remarks about Hume's education at Edinburgh:

> In addition to the required Arts course of Greek, logic and metaphysics, and natural philosophy, it may be conjectured, in the absence of any evidence, that David Hume elected, at the least, the classes in ethics and mathematics. The subject-matter of these classes at the time is unknown but may be reconstructed from the evidence of the period immediately following.[53]

Mossner speculates that Hume may have studied physics and mathematics with James Gregory or Colin Maclaurin.[54] These mathematicians were two of the most prominent Newtonians of the time, and either would have presented Newton's theory sympathetically. Maclaurin in particular was the author of *A Treatise of Fluxions, in Two Books*, published in 1742, shortly after the appearance of Hume's *Treatise* in 1739-1740. Maclaurin's theory is regarded by Jesseph and others as offering the most satisfactory solution to Berkeley's demand for a revision of the calculus to avoid Leibniz's and Newton's errors, in a return to more classical standards of mathematical rigor.[55] If Hume as Gregory's or especially Maclaurin's pupil in these subjects had a preview that Berkeley's most cogent technical objections to infinitesimals and fluxions were about to be answered, he may have decided the problem was not worth pursuing except at a higher philosophical plane.

This brings us to the second hypothesis. Notwithstanding the fate of Berkeley's internal criticisms of the calculus, Hume may have recognized that the concept of infinity could be

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[53] Mossner, *The Life of David Hume*, p. 41.

[54] Ibid., pp. 42-43. A detailed and well-documented study of Hume's education in mathematics and physical science is found in Barfoot.

[55] Maclaurin, *A Treatise of Fluxions, in Two Books*, Book I, pp. 325-363; Book II, pp. 403-415.

challenged from the standpoint of a properly humanized empiricist epistemology and philosophy of mind. If there are no ideas or adequate ideas of infinity or infinite divisibility, then there can be no ideas or adequate ideas of infinitesimals or fluxions. The more powerful criticism in Hume's opinion might then be said to strike at the heart of confusion by disallowing any form or variation of infinite quantity, magnitude, or relation. The refutation of infinitesimals and fluxions follows immediately then as a matter of course without entering into controversial and distracting technical details. I suggest that Hume, possibly with inside knowledge of the reply to Berkeley's *Analyst,* about to be issued by one of his own teachers, disappointed with Berkeley's bargain with classical geometry, and willing to proceed more radically to the source of error in the rationalist conception of superhuman abstract ideas, realized that in a single stroke he could disallow any implications of the idea of infinity and infinite divisibility, including Newton's fluxions and Leibniz's infinitesimals.[56]

When we consider Berkeley's influence on Hume's positive doctrine of sensible extensionless indivisibles, it is intriguing to find that in the *Philosophical Commentaries* Berkeley first considered developing a mathematics of *minima sensibilia* as an alternative to infinitist Euclidean geometry. Berkeley apparently believed that such a theory would be less complicated than classical geometry. In the *Commentaries,* he writes: "If ... we can make the Mathematiques much more easie & much more

[56] Hume in the *Treatise* critique of infinity does not appeal to Berkeley's refutation of abstract general ideas, as he does later in the *Enquiry.* The only point in the *Treatise* where Berkeley's theory comes indirectly into play is what is referred to below as Hume's fourth *reductio* argument from the conceivability of mathematical points. See Part Two, Chapters 4 and 7. There need be no deeper reason for Hume's reluctance to lean heavily on Berkeley's disproof of abstract general ideas than the possibility that in the early work the further significance of Berkeley's arguments simply did not occur to Hume, or that he may have wanted to offer new and independent arguments of his own.

accurate, w^t can be objected to us?"[57] A discrete geometry based on finite juxtapositions of *minima sensibilia* in place of an infinitary geometry based on continua would willingly sacrifice classical results in application for the sake of simplicity in conception and avoidance of intuitive paradoxes. But there would be interesting compensations. Berkeley touts the fact that his projected discrete geometry would prohibit incommensurable magnitudes or surds, such as the square root of 2 in the ratio of the diagonal of a square to any of its unit sides.[58] This further contradicts the Pythagorean Theorem, and, indeed, the implications of Berkeley's early proposal cascade through classical geometry from numerous fundamental but innocent-appearing differences. It may therefore have come as somewhat of a relief to Berkeley to discern in his theory of representative generality a way of reconciling potential or finite but unlimited divisibility in classical geometry with the concept of geometry as the descriptive mathematics of space whose ideas derive from the experience of sensible extension.

Although Hume would not have known Berkeley's *Commentaries*, he could hardly have overlooked Berkeley's accommodation of classical geometry in the *Principles*. Moreover, Berkeley's *An Essay Towards a New Theory of Vision*, written roughly at the same time as the *Commentaries*, contains a ready source for Hume's reflections on sensible extensionless indivisibles in a discussion devoted both to an account of *minima sensibilia* and an argument against infinite divisibility.[59] It is almost as though Hume read between the lines of Berkeley's *New Theory* to the *Commentaries* application of *minima sensibilia* as a basis for the geometry of spatial extension. From this standpoint, Hume boldly and more consistently carries forward Berke-

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[57] Berkeley, *Philosophical Commentaries*, Notebook A, *Works*, Vol. I, Comment 414, p. 52.

[58] Ibid., p. 58 (entry 469): "I say there are no incommensurables, no surds, I say the side of any square may be assign'd in numbers."

[59] Berkeley, *New Theory of Vision*, *Works*, Vol. I, p. 191. See below, Part One, note 14.

ley's program for the philosophy of mathematics and metaphysics of space, where Berkeley by the time of the *Principles* had already changed his mind about the need for a discrete geometry. If Hume understood Berkeley's reconciliation with classical geometry as unworkable, or as a betrayal of the empiricist demand for theory to be grounded in experience, it is easy to see how Hume might have regarded his doctrine of sensible extensionless indivisibles as an improvement more faithful to the guiding principles of Berkeley's philosophy.

The dispute over the intelligibility of infinity divides the history of philosophy and mathematics from the earliest papyri to the present day. Hume's quarrel with infinite divisibility falls roughly at the midpoint between ancient and contemporary debates. It reflects rationalist-empiricist antagonisms as they have played themselves out in the philosophy of mathematics and science from the beginning, and anticipates an ongoing controversy that continues to thrive. To study Hume's critique of infinity is therefore to stand at a vantage point from which to survey the entire history of the concept, looking back to the Greek origins of the concept and forward to the most recent developments in Cauchy's and Weierstrass's theory of limits, Abraham Robinson's nonstandard analysis, and Cantor's set theoretical hierarchy of transfinite cardinals. Hume does not believe that there can be an adequate idea of infinity or infinite divisibility, because the possibility is excluded by a correct theory of adequate ideas, and because putative ideas about infinity on close inspection are embroiled in contradiction. Hume's foundational problems leave no room for accepting even a false theory of infinity. Where there are no adequate ideas there can be no theory whatsoever, true or false, but at most a vain pretense at theorizing involving a meaningless string of words. The only recourse Hume sees is to give up the literal sense of the infinitist vocabulary, and if possible to reinterpret the language and mathematical symbolisms of infinity in strict finitist terms.

The details of precisely how Hume tries to effect this extraordinary upheaval in the foundations of mathematics and the metaphysics of space and time is the story to be unfolded in this book.

PART ONE

THE INKSPOT EXPERIMENT

Axiom. No reasoning about things whereof we have no idea. Therefore no reasoning about Infinitesimals.

— George Berkeley, *Philosophical Commentaries*
Notebook B §354

And to cut short all disputes, the very idea of extension is copy'd from nothing but an impression, and consequently must perfectly agree to it.

— Hume, *Treatise*, Book I, Part IV, Section V

CHAPTER 1

MINIMA SENSIBILIA

A Spot of Ink on Paper

The inkspot experiment is the basis for Hume's central argument against the infinite divisibility of extension in the *Treatise*, and for his positive doctrine of sensible extensionless indivisibles as the irreducible constituents of extension. The experiment involves the examination of a distant inkspot on paper.

> Put a spot of ink upon paper, fix your eye upon that spot, and retire to such a distance, that at last you lose sight of it; 'tis plain, that the moment before it vanish'd the image or impression was perfectly indivisible. 'Tis not for want of rays of light striking on our eyes, that the minute parts of distant bodies convey not any sensible impression; but because they are remov'd beyond that distance, at which their impressions were reduc'd to a *minimum*, and were incapable of any farther diminution.[1]

What we are supposed to conclude from the inkspot experiment is that there are size limitations for visual sense impressions. After a certain distance, a tiny object in the visual field vanishes, as we can verify experimentally. At the threshold beyond which the spot can no longer be seen, the inkspot at that distance for a particular viewer is an indivisible constituent of the visual impression of spatial extension. This is the empirical evidence for Hume's argument to establish the impossibility of infinite divisibility of finitely extended bodies, and for the existence

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[1] *Treatise*, pp. 29-30.

of sensible extensionless indivisibles or *minima sensibilia* as their fundamental components.

Limitations of Impressions and Ideas

To appreciate the force of Hume's argument, we must first understand what has often been called the copy principle for Hume's account of the origin of ideas. In disproving the infinite divisibility of extension, Hume trades heavily on a thesis presented in *Treatise*, Book I, Part I, Section I, *Of the Origin of our Ideas*, "*That all our simple ideas in their first appearance are deriv'd from simple impressions, which are correspondent to them, and which they exactly represent.*"[2] Ideas for Hume are 'faint images' of impressions of sensation or reflection, so that sense impressions are the ultimate experiential source of every idea. Hume concludes that the limitations of sense impressions of finite and finitely divisible spatial extension are also limitations of whatever ideas are mental copies of impressions, and finally of whatever ideas the mind can produce by modifying and rearranging other ideas. If Hume is correct, then there is no method by which ideas can compensate for the limitations, defects or deficiencies of the originating sense impressions of which they are only copies.

The copy principle serves as a basis for discrediting as inadequate or nonexistent ideas whose experiential origins cannot be confirmed, or for which there is a definite reason to doubt. Hume applies the criterion to the traditional metaphysical topics of space and time at the beginning of the following Section III, *Of the other qualities of our ideas of space and time*, when he declares that:

> No discovery cou'd have been made more happily for deciding all controversies concerning ideas, than that above-mentioned, that impressions always take the precedency of them, and that every idea, with which the imagination is furnish'd, first makes its appearance in a correspondent impression. These latter perceptions are all so clear and evident, that they admit of

[2] Ibid., p. 4.

> no controversy; tho' many of our ideas are so obscure, that 'tis almost impossible even for the mind, which forms them, to tell exactly their nature and composition. Let us apply this principle, in order to discover farther the nature of our ideas of space and time.[3]

As the inkspot experiment is supposed to show, finite minds cannot have sense impressions of infinitely divisible spatial extensions. There is a minimal smallness for the spatially extended objects of sense perception, which consequently are not infinitely divisible. Hume argues that as a result we cannot derive an idea of infinitely divisible extension from finitely divisible sense impressions. He further concludes that from the limitations of our ideas of the divisibility of extension into its 'most minute parts' we are entitled to conclude that extension in reality and not just in thought is subject to the same limitations as our adequate ideas of extension as at most finitely divisible. He claims that:

> ... our ideas are adequate representations of the most minute parts of extension; and thro' whatever divisions and subdivisions we may suppose these parts to be arrived at, they can never become inferior to some ideas, which we form. The plain consequence is, that whatever *appears* impossible and contradictory upon the comparison of these ideas, must be *really* impossible and contradictory, without any farther excuse or evasion.[4]

[3] Ibid., p. 33.

[4] Ibid., p. 29. Critics have noticed the fact that Hume in this passage seems to violate his own prohibition against metaphysical speculation about the existence or nature of body, or of real external entities beyond the mind's phenomenal impressions and ideas. Hume makes similar remarks quoted below, after his lengthly criticism of the concept of a vacuum, ibid., pp. 63-64. See Yolton, *Perceptual Acquaintance from Descartes to Reid*, Chapter VIII, 'Hume on Single and Double Existence', pp. 147-164. Baier, *A Progress of Sentiments: Reflections on Hume's Treatise*, Chapter 5, 'The Simple Supposition of Continued Existence', pp. 101-128. A controversy concerning the interpretation of Hume's realism versus Pyrrhonic skepticism about perception of the external world can be tracked down in the many references to some of the voluminous secondary literature on this topic

Hume thinks that a demonstration of the claim that our ideas are at most only finitely divisible is strictly unnecessary. "'Tis universally allow'd," he says, "that the capacity of the mind is limited, and can never attain a full and adequate conception of infinity: And tho' it were not allow'd, 'twou'd be sufficiently evident from the plainest observation and experience."[5] Hume then offers introspective evidence for the claim that imagination can at most successively divide an idea of spatial extension into finitely many parts. He invites us to consider the mental division and subdivision of a single grain of sand:

> ... the imagination reaches a *minimum*, and may raise up to itself an idea, of which it cannot conceive any sub-division, and which cannot be diminished without a total annihilation. When you tell me of the thousandth and ten thousandth part of a grain of sand, I have a distinct idea of these numbers and of their different proportions; but the images, which I form in my mind to represent the things themselves, are nothing different from each other, nor inferior to that image, by which I represent the grain of sand itself, which is suppos'd so vastly to exceed them. What consists of parts is distinguishable into them, and what is distinguishable is separable. But whatever we may imagine of the thing, the idea of a grain of sand is not distinguishable, nor separable into twenty, much less into a thousand, ten thousand, or an infinite number of different ideas.[6]

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in Wilson, "Is Hume a Sceptic with Regard to the Senses?". Livingston, "A Sellarsian Hume?". Wilson, "Hume's Critical Realism: A Reply to Livingston". An important earlier work in understanding this exchange is Price, *Hume's Theory of the External World*. Also, Popkin, "David Hume: His Pyrrhonism and his Critique of Pyrrhonism".

[5] *Treatise*, p. 26.

[6] Ibid., p. 27. See McNabb, ed., 'Afterword', David Hume, *A Treatise of Human Nature*), p. 367: "This proposition is true, not because, as Hume thinks, the number of parts into which the idea can be divided is less than twenty, but because it does not make sense to talk of dividing ideas into parts at all. This mistake vitiates the whole section. Hume's admission that we may be able to 'imagine' the grain of sand divided into tiny indivisible parts, suggests that something is wrong with the doctrine of impressions and ideas" (quoted in Franklin, "Achievements and Fallacies in Hume's Account

The imagination has limited ability to subdivide a spatially extended body, and no clear image or idea of an object once a certain point of division is passed. To mentally divide a grain of sand in half is possible, and those parts again, and perhaps again. But a limit is soon reached after which the imagination can no longer clearly distinguish division products from their objects. The number twenty is not magical, but by what signs, Hume asks, could the imagination be said to keep distinct the division of a grain of sand into nineteen sets of successive halves from its division into twenty?

The inkspot experiment is interpreted as showing that vision cannot receive sense impressions of infinitely divisible extension. The grain of sand thought experiment in turn is supposed to show that reflection through the agency of imagination similarly cannot provide impressions of reflection of infinite divisibility. Hume states that although he cannot clearly distinguish the division of the idea of a grain of sand into twenty let alone infinitely many ideas, he has a distinct idea of much larger numbers considered in themselves, and of proportions between, say, thousandth and ten thousandth parts. An argument is therefore needed to offset the possibility that reason or the imagination might be able to put together the clear idea of the division of a grain of sand into two parts, and the idea of the distinction between thousands, tens, and hundreds of thousands parts, and so on, extending the concept to produce an idea of the infinite division of a grain of sand. If imagination can manufacture the idea, but the mental picture is lacking, then so much the worse for an imagistic theory of mind.

There are no innate ideas for Hume, and hence no innate ideas of infinity or infinite divisibility. If we are to have an

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of Infinite Divisibility", p. 89). I do not understand McNabb's objection, nor the need for scare quotes around 'imagine' in this context. Hume simply asks us to perform the thought experiment of considering a grain of sand of realistic size, and then imagining it being divided into parts. It is not the idea Hume asks us to divide; rather, he invites us successively to replace one idea of an object with another idea of the same object subdivided into smaller objects.

idea of spatial extension, it must come from corresponding originating impressions of sensation or reflection. But at least our visual sense impressions of spatial extension are only finitely divisible, as the inkspot experiment shows. And our impressions of reflection involving imagination are also only finitely divisible, as the grain of sand thought experiment shows. The limitations of impressions in both categories are inherited by their corresponding ideas, passed along as limitations of whatever ideas are derived from them in experience. The important point for Hume's argument against infinite divisibility is that if the complex idea of extension derives from sense impressions, and if sense impressions are only finitely divisible, as the inkspot experiment and grain of sand thought experiments are supposed to show, then if the limitations of impressions apply to corresponding ideas as mental copies of impressions, the idea of extension can also be at most finitely divisible. Hume states categorically: "... since all ideas are deriv'd from impressions, and are nothing but copies and representations of them, whatever is true of the one must be acknowledg'd concerning the other."[7]

In trying the inkspot experiment, we might agree with Hume that there is a distance at which the perception of a small object vanishes, just before which it remains barely visible, and beyond which it can no longer be seen. We might also agree with Hume's conclusion that the impression of the inkspot perceived at just that point is sensible and indivisible, which is to say that the perception is a *minimum sensibilium.* The inkspot is indivisible for a given subject at a given distance in the sense that any such tiny impression could not be separated into still smaller parts while yet remaining impressions, since by hypothesis anything more miniscule will simply vanish from the perceptual field. The test of Hume's claim would be to have an assistant divide the inkspot on paper and separate or dislocate the halves, whereupon the stationary subject presumably would report that the parts of the original whole impression barely

[7] *Treatise,* p. 19.

visible at the vanishing threshold were no longer perceivable at all. When we are standing at the exact threshold, we are doing the best we can at that distance in trying to perceive an object of that size. Dividing and dislocating the object that produces the impression gives two (or more) yet smaller impressions, neither of which is visually discriminable at such a distance. Threshold impressions of the inkspot are visually indivisible.

Sensible Extensionless Indivisibles as the Constituents of Extension

What has puzzled some commentators in criticizing Hume's inkspot experiment is that although there might be a point at which a tiny mark afixed to paper and held at a certain distance is right at the threshold of vanishing from sight, this by itself does not seem to prove that the inkspot is either indivisible or extensionless. The fact that closer inspection of the mark reveals it actually to be both divisible and to have extension, suggests an appearance-reality dichotomy favoring a metaphysical idealism that Hume elsewhere is at pains to disclaim.

Yet Hume is making a more general point that is overlooked by such criticisms. We are not permitted by the experiment's constraints to approach any closer to the inkspot once we have established the vanishing point. Baxter explains: "Going in for a closer look ruins this as a model of Hume's theory of the structure of space. You cannot go in to see if the grains are divisible; you cannot go in to see if they are touching or not."[8] The idea is that for the subject to change position defeats the purpose of the inkspot experiment, which is to reveal the existence of sensible extensionless indivisibles in every visual field. Indivisibles are always present, according to Hume, but are ordinarily not discriminable, because at certain distances they blend in perfectly with their backgrounds.

It is worthwhile to think of the inkspot experiment as something like an opthamologist's method for locating ocular blindspots, the point of occluded vision where the optic

[8] Baxter, "Hume on Infinite Divisibility", p. 135.

nerve enters the retina. The inkspot experiment is first and foremost an *experiment*, designed to reveal something previously unsuspected about the nature of vision, and only derivatively provides the basis for an argument about spatial extension. Hume explains this in a passage already referred to that now receives proper emphasis: " 'Tis not for want of rays of light striking on our eyes, that the minute parts of distant bodies convey not any sensible impression; but because they are remov'd beyond that distance, at which their impressions were reduc'd to a *minimum*, and were incapable of any farther diminution."[9]

We might not have realized that there is a point after which objects of a certain size no longer produce a visual impression, just as we might not have known about the existence of ocular blindspots. But we can contrive a simple experiment in each case to demonstrate the fact. Hume's reasoning is obscure, but he seems to be saying that in every visual field, and by analogy perhaps every phenomenal field (though in the case of spatial extension he refers explicitly only to vision and touch), there are sensible indivisibles. If we approach the inkspot to examine it more closely, then of course our impression of the inkspot as a complete entity will no longer be indivisible. But there remain smaller parts of the inkspot that at any closer distance, even under the most high-resolution microscope, or viewed from a greater distance through the most powerful telescope, are at that distance *qua* sense impressions perfectly indivisible. Hume makes exactly this point in the next sentence of his discussion of the inkspot experiment:

> A microscope or telescope, which renders them [the minute parts of distant bodies] visible, produces not any new rays of light, but only spreads those, which always flow'd from them; and by that means both gives parts to impressions, which to the

[9] *Treatise*, p. 27.

> naked eye appear simple and uncompounded, and advances to a *minimum*, what was formerly imperceptible.[10]

Using instruments more sophisticated than the naked eye changes the size of or distance at which objects can be seen. Optical devices magnify small or distant objects and images by channeling lightrays through lenses, prisms, diaphragms and apertures. But the visual field that results when we peer into the eyepiece of a telescope or microscope is formally no different than that of unaided vision. It too contains indivisible impressions as parts, *minima sensibilia*, like the pixels or tiniest information units on a cathode ray screen.

The difference is that by using a microscope on the inkspot, it may be difficult or even practically impossible to identify the indivisible impressions that constitute its parts in the enhanced visual field, just as ocular blindspots cannot always be located by the naked eye except in special experimental situations. If we accept Hume's interpretation of these facts, then all that has happened is that these instruments, by disclosing a larger collection of indivisible impressions, permit the subject to experience the fine structures of objects that might otherwise have caused and appeared as a single indivisible impression, or permit otherwise imperceptible objects to be experienced as arrangements of indivisible impressions.

In every case, whether the visual field is natural or artificially enhanced, regardless of the distance between subject and perceived object, whether we are nearsighted or farsighted, or have any other defect of the senses, the phenomenal manifold for Hume inevitably consists of indivisible impressions. Indivisibles can only be detected by careful preparation, and otherwise are not perceived as indivisible. The impression of an inkspot on paper is indivisible only at a certain distance for a subject

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[10] Ibid., p. 28. I agree with Flew's criticism of Laird's objections to Hume's inkspot experiment in *Hume's Philosophy of Human Nature*, p. 68. Flew, "Infinite Divisibility in Hume's *Treatise*", p. 261. Laird appears, as Flew rightly notes, to misunderstand both the inkspot experiment and Hume's remarks about a telescope 'spreading' an image or impression in *Treatise*, p. 28.

with a certain visual acuity. If the subject moves toward the paper, the changing visual field at any point still consists entirely of indivisible impressions, which, because they no longer stand out sharply, are now practically impossible to identify. The moral of the inkspot experiment is that the impression of a tiny mark seen against a contrasting background appears to vanish at a certain distance. A cluster of adjacent indivisibles with virtually the same phenomenal properties will naturally appear divisible. This is true not only for the dark inkspot on close inspection, but for the white paper background at almost any distance. We need the experiment to satisfy ourselves that there are indivisible impressions. These we can then regard as an essential feature of every phenomenal field, even though typically we are not aware of and cannot see the indivisible components of impressions as distinct and indivisible.[11] Hume makes a similar observation about phenomenal indivisibility when he states:

> This however is certain, that we can form ideas, which shall be no greater than the smallest atom of the animal spirits of an insect a thousand times less than a mite: And we ought rather to conclude, that the difficulty lies in enlarging our conceptions

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[11] See Raynor, "'*Minima Sensibilia*' in Berkeley and Hume", pp. 196-200. Berkeley, *New Theory of Vision*, p. 191: "There is a *Minimum Tangibile* and a *Minimum Visibile*, beyond which sense cannot perceive. This every one's experience will inform him." Berkeley, *Principles*, p. 102: "If it be said that several theorems undoubtedly true, are discovered by methods in which infinitesimals are made use of, which could never have been, if their existence included a contradiction in it. I answer, that upon a thorough examination it will not be found, that in any instance it is necessary to make use of or conceive infinitesimal parts of finite lines, or even quantities less than the *minimum sensibile*: nay, it will be evident this is never done, it being impossible." Compare Berkeley, *Philosophical Commentaries*, Notebook A, Comment 441, p. 54: "Mem: before I have shewn the Distinction between visible & tangible extension I must not mention them as distinct, I must not mention M.T. [*minima tangibilia*] and M.V. [*minima visibilia*] but in general M.S. [*minima sensibilia*] etc." See also, Locke, *Essay*, pp. 201-203. The concept of *minima sensibilia* is found in Aristotle, *On Sense and Sensibilia*, 445^b33.

> so much as to form a just notion of a mite, or even of an insect a thousand times less than a mite. For in order to form a just notion of these animals, we must have a distinct idea representing every part of them; which, according to the system of infinite divisibility, is utterly impossible, and according to that of indivisible parts or atoms, is extremely difficult, by reason of the vast number and multiplicity of these parts.[12]

It is easy on Hume's theory of sensible extensionless indivisibles to form exact ideas of parts no greater (and presumably no smaller) than the smallest atom of the tiniest parts of the most diminutive creatures. This is because the ideas are of the extensions of objects consisting of the same sensible extensionless indivisible parts. Take the smallest insect and magnify it as much as possible. There, seen through the instrument's eyepiece, will appear the insect's least parts, which, under special circumstances like those required by the inkspot experiment for the naked eye, can be isolated as phenomenally indivisible. The difficulty is rather in ascending from the ideas of these parts to construct the idea of the insect itself. This is problematic in practice if there are indivisibles, because of the sheer numbers and complex relations involved. But it is utterly impossible, according to Hume, on the theory that extension is infinitely divisible. For then there can be no adequate idea of infinitely many components so configured as to constitute even the tiniest insect.[13]

Broad criticizes Hume's theory of sensible extensionless indivisibles on its own phenomenological grounds. He describes quite different experiences in performing the inkspot experiment:

[12] *Treatise*, p. 28.

[13] Berkeley in the person of Philonous describes the mite's limbs in terms of his theory of *minima sensibilia*, in *Three Dialogues*, p. 188: "A mite therefore must be supposed to see his own foot, and things equal or even less than it, as bodies of some considerable dimension; though at the same time they appear to you scarce discernible, or at best as so many visible points."

> At the earlier stages there certainly is a noticeable decrease in size, whilst the intensity of the blue colour and the definiteness of the outline do not alter appreciably. But, as I approach the limiting position, from which there ceases to be any appearance of the dot in my visual field, what I find most prominent is the growing *faintness* of the blue colour and the *haziness* of the outline. The appearance of the dot finally vanishes through becoming indistinguishable from that of the background immediately surrounding it. But, so long as I am sure that I am seeing the spot at all, I am fairly sure that the sense-datum which is its visual appearance is *extended*, and not literally punctiform. So I very much doubt whether there are punctiform visual sense-data. The case for punctiform tactual sense-data would seem to be still weaker.[14]

The objection challenges Hume's interpretation by describing a different sequence of experiences in performing the inkspot experiment. The most straightforward reply to be made in Hume's defense is that although his criterion of indivisibility and extensionlessness is phenomenal, his definition of these two concepts entails that in the situation Broad describes the inkspot remains both indivisible and extensionless. From an exact phenomenological characterization of the experience of the inkspot on paper, the impression is not divisible or extended at the vanishing threshold point. If we conclude that it is, it is doubtless because at that point the extended divisible dot itself rather than its unmediated sense impression irrepressibly comes to mind, and we reflect that if counterfactually we were to approach the inkspot, we would see that it is extended.

Broad pointedly says: 'so long as I am sure that I am seeing the spot at all, I am fairly sure that the sense-datum which is

[14] Broad, "Hume's Doctrine of Space", p. 166. Laird, *Hume's Philosophy of Human Nature*, pp. 68-69. Fogelin, *Hume's Skepticism in the Treatise of Human Nature*, p. 29. Fogelin, p. 27, offers a useful sketch of what he perceives as the principal 'stages' of Hume's reasoning about infinite divisibility. This may not be intended as a reconstruction of any particular argument in Hume, but it ignores Hume's claims about the sensory origin of the idea of extension, and mentions Hume's inkspot experiment only as an aside to what he regards as the more important divisibility limitations of imagination.

its visual appearance is *extended*, and not literally punctiform'. But the evidence that would decide the matter is phenomenal, and should not require appeal to the subject's belief about whether or not the spot is extended. The best method again is to have an assistant divide and separate parts of the inkspot, whereupon one of two possibilities will occur. If the subject at that distance can no longer see the two more finely divided spots, then the original whole inkspot in Hume's sense was extensionless after all. If the subject can still see the spots after their separation, then the subject needs to be relocated at a farther distance from the paper, and was not actually standing at the proper exact visibility-invisibility threshold for viewing the spot as an indivisible. The experiment must be arranged with some care, since even slight fluctuations of the subject's distance from the spot before and after division and separation of its parts occasioned by slight head movements or the like may cause the spot to vanish and the divided parts to remain visible, or to flicker in a blur between observability and unobservability. If the impression is at the vanishing point, and the subject's movement is controlled, then what the subject sees at that distance cannot be subdivided, for by the phenomenal criterion the separated constituents are too small to sustain an impression. This is what Hume means by describing the impression as sensible but extensionless and indivisible.

There may be room for dispute about the conclusions Hume draws from the inkspot experiment and its implications. But properly understood it is hard to find fault with Hume's claim that there are perceptual limits to the experienceable subdivisions of spatially extended figures. It is incontestable with or without the inkspot experiment that there is a threshold for sensory experience, a definite point at which objects of a certain size can yet be perceived, but where, if reduced to any smaller dimensions, they can no longer be detected. The mind in receiving sensory information from any perceptual field can only experience objects of a certain minimal size. Objects of precisely this extent and satisfying this rough characterization are *minima sensibilia*, relative to particular observers or their subjective states and circumstances. In the end, this is all Hume's argument against infinite divisibility seems to require.[15]

[15] Baxter accepts an intuitive justification of this kind for Hume's *minima sensibilia*. See his "Hume on Infinite Divisibility", p. 135: "To understand Hume's picture, do not imagine beads set next to each other. Beads are obviously divisible, so they mislead as models for indivisible parts. Imagine individual grains of sand seen at the furthest distance at which they are still visible. They should look so small that further diminution would render them invisible from that distance. When two grains of sand are set close enough that there is no discernible space between them, then they will look to be directly adjacent... With these constraints, one grain of sand will not seem extended; it will seem to occupy only a single location. Yet two grains of sand put together will seem extended." Flew, in "Infinite Divisibility in Hume's *Treatise*", similarly observes: "... Hume's next step is, as an argument, sound; and his immediate conclusion, surely, true. He proceeds to deduce from his ruinous premisses that there must be experiential minima, particularly in our ideas or images."

CHAPTER 2

AGAINST MIND-MEDIATED IDEAS OF INFINITE DIVISIBILITY

Simple and Complex Ideas

The thesis that the limitations of immediate sense impressions apply to the simple and complex ideas of which they are immediate faint replicas may well be true. But the principle by itself is insufficient for Hume's argument, unless it can also be shown that the complex idea of extension in particular is immediate, a direct ideational copy of an impression of colored or tactile points that is not mediated by memory, imagination, or reason. If not, then the possibility remains open for an adequate idea of infinitely divisible extension to be concocted from *minima sensibilia* by these faculties of mind.

Hume's heterodox rejection of infinite divisibility is an inevitable result of his uncompromising empiricism. Hume disallows putative ideas that are incompatible with his 'attempt to introduce the experimental method of reasoning into moral subjects', eliminating those for which a legitimating experiential origin is lacking. Yet Hume's arguments go beyond merely asserting that we never actually experience an infinitely divided extension, or that our finite minds never actually take in an infinite spatial expanse. Hume knows that there are ideas of other things for which we do not have immediate sense impressions, but which the mind puts together from different sorts of experientially derived cognitive raw materials. Why not suppose that the same might be true for an idea of infinite divisibility? Hume anticipates and refutes several ways of establishing an experiential origin for an adequate idea of the infinite divisibility of extension.

The copy principle for the experiential origin of ideas, as we have seen in the previous chapter, is explicitly limited to the causal derivation of simple ideas from simple impressions. Hume, in Part I, Section I, *Of the Origin of our Ideas*, distinguishes between simple and complex ideas in this way:

> There is another division of our perceptions, which it will be convenient to observe, and which extends itself both to our impressions and ideas. This division is into SIMPLE and COMPLEX. Simple perceptions or impressions and ideas are such as admit of no distinction nor separation. The complex are the contrary to these, and may be distinguished into parts. Tho' a particular colour, taste, and smell are qualities all united together in this apple, 'tis easy to perceive they are not the same, but are at least distinguishable from each other.[16]

The idea of extension for Hume is nevertheless complex. The fact that the idea of extension or of an extended body as divisible into parts necessarily excludes it from the category of simple ideas, if these, as Hume explains, are supposed to be mentally indivisible, admitting of 'no distinction nor separation'. Hume might appear to offer a counterexample to this classification in his lengthy discussion of the white marble globe in Section VII, *Of Abstract Ideas*, to which we now turn. There he writes:

> 'Tis certain that the mind wou'd never have dream'd of distinguishing a figure from the body figur'd, as being in reality neither distinguishable, nor different, nor separable; did it not observe, that even in this simplicity there might be contained many different resemblances and relations. Thus when a globe of white marble is presented, we receive only the impression of a white colour dispos'd in a certain form, nor are we able to separate and distinguish the colour from the form. But observing afterwards a globe of black marble and a cube of white, and comparing them with our former object, we find

[16] *Treatise*, p. 2.

> two separate resemblances, in what formerly seem'd, and really is, perfectly inseparable.[17]

It may seem as though Hume is saying that thought cannot distinguish the color from the shape of the white globe. If this were true, then it might suggest that the mind could also not distinguish or separate the components of a spatial extension, but could only receive it all at once as a simple sense impression producing a correspondingly simple idea.

Such an interpretation should be resisted, as we see when we place the passage in its proper context. Having previously in Part I, Section VII rejected the existence of abstract or general ideas, and argued that ideas must always be particular, Hume, now in Part II, Section V, proposes to recover an empirically respectable reinterpretation of the popular philosophical concept of a *distinction of reason*, without appealing to abstract ideas. It might seem as though a distinction between, say, the color and form of a white marble globe, would require a faculty of reasoning about abstract entities, by which the idea of the color of the globe is abstracted or considered as an abstract idea distinct from the abstract idea of its shape, while the abstract idea of the shape of the globe is similarly abstracted or considered as an abstract idea distinct from the abstract idea of its color. After calling up the image of the white marble globe, Hume resolves the problem without supposing that there must be abstract ideas by suggesting that the mind easily learns and then quickly becomes insensitive to its ability to entertain particular ideas with respect to only certain features of an object, such as the color of the globe, while ignoring others, such as its shape, or the reverse. As he often does in deflating similar philosophical speculations, and, indeed, as is his purpose in one sense throughout the book, Hume substitutes phenomenological psychology for metaphysics. Thus, he says:

> When we wou'd consider only the figure of the globe of white marble, we form in reality an idea both of the figure and colour, but tacitly carry our eye to its resemblance with the globe

[17] Ibid., p. 25.

> of black marble: And in the same manner, when we wou'd consider its colour only, we turn our view to its resemblance with the cube of white marble. By this means we accompany our ideas with a kind of reflexion, of which custom renders us, in a great measure, insensible. A person, who desires us to consider the figure of a globe of white marble without thinking on its colour, desires an impossibility; but his meaning is, that we shou'd consider the colour and figure together, but still keep in our eye the resemblance to the globe of black marble, or that to any other globe of whatever colour or substance.[18]

Hume, in any case, does not say categorically that the idea of the white marble globe presents itself to the mind as simple, but rather as a complex idea that is phenomenologically separable into the distinct ideas of its color, shape, and material. The idea of the white marble globe is no different in this respect than the complex idea of the apple with which Hume introduces the distinction between simple and complex ideas, separable, also, as he says, into the apple's color, taste, and smell. His point is that in separating ideas of the color, shape, and material of the white marble globe the mind is not thereby entertaining abstract ideas. The idea of the white marble globe remains particular and complex, and the mind in separating its component ideas limits its attention to one or other of these, alternatively to the particular idea of the whiteness or marble material or globe shaped figure of the particular idea of the white marble globe.

Hume claims only subjunctively, as added emphasis more clearly indicates, that 'the mind *wou'd* never have dreamed of distinguishing a figure from the body figur'd ... *did it not* observe, that even in this simplicity there might be contain'd many different resemblances and relations'. Hume then proceeds to indicate the kinds of experiences that can enable the mind to make the separations of particular ideas in distinctions of reason between the colors and shapes of black and white marble globes and cubes. It helps to have a white

[18] Ibid.

and black marble globe to compare in attending only to the shape of the globes, just as it helps to compare a white globe with a white cube in attending only to their color. In lieu of such comparisons, the mind might 'never have dream'd' of making such distinctions of reason. But the distinctions do not involve abstract ideas, and the idea of the white marble globe is not simple but complex, because it is 'impossible', according to Hume, to think of the shape or 'figure' of the white marble globe without thinking of its color.[19]

Hume does not explicitly describe the idea of extension as 'complex' in his official terminology, although he refers to it as 'compound'.[20] The distinction is important to Hume's argument, when we further introduce the distinction between immediate or primary and secondary or mind-mediated ideas. Hume explains that:

> ... besides this exception, it may not be amiss to remark on this head, that the principle of the priority of impressions to ideas must be understood with another limitation, *viz.* that as our ideas are images of our impressions, so we can form secondary ideas, which are images of the primary; as appears from this very reasoning concerning them. This is not, properly speaking, an exception to the rule so much as an explanation of it. Ideas produce the images of themselves in new ideas; but as the first ideas are supposed to be derived from impressions, it still remains true, that all our simple ideas proceed either mediately or immediately, from their correspondent impressions.[21]

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[19] For a careful account of Hume's discussion of distinctions of reason as applied to the white marble globe example, see Butler, "*Distinctiones Rationis*, or The Cheshire Cat Which Left Behind its Smile". An unsuccessful contra-Humean effort to apply Hume's discussion of distinctions of reason to a variety of perceptions to abstract ideas of space and time is ventured by Tweyman, "Hume on Separating the Inseparable".

[20] *Treatise*, p. 38. See Mijuskovic, "Hume on Space (and Time)", p. 387. Kemp Smith, *Philosophy of David Hume*, p. 227. Newman, "Hume on Space and Geometry".

[21] *Treatise*, pp. 6-7.

Immediate and Mind-Mediated Ideas of Extension

All ideas, according to Hume, regardless of whether they 'proceed' immediately or mediately from 'correspondent impressions', remain subject to the copy principle as deriving ultimately from corresponding simple or complex sense impressions. The limits of ideas derived from impressions and considered individually can sometimes be corrected if they are combined into more complex ideas by other faculties of mind. Hume says, in Part I, Section III, *Of the ideas of the memory and imagination*:

> The same evidence follows us in our second principle, *of the liberty of the imagination to transpose and change its ideas*. The fables we meet with in poems and romances put this entirely out of question. Nature there is totally confounded, and nothing mentioned but winged horses, fiery dragons, and monstrous giants. Nor will this liberty of the fancy appear strange, when we consider, that all our ideas are copy'd from impressions, and that there are not any two impressions which are perfectly inseparable. Not to mention, that this is an evident consequence of the division of ideas into simple and complex. Where-ever the imagination perceives a difference among ideas, it can easily produce a separation.[22]

The mind, in principle, then, might be able to manufacture from its immediate complex sense impressions of finite extension and finite divisibility a mediated complex idea of infinite divisibility. It could do so in somewhat the way that imagination puts together part of the impression-derived complex idea of a horse with part of the impression-derived complex idea of the head and torso of a man, to create the mediated complex idea of a centaur. If the mind's cut and paste operations do not enter into the mediated synthesis of a complex idea from experience of its originating complex impressions, then there is neither opportunity nor mechanism for the limitations of finite sense impressions to be overcome in a modified idea of the infinite divisibility of extension. If the complex idea of ex-

[22] Ibid., p. 10.

tension can be mediated by mind as a secondary idea, then Hume's distinctions do not yet preclude the possibility of an idea of infinitely divisible extension assembled from immediate complex sense impressions of finitely extended finitely divisible extension, together with the idea of the privation, opposition, negation, or complementation of a finite succession of divisions.

Hume certainly seems to regard extension as an immediate complex idea, rather than a mediated complex idea. As the argument develops in Part II, Section III, Hume indicates that immediate sense impressions of spatial entities without intervention by other faculties of mind are sufficient by themselves to produce the idea of extension. Hume maintains:

> The table before me is alone sufficient by its view to give me the idea of extension. This idea, then, is borrow'd from, and represents some impression, which this moment appears to the senses. But my senses convey to me only the impressions of colour'd points, dispos'd in a certain manner. If the eye is sensible of any thing farther, I desire it may be pointed out to me. But if it be impossible to shew any thing farther, we may conclude with certainty, that the idea of extension is nothing but a copy of these colour'd points, and of the manner of their appearance.[23]

The danger to Hume's theory of space is that the finite divisibility of impressions of extended bodies might not transfer to complex mind-mediated secondary ideas of extension. The threat can be avoided if Hume is able to prove that complex ideas of extension are immediate rather than mediated. His remark that the immediate impression of an extended object such as a table is enough to confer on the mind the idea of extension indicates only that the complex idea of extension might be immediate, or that it is immediate in certain

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[23] Ibid., p. 34. See pp. 239-240: "And to cut short all disputes, the very idea of extension is copy'd from nothing but an impression, and consequently must perfectly agree to it." Hume makes precisely parallel remarks about the immediacy of the idea of duration in time; ibid., p. 36: "Five notes play'd on a flute give us the impression and idea of time..." See Locke, *Essay*, pp. 266-267.

experiences of spatial phenomena. This does not preclude the possibility, as his position requires, that the idea is necessarily immediate or immediate in every case, nor that there cannot also be another complex mediated idea of infinitely divisible extension fabricated from impressions and ideas of sensation and reflection by the mind's manipulations of its experientially derived cognitive data. The challenge to Hume's refutation of infinite divisibility from this direction can only be removed by mounting a successful counterargument.

Hume does not dispute the possibility of a mind-mediated complex idea of extension. Presumably, the mind could produce such an idea by putting together two or more of the sensible extensionless indivisibles encountered in Hume's discussion of the inkspot experiment, that for Hume are supposed to constitute space.[24] But Hume's description of how we might obtain the idea of extension from an immediate sense experience of a table top suggests that he does not believe that the mind needs to or even that it ordinarily does in fact hammer together its idea of extension out of component ideas. It would appear that the usual course is for the mind to receive an immediate complex idea of extension from immediate sense impressions of spatially extended phenomena. Whether for Hume there could be a mediated idea of infinitely divisible extension turns on the question of whether the mind can assemble the idea from available component impressions and ideas. The ideas of extension and divisibility are already at hand, so the problem comes back to whether the mind can have access to an idea of infinity. The most promising proposal for a finite mind to fashion an idea of inifinity might then be to consider the privation, opposition, negation, or complementation of the idea of finite divisibility, or of a finite set or series. In principle, the idea of finite divisibility might be extrapolated beyond finite subdivisions to an infinite divisibility, similar to the mind's ability, which Hume

[24] *Treatise*, p. 27. See also p. 42. Jacquette, "Hume on Infinite Divisibility and Sensible Extensionless Indivisibles".

elsewhere allows, for reason to interpolate a missing shade of blue.[25]

Locke's Category of Negative Ideas

To investigate the possibility of a neo-Humean rehabilitation of infinite divisibility involving the negation of finite divisibility rigged together with a complex idea of extension requires a brief excursion into Locke's category of negative ideas. It will be necessary also to consult the only context in which

[25] *Treatise*, p. 6: "Suppose therefore a person to have enjoyed his sight for thirty years, and to have become perfectly well acquainted with colours of all kinds, excepting one particular shade of blue, for instance, which it never has been his fortune to meet with. Let all the different shades of that colour, except that single one, be plac'd before him, descending gradually from the deepest to the lightest; 'tis plain, that he will perceive a blank, where that shade is wanting, and will be sensible, that there is a greater distance in that place betwixt the contiguous colours, than in any other. Now I ask, whether 'tis possible for him, from his own imagination, to supply this deficiency, and raise up to himself the idea of that particular shade, tho' it had never been conveyed to him by his senses? I believe there are few but will be of opinion that he can; and this may serve as a proof, that the simple ideas are not always derived from the correspondent impressions; tho' the instance is so particular and singular, that 'tis scarce worth our observing, and does not merit that for it alone we should alter our general maxim." Commentators have been profoundly dissatisfied with this answer to the problem, since the missing shade of blue seems to violate Hume's copy principle. It looks as though Hume tries to paper over a serious counterexample by acknowledging the problem while downplaying its damaging consequences. I am inclined to accept a version of Noonan's solution to the problem in his book, *Hume on Knowledge.* Noonan, pp. 64-70, distinguishes between 'meaning empiricism' and 'genetic empiricism', and acknowledges that the missing shade of blue is a counterexample to Hume's copy principle, but only as a thesis of genetic empiricism, that seeks to explain the origin of every concept as one that is actually encountered in experience. Noonan explains Hume's insouciance about the missing shade of blue on the grounds that Hume in fact accepts the copy principle instead as a thesis of meaning empiricism, according to which not every concept need derive directly from sensation, provided that it is a concept of something that might actually be experienced by virtue of having been constructed out of other ideas originating from immediate sense impressions.

Hume discusses negative ideas of space, not of infinity or infinite divisibility, but of a vacuum. We may nevertheless have come far enough to see that there is no other possibility within the resources of Hume's empiricism to carve out a niche for infinite divisibility that will satisfy his principle of the experiential origin of simple, complex, immediate or mediated ideas in impressions of sensation or reflection, if he cannot further avail himself of a negative idea of infinity.

In *An Essay Concerning Human Understanding*, Locke argues that the mind as a finite intelligence cannot possess what he describes as a 'positive' idea of infinite extension, but at best a 'negative' idea. Locke characterizes the negative idea of infinity as indeterminate and confused, because it is only incompletely comprehended:

> So much Space as the Mind takes a view of, in its contemplation of Greatness, is a clear Picture, and positive in the Understanding: But Infinite is still greater... *The Idea of so much greater, as cannot be comprehended* ... *is plain Negative*; Not Positive. For he has no positive clear Idea of the largeness of any Extension, (which is that sought for in the *Idea* of Infinite,) that has not a comprehensive *Idea* of the Dimensions of it: And such, no body, I think, pretends to, in what is infinite... So that what lies beyond our positive *Idea* towards Infinity, lies in Obscurity; and has the indeterminate confusion of a Negative *Idea*, wherein I know, I neither do nor can comprehend all I would, it being too large for a finite and narrow Capacity...[26]

For Locke, a negative idea of infinity is that of the unbounded or unlimited, in number, extent, divisibility, or the like. This definition reflects the derivation of the English word from the Latin *infinitas* by way of the French *infinité*. The mind is incapable of a positive idea of infinity, but insofar as it appears to think of the infinite, Locke suggests it does so negatively by imaginatively considering sets, series, or relations that are supposed to continue without beginning or without end.

[26] Locke, *Essay*, p. 218.

The negative idea of infinity or infinite divisibility cannot be advanced without presupposing infinite time or some other infinite domain, and thereby requiring that we already understand infinity at some level in order to be capable of entertaining the concept even as a negative idea. Although Locke appears to devalue negative ideas in comparison with positive ones, he admits that the mind might at least have a genuine negative idea of infinity. His main purpose in the discussion, like Hume's, is to argue that even in the case of infinity, the (in this case, at most, negative) idea derives ultimately from impressions of sensation and reflection. Locke continues:

> §22. If I have dwelt pretty long on the Considerations of Duration, Space, and Number; and what arises from the Contemplation of them, Infinity, 'tis possibly no more, than the matter requires, there being few simple *Ideas*, whose Modes give more exercise to the Thoughts of Men, than these do. I pretend not to treat of them in their full Latitude: it suffices to my Design, to shew, how the Mind receives them, such as they are, from *Sensation* and *Reflection*; And how even the *Idea* we have of *Infinity*, how remote soever it may seem to be from any Object of Sense, or Operation of our Mind, has nevertheless, as all our other *Ideas*, its Original there. Some Mathematicians, perhaps, of advanced Speculations, may have other ways to introduce into their Minds *Ideas* of Infinity: But this hinders not, but that they themselves, as well as all other Men, got the first *Ideas*, which they had of Infinity, from Sensation and Reflection, in the method we have here set down.[27]

The method of which Locke speaks is that of beginning with sense impressions of extension and the divisibility of extension, and then adding to these conceptions the negative idea of unbounded or unlimited continuation, to produce the negative ideas of infinity and infinite divisibility. Locke criticizes attempts to reconstrue such constructions as positive ideas by maintaining that the end of a series or sequence is something

[27] Ibid., p. 223.

intrinsically negative, on the grounds that negating a negative idea in thought leaves the mind with a positive idea of infinity. Locke objects to the suggestion on intuitive grounds, holding that the idea of an end is inherently positive rather than negative:

> §14. They who would prove their *Idea of Infinite to be positive*, seem to me to do it by a pleasant Argument, taken from the Negation of an end; which being negative, the Negation of it is positive. He that considers, that the end is in Body but the extremity or superficies of that Body, will not, perhaps, be forward to grant, that the end is a bare negative: And he that perceives the end of his Pen is black or white, will be apt to think, that the end is something more than a pure Negation. Nor is it, when applied to Duration, the bare Negation of Existence, but more properly the last moment of it. But if they will have the end to be nothing but the bare Negation of Existence, I am sure they cannot deny, but that the beginning is the first instant of Being, and is not by any Body conceived to be a bare Negation; and therefore by their own Argument, the *Idea* of Eternal *à parte ante*, or of a Duration without a beginning, is but a negative *Idea*.[28]

[28] Ibid., p. 217. See pp. 216-217: "Though it be hard, I think, to find any one so absurd, as to say, he has the positive *Idea* of an actual infinite Number; the Infinity whereof lies only in a Power still of adding any Combination of Unites [*sic*] to any former Number, and that as long, and as much as one will; the like also being in the Infinity of Space and Duration, which Power leaves always to the Mind room for endless Additions; yet there be those, who imagine they have *positive* Ideas *of infinite* Duration and Space. It would, I think, be enough to destory any such positive *Idea* of infinite, to ask him that has it, whether he could add to it or no; which would easily shew the mistake of such a positive *Idea*." Locke discusses the vacuum in *Essay*, Book II, XIII, on *The Simple Modes of Space*. Kemp Smith documents the original sources for Hume's criticisms of the vacuum in Bayle, Locke, and Barrow, in *Philosophy of David Hume*, Appendix C, 'The Influence of Bayle', pp. 325-338, and Appendix E, 'Isaac Barrow on Congruity as the Standard of Equality and on What Can be Meant by a "Vacuum"', pp. 343-346. A useful discussion of Locke's efforts to domesticate a negative idea of infinity as no threat to empiricism in its historical context is offered by Rogers, *Locke's Enlightenment: Aspects of the Origin, Nature and Impact of his Philosophy*, pp. 55-60.

The influence of Locke on Hume's empiricism is qualified but profound. Yet it is remarkable that Hume does not avail himself of Locke's category of negative ideas in order to allow for the possibility of a mediated negative idea of infinity, as Locke explicitly does, even if it can only be indeterminate or confused. The reason seems to be that Hume rejects negative ideas altogether, which he does not even consider in the case of the infinite divisibility of extension, but only in application to another well-entrenched concept of ancient classical and Newtonian physics and natural philosophy, the negative idea of a vacuum.

This leaves only memory, imagination, and reason to combine and reassemble the raw material of simple ideas into more complex constructions. The implication is that the limitations of impressions that cannot be compensated by these faculties. Imagination, memory, and reason cannot make more out of the materials of the senses than their materials allow. Interestingly, however, Hume does not follow Locke's suggestion of permitting a special category for a Lockean negative idea of infinity. It would be gratifying if Hume's critique could be read as carrying Locke's objections to infinity one step further. Locke denies the possibility of the mind having a positive idea of infinity, but permits less cogent negative ideas. Hume might then be seen as completing the program by rejecting even negative ideas of infinity. Unfortunately, Hume's text does not directly lend itself to such an interpretation.

Hume's Objections to the Negative Idea of a Vacuum

Hume nowhere considers negative ideas of infinity or infinite divisibility. The fact that he leaves the proposal unexplored may indicate either that he does not take it seriously or that the objection simply did not occur to him. It is significant that the *Treatise* never mentions Locke's attack on positive ideas of the infinite by name, nor Locke's account of the mind's origination of a negative idea of infinity based on impressions and combined with the negative idea of the lack or absence of any end, limit, or boundary. Hume does, however, consider whether there could be a complex negative idea of a vacuum. The question arises because it is a consequence of his theory of space as a distribution of sensible extensionless indivisibles that there can be no adequate idea of a true vacuum. Hume explains:

> If the second part of my system be true, *that the idea of space or extension is nothing but the idea of visible or tangible points distributed in a certain order*; it follows, that we can form no idea of a vacuum, or space, where there is nothing visible or tangible.[29]

The inference is clear enough. If the idea of extension is nothing but the idea of an arrangement of adjacent sensible extensionless indivisibles, then there can be no idea of a part of space that contains nothing sensible, as the concept of a true vacuum requires. Hume considers the criticism that the idea of a vacuum might result from the negation of the ideas of light and motion in a plenum. The resulting complex idea, approximating the concept of a vacuum, would then be that of annihilation, absolute rest or immobility, and total darkness.[30]

The argument is divided into two parts. The first seeks to prove that the idea of absolute darkness, together with the annihilation of all visible and tangible objects, does not provide an adequate idea of a vacuum. The same conclusion is then supported by the assumption of absolute darkness containing visible tangible things. Hume answers three classical arguments in support of the idea of a vacuum: (1) that longstanding

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[29] *Treatise*, p. 223. An excellent account of the concept of vacuum in ancient and later classical Newtonian sources in the history of science is given by Grant, *Much Ado About Nothing: Theories of Space and Vacuum from the Middle Ages to the Scientific Revolution.* More contemporary definitions and approaches to the problem of the vacuum are represented by the papers contributed to the recent collection, *The Philosophy of Vacuum*, ed. by Saunders and Brown.

[30] See Laird, *Hume's Philosophy of Human Nature*, pp. 64-83. Maund, *Hume's Theory of Knowledge: A Critical Examination*, pp. 198-213. Cummins, "Bayle, Leibniz, Hume and Reid on Extension, Composites and Simples".

disagreements about the vacuum demonstrate the existence of an idea of a vacuum as the subject of dispute; (2) that even if space is in fact a plenum, the idea of a vacuum as a possibility is proved by imagining that all the matter within the walls of a chamber might be annihilated; (3) that the idea of a vacuum is necessary in order to explain motion in space, by providing an empty place for objects to enter. Instead of trying to answer the three arguments individually, Hume attacks the possibility, even as a negative idea, of a pure extension void of any sensible entity. This part of Hume's theory of space offers a revealing picture of his negative attitude toward negative ideas, and of the mind's inability to fabricate adequate ideas by privation, opposition, negation, or complementation.

Hume begins with a preamble that serves to remind the reader that the investigation concerns the conditions under which the mind is able to assemble complex ideas from simpler component ideas. "In order to answer these objections," Hume warns, "we must take the matter pretty deep, and consider the nature and origin of several ideas, lest we dispute without understanding perfectly the subject of the controversy."[31] The idea of absolute total darkness is dismissed by Hume as the mere negation of a positive idea:

> 'Tis evident the idea of darkness is no positive idea, but merely the negation of light, or more properly speaking, of colour'd and visible objects. A man, who enjoys his sight, receives no other perception from turning his eyes on every side, when entirely depriv'd of light, than what is common to him with one born blind; and 'tis certain such-a-one has no idea either or light or darkness. The consequence of this is, that 'tis not from the mere removal of visible objects we receive the impression of extension without matter; and that the idea of utter darkness can never be the same with that of vacuum.[32]

[31] *Treatise*, p. 55.
[32] Ibid., pp. 55-56.

There is more even to the negative idea of darkness than mere absence of light. Hume argues that the blind also suffer a total privation of light, but cannot reasonably be supposed to have an idea of either darkness or light. Hume equates the conceptual plight of the blind with the situation of the sighted, who, in the total absence of light, are equally unable to obtain any idea-producing impressions by visual perception. The point is not that the sighted unlike the blind can acquire the idea of darkness because they experience light and thereby participate in at least part of the light-darkness polarity required for complementary ideas both of darkness and light. Hume's position is rather that neither the sighted nor the blind can have an idea of absolute darkness, because absolute darkness affords no opportunity for the occurrence of idea-producing visual impressions. The blind are mentioned only to dramatize the conclusion, since they are permanently deprived of light, and as such more obviously lack the opposite idea of darkness, having no visual sense impressions by which to discriminate darkness from light.

Absolute darkness must be distinguished from partial or less-than-absolute darkness in order for Hume's argument to hold. Sighted perceivers intuitively have an adequate idea of 'darkness' in what is arguably the proper sense of the word. They derive the idea from impressions of greatly diminished light such as that experienced at twilight, in which, as the word itself suggests, there remains at least some dim trace of light. The experience of partial darkness is correctly designated as 'darkness' only if terms of the kind are meant to refer exclusively to things of which there are corresponding ideas, while what is called absolute darkness turns out on Hume's theory to be a philosophical fiction. If so, it may help to interject 'absolute darkness' wherever Hume in the argument refers merely to 'darkness'. There are related confusions to be clarified if Hume's argument is to be understood. It must first be resolved whether Hume is trying to prove that the total absence of light is insufficient to determine or convey to the mind an idea, or positive, adequate, or adequate positive idea of absolute darkness. The possibilities for interpreting Hume's remarks on negative ideas of darkness for the sighted fall into these categories:

(1) There can be a positive idea of (absolute) darkness.
(2) There can only be a negative idea of (absolute) darkness.
(3) There can be an adequate positive idea of (absolute) darkness.
(4) There can only be an adequate negative idea of (absolute) darkness.

Hume can reasonably be understood as agreeing or disagreeing with the content of the statements, depending on whether or not he accepts associated philosophical propositions about the nature and relationships between ideas and negative ideas, including in the first place whether negative ideas are really ideas. Hume might refuse to countenance negative ideas by claiming that negative 'ideas' are not genuine ideas, or by arguing that negative ideas are inherently inadequate, in the sense that they are inadequate *as* ideas, not merely in terms of their content. Such an argument would require more careful development. But for present purposes it is sufficient to see how the question whether Hume accepts some version of its conclusion might bear on his attitude toward the four proposals concerning the idea of absolute darkness.

Whether or not Hume is prepared to deny that negative ideas are inherently inadequate, he must surely reject alternatives (1), (3), and (4) above. He will not allow that there could be a positive idea of absolute darkness, nor an adequate idea, whether positive or negative. If he accepts the conclusion that negative so-called ideas are not really ideas, then he must also deny (2), however, since the conclusion precludes the existence of negative ideas. If, on the other hand, Hume grants that negative ideas are genuine ideas, then he might affirm (2), that there could be a negative idea of absolute darkness, though obviously, from what he says, not an *adequate* negative idea.

The purpose of considering these alternatives is not to pin Hume down to any particular choice. The point is rather to compare the consequences of each possibility for his elimination of the idea or adequate idea of a vacuum, and, ultimately, for the problem of whether the mind can construct a complex mediated negative idea of infinite divisibility. To refute the idea or adequate idea of a vacuum altogether, Hume must accordingly either deny that negative so-called 'ideas' are really ideas, in order to support the strong proposition that there can be no negative idea of absolute total darkness, or else fall back on the attempt to prove the weaker possibility that there can be no adequate negative idea of absolute total darkness.

Hume draws two conclusions from the nonexistence of an (adequate) idea of (absolute) darkness. The first is that the mind cannot attain an idea of extension without matter by negative abstraction, imagining the removal or annihilation of visible objects. The second is that even if we had a negative idea of absolute darkness, it would not be the idea of a vacuum. Simply negating or imagining the 'removal' or total annihilation of all sensible objects in space does not produce the 'impression' of 'extension without matter', in the standard way of understanding the concept of a true vacuum. This is supposed to be true for the same reason that the total absence of light does not convey by way of impressions of sensation or reflection an idea of absolute darkness to the blind. Hume's argument that the idea of absolute darkness is not the same as the idea of a vacuum is similarly structured in its appeal to negation, mere absence or privation, as inadequate to provide an idea-originating sense impression or mind-mediated idea.

The justification for Hume's second conclusion is more obscure, because the analogy it requires with a blind person's idea of light and darkness no longer holds. Hume claims that the idea of (absolute) darkness is merely negative. It appears that the sighted, unlike the blind, by virtue of experiencing a light-dark complementarity, might have a negative even if inadequate idea of absolute darkness. What is unclear is why Hume believes that the idea of darkness is a negative idea even for the sighted. The only way to maintain the proposition may be to presuppose something Hume does not try to make explicit, that idea-originating impressions of sensation must be caused in every instance by something active, an external object that literally impresses its image on sense organs, in the manner of a stylus on a wax tablet. Light rays and lighted objects can do this by physically acting on the eye. But absolute darkness, considered as a thing of which the mind might strive to attain an idea, cannot. In this sense, absolute darkness is something inherently negative, a mere absence of impression-activating light. This is more especially true if darkness is supposed to provide the idea of a vacuum as extension without sensible entities, as in the first part of Hume's argument. For, in that case, there is nothing present to act on the perceiver's sense organs or causally effect a change in their condition or state. If there is no idea-originating sense impression of absolute darkness, then, on Hume's thesis of the experiential origins of ideas, there can similarly be no adequate mind-mediated complex idea of absolute total darkness.

Hume then complicates the question in an interesting way. If absolute total darkness provides no impression of empty space, what about impressions of empty space surrounding visible things? Hume accordingly considers the possibility of deriving an idea of a vacuum from perceptions of the darkness of spaces occurring in-between dispersed illuminated phenomena. He writes:

> Since then it appears, that darkness and motion, with the utter removal of every thing visible and tangible, can never give us the idea of extension without matter, or of a vacuum; the next question is, whether they can convey this idea, when mix'd with something visible and tangible?
>
> 'Tis commonly allow'd by philosophers, that all bodies, which discover themselves to the eye, appear as if painted on a plain surface, and that their different degrees of remoteness from ourselves are discover'd more by reason than by the senses. When I hold up my hand before me, and spread my fingers, they are separated as perfectly by the blue colour of

> the firmament, as they cou'd be by any visible object, which I cou'd place betwixt them. In order, therefore, to know whether the sight can convey the impression and idea of a vacuum, we must suppose, that amidst an entire darkness, there are luminous bodies presented to us, whose light discovers only these bodies themselves, without giving us any impression of the surrounding objects.[33]

Hume, as he should, also considers a parallel mixed case about the motion of tangible entities dispersed throughout an otherwise intangible empty space.[34] For simplicity sake, I shall limit my analysis primarily to Hume's version of the argument involving only visible lighted objects separated by total absolute darkness in space, and forgo detailed discussion of the sensation of the movement of tangibles. The argument now follows a convoluted path, as Hume explores exactly what is involved in perceiving the mixture of sensible objects and intervening space in the visual or tactile field. Hume proceeds to divide and conquer:

> To begin with the first case; 'tis evident, that when only two luminous bodies appear to the eye, we can perceive, whether they be conjoin'd or separate; whether they be separated by a great or small distance; and if this distance varies, we can perceive its increase or diminution, with the motion of the bodies. But as the distance is not in this case any thing colour'd or visible, it may be thought that there is here a vacuum or

[33] Ibid., p. 56.

[34] Ibid., pp. 56-57: "We must form a parallel supposition concerning the objects of our feeling. 'Tis not proper to suppose a perfect removal of all tangible objects: we must allow something to be perceiv'd by the feeling; and after an interval and motion of the hand or other organ of sensation, another object of the touch to be met with; and upon leaving that, another; and so on, as often as we please. The question is, whether these intervals do not afford us the idea of extension without body?" Hume's treatment of touch in space is more complicated than in the case of vision, but his conclusions like his assumptions agree in refuting the possibility of deriving an idea of a vacuum from the perception of lighted or tangible objects in space.

> pure extension, not only intelligible to the mind, but obvious to the very senses.
>
> This is our natural and most familiar way of thinking; but which we shall learn to correct by a little reflexion. We may observe, that when two bodies present themselves, where there was formerly an entire darkness, the only change, that is discoverable, is in the appearance of these two objects, and that all the rest continues to be as before, a perfect negation of light, and of every colour'd or visible object. This is not only true of what may be said to be remote from these bodies, but also of the very distance; which is interpos'd betwixt them; *that* being nothing but darkness, or the negation of light; without parts, without composition, invariable and indivisible. Now since this distance causes no perception different from what a blind man receives from his eyes, or what is convey'd to us in the darkest night, it must partake of the same properties: And as blindness and darkness afford us no ideas of extension, 'tis impossible that the dark and undistinguishable distance betwixt two bodies can ever produce that idea.[35]

The mixed case of darkness peppered with lighted objects does not add anything of metaphysical significance to the pure case of total absolute darkness. Neither the total darkness that by hypothesis obtains between lighted objects, nor the distance that vision thereby reveals as separating them, constitute an idea of a pure extension or true vacuum. The fact that the darkness in this case is accented with distributed light does not enable perception of the lighted objects surrounded by darkness to arrive at an idea of anything more than the lighted objects themselves. Hume claims, however, that such experiences can give rise to mistakenly imagining the possibility of an idea of a vacuum:

> But tho' motion and darkness, either alone, or attended with tangible and visible objects, convey no idea of a vacuum or extension without matter, yet they are the causes why we falsely imagine we can form such an idea. For there is a close relation

[35] Ibid., p. 57.

> betwixt that motion and darkness, and a real extension, or composition of visible and tangible objects.[36]

The question that remains is how we are to think of such mixed impressions in which lighted or moving objects are scattered about in an otherwise apparently empty space. The answer, which Hume does not explicitly articulate in his otherwise instructive example, appears to be that in such cases ambient sensation sources make it possible for a perceiver also to experience the colored background of the objects in space, considered as a plenum of sensible extensionless indivisibles, as much when the darkened visual background is pitch black as when it appears as the blue sky between the perceiver's outstretched fingers. This seems to be the meaning behind Hume's cryptic pronouncement that:

> We may observe, that two visible objects appearing in the midst of utter darkness, affect the senses in the same manner, and form the same angle by the rays, which flow from them, and meet in the eye, as if the distance betwixt them were fill'd with visible objects, that give us a true idea of extension. The sensation of motion is likewise the same, when there is nothing tangible interpos'd betwixt two bodies, as when we feel a compounded body, whose different parts are plac'd beyond each other.[37]

If this is a correct interpretation of Hume's conclusion, then there is no metaphysically important difference between the following three situations with respect to the ideas of extension conveyed by sensations of impression:

(1) Looking up at the enclosed ceiling of a room, and seeing the solid arrangement of sensible objects that constitute the interior surface of the ceiling.
(2) Looking up at one's outspread fingers, and seeing a solid arrangement of sensible objects that constitute the flesh

[36] Ibid., p. 58.
[37] Ibid., pp. 58-59.

of the fingers and the blue sky background in which the fingers are visually framed.

(3) Looking up at the clear night sky, and seeing a solid arrangement of sensible objects that constitute the stars or planets and the pitch black totally darkened sky background in which the stars and planets are visually framed.

None of these experiences is sufficient to occasion an idea of a true vacuum as space devoid of any sensible object, and, in particular, of any sensible extensionless indivisible object, the arrangement of which according to Hume constitutes our only idea of space. To appreciate the advantages of this account, it will be useful to consider a criticism of Hume's argument against the vacuum, based on a significantly different exegesis.

Kemp Smith's Analysis

Norman Kemp Smith, in his monumental work, *The Philosophy of David Hume: A Critical Study of its Origins and Central Doctrines*, in Chapter XIV, 'Hume's Version of Hutcheson's Teaching That Space and Time are Non-Sensational', and Appendix A, 'Hume's Discussion of the Alleged Possibility of Infinite Divisibility and of a Vacuum, in Part ii of Book I', offers the following exposition of Hume's argument of the mixed case of lighted or moving objects in apparently empty space:

> But what, Hume asks, if the darkness and motion be 'mixed' with something visible and tangible? Can the darkness and the motion then yield the idea of extension without matter? This question he finds to be more difficult of answer, and discusses it in elaborate detail.
>
> First, he states the issue more precisely. We are supposed, amidst an entire darkness, to have luminous bodies presented to us, whose light discovers only those bodies, without our having any *impression* of objects surrounding them or space intervening between them. The question is whether we thereby acquire the idea of extension without body. So also in respect of motion mixed with tangibles. We are supposed to have an impression of touch and after an interval occupied by motion of the hand

> or other organ of touch another tactual impression, and upon leaving that, another, and so on. Do these intervals then afford us the idea of a vacuum?[38]

Kemp Smith criticizes Hume's argument, and then concludes with what seems to be a misinterpretation of Hume's objections to the idea of a vacuum. Kemp Smith first complains about what he sees as the inappropriateness of Hume's use of the word 'darkness' in refuting the three arguments for the existence of a vacuum. He explains:

> The first of idea of which [Hume] treats is that of "darkness" — an unfortunate, very misleading term in this connexion. For, by it Hume means the total absence of all *visibilia*, and therefore of shade as well as of light, of black as well as of any brightness... (This, surely, is a very strange and perverse way of asserting that 'darkness' is being taken as signifying simply the absence of *any* type of visual experience, and therefore of *any* apprehension of extension; and that the idea of 'darkness' cannot, consequently, be regarded as being a possible source of the idea of a vacuum, i.e. of *extension* without matter.)[39]

Then Kemp Smith explains Hume's arguments in a way that creates unnecessary paradoxes, and paints Hume as stubbornly maintaining a conclusion he has done everything in his power to contradict. Kemp Smith continues:

> Beginning with the visual impressions, Hume makes admissions which appear to grant all that is asked in the objection. It is evident, Hume allows, that when two luminous bodies appear to the eye, we can perceive whether they are conjoined or separate, and whether they are separated by a great or a small distance. If the distance varies upon motion of the bodies, we can perceive its increase or diminution. By supposition, the distance is not anything coloured or in any way visible. Surely, then, what we have here is a vacuum, a pure extension, obvious to the very senses, as well as intelligible to the mind.

[38] Kemp Smith, *Philosophy of David Hume*, p. 309.
[39] Ibid., pp. 308-309.

> In refusing to accept this conclusion, while yet making the above admissions, Hume asks us to recall what has been already established. When two bodies present themselves, where formerly there was an entire 'darkness', the only change is in the appearance of the two bodies. All the rest continues as before, a complete negation of light, and therefore of everything visible. And this is true of the very distance interposed between them. It remains nothing but the negation of the visible, "without parts, without composition, invariable and indivisible"... If then, as already shown, "blindness and darkness afford us no ideas of extension, 'tis impossible that the dark and undistinguishable distance betwixt two bodies [indistinguishable, i.e. as allowing of no distinguishable differences, and therefore of no divisions] can ever produce that idea." Again Hume is using his terms in a very bewildering manner; but the intention of his argument is sufficiently clear.[40]

It is unclear, however, whether Kemp Smith is right in the first place to criticize Hume for his choice of the term 'darkness' in the argument. Assuming always that by this term Hume means absolute total darkness, either by itself or in the interstitia between lighted objects, the word seems perfectly appropriate.

Kemp Smith seems to believe that perception of inky blackness is itself an impression that must be caused by material entities, whereas Hume is considering a vacuum to be altogether devoid of any *sensibilia.* This is precisely Hume's point, that when we perceive absolute total darkness surrounding lighted objects we are in fact experiencing an extension of space composed of pitch black *minima sensibilia,* since that is just what so-called empty space consists of when it is unoccupied by *other* sensible entities. The spaces that are said to be empty are not really empty at all, for there is no true vacuum if space is a distribution of colored, albeit inky-black-colored, sensible extensionless indivisibles. Darkness, just as much as light and lighted objects in space, is composed of colored 'points', and, for this reason, total absolute darkness

[40] Ibid., p. 310.

does not provide an idea of a true vacuum, in the sense Hume is concerned to refute, as a total absence of any sensible thing.

Nor does it seem correct, as Kemp Smith declares, by way of avoiding rather than resolving the difficulty he has foisted onto Hume's text, that, although Hume's usage of terms is 'very bewildering', 'the intention of his argument is sufficiently clear'. The intention of the argument is anything but clear on Kemp Smith's reading, unless by this Kemp Smith means the outcome rather than the inference by which Hume, apparently despite himself, concludes that the dispersal of luminous bodies in darkness and tangible bodies in motion separated by distances is inadequate to provide an idea of a true vacuum as the absence of any sensible phenomena in space. For Kemp Smith says that Hume's admissions in the argument ought to support the contrary conclusion that our perception of the objects in question provide an idea of a 'vacuum as a pure extension, obvious to the very senses, as well as intelligible to the mind'. What is so clear about Hume's obstinately 'refusing to accept this conclusion, while yet making the above admissions'? Kemp Smith merely falls back on Hume's previous argument against the vacuum, as 'Hume asks us to recall what has been already established'. But what 'has been already established' appears flatly inconsistent with Kemp Smith's portrayal of Hume's argument concerning mixed or dispersed illuminated visual and moving tangible phenomena in space. The implication points toward Kemp Smith's account of Hume's argument as unintelligible.

If my interpretation is correct, on the other hand, then it also explains why Hume's discussion of the mixed phenomena of total darkness punctuated by lighted objects in space is really no different than the image of outspread fingers surrounded by blue sky. Inky black or sky blue as the color of a spatial extension, what is the difference as far as Hume's theory of space is concerned? The stars and planets embedded in a matrix of total darkness in distant parts of space are just as much extended by virtue of their separation from one another by definite stretches of utterly black *minima sensibilia* as my fingers are by definite stretches of heavenly blue. The blackness of these extensions, considered as a distribution of colored extensionless indivisibles, is, therefore, no better basis for maintaining an experientially justified idea of a true vacuum than the blue between my fingers, for the simple reason that in neither case on Hume's theory of space can we be said to have an idea of space that is altogether empty of sensible entities. The idea of absolute total darkness is not a counterexample to Hume's theory of space, therfore, because it is not an idea of a true vacuum void of anything sensible that is capable of making a sensation on the mind.

As I understand Hume's argument, he means to distinguish between total absolute darkness and total absolute rest or absence of motion in space in pure and mixed cases. Where there is no light or no motion whatsoever, there can be no visual or tactile perceptions, and hence no ideas of space or anything spatial, and hence no idea of a true vacuum as pure extension or totally empty space containing no sensible phenomena. In Hume's epistemology and philosophy of mind, this conclusion seems inescapable. But where there is at least some light or at least some motion involving lighted or moving sensible things surrounded by space, the situation is exactly like that of the perceiver's outstretched fingers framed by blue sky, even if the surrounding space is colored pitch black, as when we look up at the distant stars on a clear night. This is why Hume says that 'the idea of utter darkness can never be the same with that of vacuum'. We can have at least a negative idea of darkness, but not of a vacuum, because the negative idea of darkness is not enough to provide us with an idea of pure extension or totally empty space.

It should suffice to read Hume's statement as implying only that the negative idea of absolute total darkness can never be the same as the idea of a true vacuum sought for or needed by its proponents. Such an interpretation is supported by Hume's charge two paragraphs later that: "... it appears, that darkness and motion, with the utter removal of every thing visible and tangible, can never give us the idea of extension without matter, or of a vacuum..."[41] The concept of extension without matter is first identified by Hume with the idea of a vacuum. He then asserts that imagining the annihilation of every visible and tangible thing cannot provide an idea either of a vacuum or of extension without matter. Of course, there can be no light-emitting vacuum in this sense, but there can presumably be nonvacuums, spatial extensions occupied by material entities, shrouded in absolute darkness. Hume's previous remarks preclude the possibility that his use of 'or' in speaking of 'extension without matter, or of a vacuum' is the 'or' of explication rather than the 'or' of inclusive logical alternation. The implication is that the previous arguments do not merely establish that the idea of absolute darkness is not the same as the idea of a vacuum, but that the inadequacy of the negative idea of absolute darkness shows that there can be no idea of a true vacuum.[42]

Frasca-Spada's Hume on Infinite Divisibility and the Vacuum

The only more recent study to relate Hume's criticism of infinite divisibility to his objections to the idea of a vacuum is Marina Frasca-Spada's *Space and the Self in Hume's Treatise.* Frasca-Spada's conclusions about Hume's philosophy of space are interesting, but also different from those proposed here in important ways. Frasca-Spada emphasizes passages in which Hume maintains that in the absence of adequate ideas philosophers sometimes make use of empty words and phrases that do not correspond to any experience. This is true, and in a way obvious. But while Hume evidently must say this kind of thing where words do not represent adequate ideas with respectable experiential credentials, the observation is entirely general, and has no special application to the problems of space. Frasca-Spada writes:

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[41] *Treatise*, pp. 55-56. See Wright, *The Sceptical Realism of David Hume*, pp. 100-107. Frasca-Spada, "Some Features of Hume's Conception of Space", pp. 406-411. Frasca-Spada, *Space and the Self in Hume's Treatise*, pp. 157-193, 194-198.

[42] *Treatise*, p. 56.

> So, when we talk about a vacuum we (literally) do not know what we are talking about: we merely use words, and we take it for granted that they stand for ideas. In fact, we only have what we may call a pseudo-idea of a vacuum, which is the result not of our experience in the ordinary sense, but of our use of language. But then, what matters about the vacuum in this connection, and also what makes Hume's long discussion worthwhile, is not to know whether it exists or not, either within or without our mind, but rather that there is a way we commonly talk about it. At this point, however, it is worth noting that something starts to assume a new significance: the satirical mood Hume displays in this section.[43]

Wherever corresponding ideas are lacking, Hume will want to say and do his best polemically to drive home the fact that we are only playing with words. The question is whether and how Hume has first satisfactorily shown that what passes for an idea of the vacuum or of the infinite divisibility of extension is, in Frasca-Spada's helpful term, at most only a pseudo-idea. Frasca-Spada identifies three strands of Hume's argument against the vacuum in Part II, Section V, *The same subject continu'd*, ranging over twelve pages of Hume's text. These are the arguments in which Hume tries to disprove the above mentioned three reasons frequently offered in support of the vacuum. She explains:

> ... the standard arguments for the conceivability and existence of empty space turn out to be objections to Hume's theory [of space as a distribution of extensionless indivisible colored points]. Hume examines three of them. First, 'the very dispute is decisive concerning the idea': the fact that we talk about empty space is itself evidence that we have an idea of it (*T*/54). Secondly, the idea of a vacuum is 'a necessary and infallible consequence' of the 'two possible ideas of *rest* and *annihilation*'... Thirdly, motion would be not only inconceivable, but also altogether impossible, 'without a vacuum, into which one body must move in order to make way for another' (*T*/55).[44]

[43] Frasca-Spada, *Space and the Self in Hume's Treatise*, p. 187.
[44] Ibid., p. 160.

It is true that Hume, in what Frasca-Spada calls this 'very difficult piece of writing', recognizes the need to refute these three classic arguments in support of the concept of a vacuum, in order to vouchsafe his own theory of space which it plainly contradicts. But this can only be part of Hume's task. Even if Hume is successful in proving that every argument offered to defend the idea of a vacuum fails, their failure will still not justify the stronger conclusion Hume needs in order to uphold his theory of space, that there definitely is not or cannot be a vacuum, or that there is not or cannot be an idea or adequate idea of a vacuum.

Hume needs to take the offensive more proactively by impugning the idea of a vacuum. This he does in his vigorous attack on Lockean negative ideas of the vacuum. Frasca-Spada only briefly mentions the fact that: 'The main point of Hume's argument is that the absence of visual or tactile impressions, being a mere negation, does not and cannot give rise to the idea of a vacuum or extension without matter'.[45] But she does not address what I take to be the vital part of Hume's strategy in rejecting negative ideas of the vacuum, stamped with Locke's authority, as a special category of idea, comparable otherwise in principle to a simple or complex, immediate or mind-mediated idea. All reference to a vacuum may just be empty talk about empty space for Hume. But his criticism is insufficiently motivated without a better understanding of why he rejects even the possibility that there could be a Lockean negative idea of a vacuum. Without this important piece of the puzzle, we cannot fully appreciate even if we fully agree with Frasca-Spada's insight that the critique of infinite divisibility and the vacuum stand or fall together in Hume's theory of space.

Frasca-Spada concludes that Hume's theory of space, and in particular his rejection of infinite divisibility and the vacuum, should be interpreted in terms of the spatiotemporal situation of the physically embodied psychological subject. She draws

[45] Ibid., p. 161.

an interesting series of connections between Hume's theory of space and the self:

> My overall aim has been to show that Hume's treatment of the idea of space is tightly linked with other, and for us more central, themes in the *Treatise*: The question of external objects, the problem of the unity of the self, the relation between knowledge and belief, and abstract ideas and distinctions of reason. I have argued that to understand Hume's sections on the idea of space we have to read them in the light of these other central issues in the *Treatise*, and, conversely, that the treatment of the idea of space may itself be used to cast light on these other issues, and thus on the philosophical project of the *Treatise* as a whole.[46]

The exact connection between ideas of space and the self Frasca-Spada postulates in this way, as falling under the category of 'manners':

> We have seen that the idea of space has something in common with the notion of belief: they are both 'manners' in which perceptions appear or are conceived. Both seem to be peculiar hybrids: they certainly cannot be ideas with correspondence with impressions, they do not seem to derive from any distinct impression of the senses, they do not exactly derive from passions or impressions of reflection either. This is why I have suggested they may be regarded as traces, among mental contents, of mental operations and activities. Similarly, I have suggested that certainty depends on a reflexive quality of a particular kind of acts of the mind, and that another such reflexive quality is guiding the formation of our pseudo-idea of an independent space and of a vacuum. So Hume's sections on the idea of space may be regarded as investigations into the 'powers and qualities' of the mind. Hume regards metaphysics as a 'science of human nature', and the 'experience' constituting its object includes both 'impressions' and 'manners of appearance', both 'ideas' and 'manners of

[46] Ibid., p. 194. The discussion is prefigured in Frasca-Spada, "Reality and the Coloured Points in Hume's *Treatise*, Part 1: Coloured Points", and "Reality and the Coloured Points in Hume's *Treatise*, Part 2: Reality".

> conception'; perceptions are its only objects, and the world and the mind are its two poles.[47]

There is something right and something misleading about this characterization of Hume's philosophy. It is to be expected that Hume's empiricism will take notice of the effect of spatial relations between the subject and its cognitive objects. There is also a sense in which Hume's study of space is a study of the powers and qualities of the mind. But the same might be said for Hume's study of any metaphysical topic, notably, body and causation; or, rather, since this is all that Hume will admit philosophy is competent to consider, the mind's ideas of space, time, body, and causation.

Frasca-Spada seems to go too far in suggesting that for Hume space and time can be regarded as 'traces' among mental contents, of mental operations and contents. Such an interpretation implausibly makes Hume's theory of space and time a psychological presursor of Kant's pure forms of intuition in the Transcendental Aesthetic. One would minimally want to see the account developed more completely than Frasca-Spada does in her tantalizing conclusion on space and self. For it is also true, as Frasca-Spada observes, that: "the connection between space and the self is merely implicit in Book I of the *Treatise*"[48], and, as I would add, it takes a great deal of reading between the lines even where "it is made fully explicit

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[47] Frasca-Spada, *Space and the Self in Hume's Treatise*, pp. 194-195. See Falkenstein, "Hume on Manners of Disposition and the Ideas of Space and Time". Falkenstein puts the point more carefully with respect to the relation between space and time, ideas of space and time, and manners of spatiotemoral order among ideas of space and time, when he writes, pp. 179-180: "What chiefly causes concern is Hume's surprising conclusion, that what our abstract ideas of space and time really represent — and what gives a characteristic content to all of our peculiar ideas of individual spaces and times — is not any one distinct impression, or even a collection of such impressions, but rather the manners of placement or "disposition", as Hume puts it, of impressions and their parts."

[48] Frasca-Spada, *Space and the Self in Hume's Treatise*, p. 195.

in Book 2, devoted to the passions"[49], in order to agree with Frasca-Spada that Hume understands space and time as traces among mental contents of mental operations and contents.

To say this with respect to space and time as opposed to our ideas of space and time is precisely to try to take Hume beyond the limits of our ideas of space and time to posit a metaphysical theory, which he is consistently unwilling to do. If we reinterpret Frasca-Spada's message as saying that it is the *ideas* of space and time that are traces among mental contents, even then we may be at a loss to hold that space and time for Hume are supposed to be traces of mental contents rather than mental contents themselves. For in the section of the *Treatise* where Hume explicitly treats these topics, he argues, as Frasca-Spada knows, that space is a distribution of sensible extensionless indivisibles, which are either ideas or imaginably external objects of which we have ideas, rather than mere traces of mental operations among ideas.

The only context relevant to the metaphysics of space and time in which Hume refers to 'traces' of mental operations is where he writes:

> I shall therefore observe, that as the mind is endow'd with a power of exciting any idea it pleases; whenever it dispatches the spirits into that region of the brain, in which the idea is plac'd; these spirits always excite the idea, when they run precisely into the proper traces, and rummage that cell, which belongs to that idea. But as their motion is seldom direct, and naturally turns a little to the one side or the other; for this reason the animal spirits, falling into the contiguous traces, present other related ideas in lieu of that which the mind desir'd at first to survey.[50]

Kemp Smith, in quoting part of this passage, prefaces it with the desultory comment that: "Some of Hume's strangest statements in regard to the principles of association occur in

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[49] Ibid.

[50] *Treatise*, pp. 60-61.

this connexion. It is here that he propounds his physiological explanation of the deceptive effects of resemblance...".[51]

A glance at Hume's reference to neurophysiological 'traces' indicates, first, that he is speaking specifically of engrams of animal spirits rather than mental operations in anything like the phenomenological sense Frasca-Spada seems to have in mind. Second, that Hume enters into this aside about how easily certain kinds of confusions and mistakes arise according to an associationist psychology, taking his terminology directly from Locke, for whom 'two kinds of distance' through habit and the slippage that can occur from one neurophysiological trace to another adjacent to it, distinguishing "betwixt that distance, which conveys the idea of extension, and that other, which is not fill'd with any colour'd or solid object."[52] It is the latter idea of extension that mistakenly suggests the possibility of a vacuum, but it is only the former that can properly provide an idea of extension. Hume digresses to explain how such confusions can arise, where 'an imaginary dissection of the brain' would reveal, "why upon our conception of any idea, the animal spirits run into all the contiguous traces, and rouze up the other ideas, that are related to it."[53] This is a fascinating if bizarre naturalization of associationist psychology, but it hardly commits Hume, as Frasca-Spada contends, to a theory of space as reducible to traces among mental contents of mental activities and operations. Hume's engram 'traces' are neurophysiological, occurring in the brain, and as such are themselves spatiotemporal, rendering viciously circular any attempt to understand his theory as reducing space and time to what he means by 'traces'.

When Frasca-Spada says in the quotation above that the idea of space and 'beliefs' '... certainly cannot be ideas with correspondence with impressions, they do not seem

[51] Kemp Smith, *Philosophy of David Hume*, p. 312.
[52] *Treatise*, p. 59.
[53] Ibid., p. 60.

to derive from any distinct impression of the senses', she also seems to have lost sight of Hume's previously cited assertion that: 'The table before me is alone sufficient by its view to give me the idea of extension. This idea, then, is borrow'd from, and represents some impression, which this moment appears to the senses'. Similarly, with respect to Hume's claim that: "The ideas of space and time are therefore no separate or distinct ideas, but merely those of the manner or order, in which objects exist: Or, in other words, 'tis impossible to conceive either a vacuum and extension without matter, or a time, when there was no succession or change in any real existence."[54] It thus appears that Frasca-Spada may have misconstrued Hume's remarks about ideas of space and time between ideas *of* the manner or order in which the ideas of objects appear with the manner or order itself of the object's appearance in thought. This would assuredly be something mental, to be characterized, perhaps, in a term Hume seldom uses, as a 'trace' of mental operations in surveying the ordering of spatiotemporal phenomena. But Hume is adamant that we can infer from the limitations of ideas about space and time, as he says in a passage already reproduced, 'that whatever *appears* impossible and contradictory upon the comparison of these ideas [concerning the most minute parts of extension], must be *really* impossible and contradictory, without any farther excuse or evasion'. This does not seem like a very promising methodology on which to build a theory of space and time as reducible to the powers and qualities of the mind. Could Frasca-Spada's interpretation of Hume's theory of space and time as a reduction to mental traces of mental operations nevertheless provide a basis for restoring negative Lockean mind-mediated ideas of infinite divisibility, if not also of the vacuum?

[54] Ibid., p. 40.

Hume on the Idea of a Vacuum and Complex Mediated Ideas of Infinite Divisibility

A Lockean negative idea of infinite divisibility might be produced by reason or imagination from the opposite idea of a finitely divisible extension, or from the adequate idea of a finitely divisible extension, together with the idea or mental operation of privation, opposition, negation, or complementation. Applying Hume's reasoning about the negative idea of absolute darkness to a negative idea of infinite divisibility, it seems likely that Hume would raise similar objections there as in his arguments against the idea of a vacuum.

If Hume discounts the negative idea of absolute darkness as providing an idea of the vacuum, then what Locke calls a negative idea can only exist for Hume where there is relevant sensory input, as in the mixed case of lighted or moving objects in space. Hume might rightly insist, as a result, that, unlike the negative idea of darkness in the case where darkness is combined with the lighting of at least some sensible things, there can be no comparable negative idea of infinite divisibility, for which there is evidently no counterpart basis for a mixed case involving some if not total absolute infinite divisibility — an extension is either infinitely divisible or it is not. A negative idea of infinite divisibility, in the nature of things, can have no experiential cause, and as such, unlike the negative idea of darkness, cannot occur even as an inadequate negative idea to thought.

The conclusion for Hume is that so-called negative ideas of infinite divisibility or the vacuum lack legitimating empirical origins in impressions of sensation or reflection. Hume's rejection of the negative idea of a vacuum by parity of reasoning entails that there also cannot be a negative mind-mediated complex idea of infinity or infinite divisibility. The privation, opposition, negation, or complementation of the finite divisibility of extension by itself cannot produce an idea or adequate idea of infinite divisibility, positive or negative, just as the mere privation of light is inadequate to convey an idea of absolute darkness to the blind. The negation of the idea of finite divisibility underdetermines the concept of infinite divisibility, because the mere absence or lack of finite divisibility applies equally to whatever is indivisible rather than infinitely divisible. The idea of sensible extensionless indivisibles, in Hume's theory of space, on the other hand, is supported in principle by positive originating sense impressions, as in the experiences Hume describes of the inkspot experiment.

If the argument is correct, it proves that there can be no simple or complex, immediate or mind-mediated, positive or negative, adequate or inadequate idea of infinite divisibility. Hume's arguments against negative ideas of a vacuum equally apply to negative ideas of infinity and infinitely divisible extension. Within his experiential theory of knowledge, Hume proves that there is no way for the mind with its limited faculties and resources in impressions of sensation and reflection to concoct an adequate complex idea of infinitely divisible extension by subjecting possible impressions to the combined agencies of mind. There is, consequently, no alternative within Hume's theory but to acknowledge the concept of extension as an immediate complex idea that must inherit the finite divisibility limitations of the finitely divisible sense impressions from which it derives.

Hume's Inkspot Argument

What I shall refer to as Hume's inkspot argument is directly based on the inkspot experiment. It relies on the copy principle that ideas derive from impressions, and interprets the inkspot experiment as showing that impressions and ideas of extension are only finitely divisible. The conclusion, then, is that there can be no adequate idea of infinite divisibility. The simplest version of the inkspot argument has this form:

Hume's Simplified Inkspot Argument

1. Ideas are mental copies of impressions, and as such are subject to the same limitations as the impressions from which they derive.

2. Impressions of extension are at most finitely divisible into *minima sensibilia*, as the inkspot experiment shows.
3. Ideas of extension are adequate representations of the most minute parts of extended things, so that extended things in reality cannot be more finely divided than their corresponding adequate ideas.

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4. There is no idea of infinitely divisible extension, and extension in reality is not infinitely divisible; space is a finite and finitely divisible distribution of sensible extensionless indivisibles or *minima sensibilia.* (1,2,3)

This version of the argument offers a good if oversimplified synopsis of Hume's reasoning. It covers both aspects of the two-fold task of his critique of infinity, relating the inkspot experiment to the rejection of infinite divisibility and the positive theory of space as a distribution of sensible extensionless indivisibles.

The inkspot experiment shows that the idea of extension obtained through sense impressions cannot be further subdivided in sense, as the grain of sand thought experiment shows that it cannot be further divided in the imagination, beyond the limits of *minima sensibilia*, and hence are certainly not infinitely divisible. At the same time, and by means of the same phenomenal evidence, the inkspot experiment also reveals that spatially extended objects and ultimately space itself are composed of *minima sensibilia*, thereby supporting the positive flank of Hume's metaphysics of space.

The argument as stated unfortunately does not take into account Hume's distinction between simple and complex impressions and ideas, and between immediate and mind-mediated ideas. Nor does it consider his anticipation of the suggestion that a Lockean negative idea of infinity could be adopted as a component to be put together with the idea of divisibility by reason or imagination in constructing a mind-mediated complex idea of infinite divisibility. First, we recapitulate the main points of Hume's rejection of Lockean negative ideas of infinity, and then apply his results to the refutation of any mind-mediated complex idea of infinite divisibility.

Against Lockean Negative Ideas of Infinity

1. A negative idea requires at least some experiential basis, and cannot result merely by thinking away or abstracting from all aspects of a positive idea; hence, as an examination of analogous putative ideas shows, there can be no negative idea of total absolute darkness or of a true vacuum.
2. There is no experiential basis for a Lockean negative idea of infinity, as the simplified version of the inkspot argument shows in confirming the common sense recognition that finite minds cannot entertain an adequate idea of infinity or infinite divisibility.

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3. There can be no Lockean negative idea of infinity. (1,2)

Against Mind-Mediated Ideas of Infinite Divisibility

1. If the mind is to produce a complex idea of infinitely divisible extension, it can only do so by modifying ideas of finite and finitely divisible extension through the faculties of memory, reason, or imagination.
2. Memory obviously cannot transform an idea of finitely divisible extension into a complex mind-mediated idea of infinitely divisible extension, because memory is just a replication or reproduction of an existing idea, bringing it forward for conscious consideration after having been temporarily forgotten.
3. Reason and imagination also cannot modify a complex mind-mediated idea of finitely divisible extension in the required way, because reason and imagination are limited to the finite divisibility of an idea of extension. This result is demonstrated by the grain of sand thought experiment as a limitation of reason and imagination for

impressions of reflection that precisely parallels the finite divsbility limitations revealed by the inkspot experiment for impressions of sensation.

4. If reason and imagination are to produce a complex mind-mediated idea of infinitely divisible extension, it can only be by combining a Lockean negative idea of infinity with the complex idea of extension or the divisibility of extension.
5. But for reasons explained above, there can be no Lockean negative idea of infinity.

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6. There can be no mind-mediated complex idea of infinite divisibility, of the infinite divisibility of extension, or of extension as infinitely divisible. (1-5)

The inkspot argument in its full glory has a rather more complicated structure than the simplified version. Hume does not collect all the inferences in a single statement, which we can now represent as follows.

Hume's Inkspot Argument

1. Ideas are mental copies of impressions, and as such are subject to the same limitations as the impressions from which they derive.
2. The limitations of impressions cannot be avoided, perfected, or otherwise compensated for by corresponding ideas, unless they are enhanced by the mind's faculties of memory, imagination, or reason, as in combination with Lockean negative ideas.
3. The inkspot experiment shows that complex sense impressions of extension are limited to *minima sensibilia*, beyond which sensation cannot discriminate any smaller parts.
4. Complex ideas of extension are immediate mental copies of complex sense impressions of expanses or distributions of *minima sensibilia* unmodified by memory, reason, or imagination.

5. There can be no Lockean negative idea of infinity or infinite divisibility for thought to combine with its idea of divisibility to produce the complex mind-mediated idea of infinite divisibility, just as there can be no Lockean negative idea of a true vacuum.
6. Complex ideas of extension are adequate representations of the most minute parts of extended things, so that extended things in reality cannot be more finely divided than their corresponding adequate complex ideas.

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7. Complex impressions of extension are at most finitely divisible into finitely many *minima sensibilia*, as the inkspot experiment shows. (1,2)
8. Complex ideas of extension are mental copies of sense impressions of expanses or distributions of *minima sensibilia*. (3,7)
9. Complex ideas of extension are also at most finitely divisible into *minima sensibilia*, as sensible least extended parts or indivisibles, because they partake of the limitations of the corresponding complex sense impressions from which they derive; they cannot be modified by the mind's faculties of memory, reason, or imagination, or by a Lockean negative idea of infinity, to produce a complex mind-mediated idea of infinite divisibility, as the grain of sand thought experiment also confirms for complex impressions of reflection in the imagination. (4,5,8)
10. Spatial extension in reality, adequately represented by ideas of the most minute parts of extended things, is also composed of *minima sensibilia*; space in reality is at most finitely divisible. (6,9)

The conclusions in (7) and (8) restate the main results of Hume's simplified inkspot argument against infinite divisibility. The conclusion in (9) reinforces the limitation of ideas of extension to finite divisibility by extending it also to mind-mediated complex ideas involving memory, reason, and imagination, even in combination with Lockean negative ideas of infinity. Hume would seem to discount in the same way that he explicitly rejects Lockean negative ideas of total absolute darkness or a true vacuum. This completes the first stage of Hume's argument. It proves that the finite divisibility limitations of sense impressions of extended bodies transfer to corresponding immediate and mind-mediated complex ideas of extension. The conclusion is unavoidable once it is admitted that unmediated ideas inherit the limitations of the sense impressions from which they derive, that the inkspot experiment establishes *minima sensibilia* as finite divisibility limitations for sense impressions, and that memory, reason, and imagination, even in combination with Lockean negative ideas, cannot compensate for the defects of finite and finitely divisible idea-originating impressions of sensation and reflection. The second stage of the inference is to derive conclusion (10) from (6) and (9). Here Hume seeks to show that the finite divisibility limitations even of mind-mediated complex ideas of spatial extension imply the finite divisibility of spatially extended objects in reality.

This completes the first, negative, part of Hume's twofold task. If the argument is correct, it proves in the first place that there can be no simple or complex, immediate or mind-mediated, positive or negative, adequate or inadequate idea of infinite divisibility. Hume has shown, in that case, that there can be no idea of infinite divisibility. Within his empiricist theory of knowledge, Hume proves that there is no way for the mind with its limited faculties and resources in impressions of sensation and reflection to concoct an idea of infinitely divisible extension. There is no alternative within Hume's theory but to acknowledge the concept of extension as a complex idea that by the copy principle inevitably inherits the finite divisibility limitations of the finitely divisible sense impressions from which it derives.[55]

[55] Hume's first *Enquiry* contains another basis for rejecting negative complex mediated ideas of infinite divisibility, but which has force only in conjunction with the *reductio* arguments against infinite divisibility in the later parts of the *Treatise* and the note to *Enquiry* section 124. This is the argument that negations of fact are conceivable, but not negations of the propositions of 'the sciences, properly so called'. Hume declares in the first *Enquiry*, pp. 163-164: "Whatever *is* may *not be.* No negation of a fact can involve a contradiction. The non-existence of any being, without exception, is as clear and distinct an idea as its existence. The proposition, which affirms it not to be, however false, is no less conceivable and intelligible. The case is very different with the sciences, properly so called. Every proposition, which is not true, is there confused and unintelligible." The example which follows makes it obvious that by this Hume intends what are otherwise called *a priori* propositions, which he distinguishes from contingent matters of fact. If the *reductio* proofs against infinite divisibility are effective, then infinite divisibility is a necessarily false thesis, and its refutation part of science proper, so that the negation of finite divisibility is inconceivable. This in turn precludes the possibility of the mind's creating an adequate mediated complex idea of infinite divisibility by negating the received idea of finite divisibility. See below, Chapter 7, and Jacquette, "Infinite Divisibility in Hume's First *Enquiry*".

CHAPTER 3

HUME'S INKSPOT METAPHYSICS OF SPACE: FINITE DIVISIBILITY OF EXTENSION INTO SENSIBLE EXTENSIONLESS INDIVISIBLES

Idea and Reality

Hume is officially skeptical both about realism and idealism. The naturalist strain in his philosophy implies that imagination is compelled without justification to draw a commonsense distinction between the objects of thought and the external world. It is mind-independent reality, however unsupported on this view, that causes impressions of sensation and makes them available to be copied as ideas and transformed by the faculties of mind. Hume insists that from a skeptical, empirically more circumspect methodology, we can only look to impressions and ideas for truths about the world.[56] This is idealism of a sort, but only in the service of a mitigated skepticism. Hume nowhere accepts a positive idealist metaphysics like Berkeley's, by which only mental entities, minds and ideas as the contents of minds, exist. Hume regards ontic idealism and realism alike as incapable of philosophical justification.[57]

There is an apparent inconsistency in the realist and idealist tendencies in Hume's philosophy. Yet the contradiction runs no deeper than the disparity in Hume's thought between natural belief, which inclines opinion toward acceptance of an external, mind-independent world, and skeptical philosophical reasoning,

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[56] *Treatise*, p. 463. *Enquiry*, p. 287.

[57] Tipton, *Berkeley: The Philosophy of Immaterialism*. Warnock, *Berkeley*, pp. 86-125.

which disavows commitment to the existence of anything not encompassed in the ideational domain. The transitions in Hume's discussion from the properties and limitations of ideas to the properties and limitations of things outside the mind manifests itself in several ways. This is seen in the title of Part II of the *Treatise*, *Of the ideas of space and time*, followed as though in transition from idea to reality and back again in successive subsections, *Of the infinite divisibility of our ideas of space and time*, *Of the infinite divisibility of space and time*, *Of the other qualities of our ideas of space and time.*

It is no exaggeration to say, as several commentators have lately held, that Hume's philosophy can only be understood as riding the tension between a realism dictated by natural belief, and a methodological idealism sustained by a skeptical appreciation of the limits of reason in metaphysical inquiry. Affirming a realist natural belief thesis about the existence of mind-independent reality, Hume succinctly states: "['T]is in vain to ask, *Whether there be body or not?* That is a point, which we must take for granted in all our reasonings."[58] For Hume, it must be recalled, belief is involuntary, and natural belief in an external reality may in principle be erroneous, something the mind is compelled to accept without adequate philosophical rationale.[59] The idealist counterpoint in Hume's philosophy is expressed in so many places and so many ways that the key to resolving the conflict must be found if Hume's criticisms of infinite divisibility are to be understood as applying merely and exclusively to ideas of extension, as opposed to real extension in the external world. Hume maintains: " 'Tis impossible, therefore, that from the existence of any of the qualities of [perceptions], we can ever form any conclusion concerning the existence of [external objects], or ever satisfy our reason in this particular."[60] In the *Enquiry*, he similarly argues:

[58] *Treatise*, p. 187.
[59] Ibid., pp. 153-154; 624.
[60] Ibid., p. 212.

> It is a question of fact, whether the perceptions of the senses be produced by external objects, resembling them: how shall this question be determined? By experience surely; as all other questions of a like nature. But here experience is, and must be entirely silent. The mind has never anything present to it but the perceptions, and cannot possibly reach any experience of their connexion with objects. The supposition of such a connexion is, therefore, without any foundation in reasoning.[61]

The contradiction in Hume's theory is only apparent. Throughout his philosophical development, from the early to the later period, Hume consistently upholds the real existence of external objects, while maintaining with equal conviction that any such belief is unjustified by metaphysical reasoning. From the standpoint of his Academic skepticism, rooted in what he takes to be the only correct, empiricist or humanized experiential methodology, there is no sound philosophical foundation for Hume in extending belief beyond the contents of impressions and ideas.

The essential contrast is expressed as a distinction between what we in fact believe, and what we may be entitled by reason to believe. If we ask, 'What does Hume believe about the existence of external, mind-independent reality?', the answer is that he is a realist, and does in fact accept the existence of real external objects beyond or outside the realm of ideas and impressions present to the mind. He believes that we have no choice in the matter, but are psychologically compelled as part of human nature to accept their existence. If, by contrast, we ask, 'What does Hume believe he is entitled by reason and the data of phenomenal experience to believe about the reality of an external, mind-independent world?', then the answer is that here he is skeptical, and that his skepticism takes the form of an even more austere idealism than Berkeley's. Hume limits philosophical investigations to whatever reason can make of impressions and ideas. Unlike Berkeley, Hume does not even admit the existence of a unified substantial

[61] *Enquiry*, p. 153.

self, since he claims not to discover the subject in experience. The interplay of realism and idealism in Hume's theory of extension reflects a more pervasive discord between natural belief and skeptical metaphysical reason in his philosophy. Hume, moreover, again in sharp contrast with Berkeley, is notoriously skeptical about the existence, let alone the rigorous philosophical demonstrability, of the existence of God.

Hume acknowledges the conflict between natural belief and skeptical reason in the *Treatise*, when he observes:

> Thus there is a direct and total opposition betwixt our reason and our senses; or more properly speaking, betwixt those conclusions we form from cause and effect, and those that persuade us of the continu'd and independent existence of body. When we reason from cause and effect, we conclude, that neither colour, sound, taste, nor smell have a continu'd and independent existence. When we exclude these sensible qualities there remains nothing in the universe, which has such an existence.[62]

The conflict of natural belief and philosophical reason seems to place natural belief at a disadvantage in elaborating a metaphysics of external or mind-independent reality, especially in the theory of extension. Hume holds that there can be no philosophical justification for belief in the properties of real extension beyond the properties of ideas of extension. Seated at the backgammon table or dining with his friends, Hume escapes from the skepticism of 'cold, and strain'd' speculations of metaphysical reason, for which nature herself provides the best remedy, and which for Hume, it must be recalled, is and ought only to be the slave of the irrational ('violent' as opposed to 'calm') passions.[63]

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[62] *Treatise*, p. 231.

[63] Ibid., p. 269: "Most fortunately it happens, that since reason is incapable of dispelling these clouds, nature herself suffices to that purpose, and cures me of this philosophical melancholy and delirium, either by relaxing this bent of mind, or by some avocation, and lively impression of my senses, which obliterate all these chimeras. I dine, I play a game of back-gammon, I converse, and am merry with my friends; and when

The compromise between these two opposed forces in Hume's thought is evident throughout his theory of human nature. It is prominent in his statement that: "We may observe, that 'tis universally allow'd by philosophers, and is besides pretty obvious of itself, that nothing is ever really present with the mind but its perceptions of impressions and ideas, and that external objects become known to us only by those perceptions they occasion."[64] Here in one breath Hume combines the idealist limitations of his disciplined empiricist methodology to perceptions present to the mind as impressions and ideas, with the unmistakable assertion that real external objects nevertheless 'become known to us' by means of the impressions and ideas they 'occasion'.[65]

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after three or four hour's amusement, I wou'd return to these speculations, they appear so cold, and strain'd, and ridiculous, that I cannot find in my heart to enter into them any farther." And p. 415: "We speak not strictly and philosophically when we talk of the combat of passion and of reason. Reason is and ought only to be the slave of the passions, and can never pretend to any other office than to serve and obey them."

[64] Ibid., p. 67. *Enquiry*, p. 152: "[T]he slightest philosophy ... teaches us, that nothing can ever be present to the mind but an image or perception, and that the senses are only the inlets, through which these images are convey'd, without being able to produce any immediate intercourse between the mind and the object." For comparison, see also *Treatise*, pp. 191-193, 210-211, 230, 237-239; *Enquiry*, p. 151.

[65] There are several explanations of the conflict and resolution of the conflict between natural belief and skeptical reason in Hume's thought. Lengthy discussion and critical evaluation of the alternatives is atopical here, but a few of the more noteworthy accounts in recent philosophical commentary can be mentioned. Bricke, *Hume's Philosophy of Mind*, writes, p. 24: "Clearly enough, Hume's radical skepticism is not designed to advance the thesis that there are no physical objects. I suggest that its object is, rather, to put a rein on radical departures from the plain man's metaphysics. This it purports to do by displaying the irremediably antinomic character of the human imagination when its subject is the physical world... Despite his own sceptical arguments, then, Hume contends that one must be a plain man and believe in bodies that have distinct and continued existence... In the ordinary run of things the plain man's beliefs are unavoidable." Bricke, despite acknowledging Hume's commitment to unavoidable natural belief in an external reality, claims, pp. 6-9, that

Hume is a realist about the existence of an external, mind-independent reality, and an idealist in philosophical methodology. He limits knowledge about the properties of objects to what can be discovered by the mind's reflections on its impressions and ideas. This regrettably leaves the problem of the real properties of extension high and dry, including the question whether extension or finitely extended objects in reality are infinitely or only finitely divisible. Hume's skeptical realism implies only the philosophically unjustified belief in the bare existence of external entities, but goes no distance to quench our natural desire for knowledge of their real properties. From the fact, if it is a fact, that the idea of extension is not infinitely divisible, it would seem that it should not necessarily follow that extension itself or any real finitely extended body is not infinitely divisible. According to Hume, after all, there is no philosophically tenable way to know whether or not idea and reality agree.

It is precisely at this point that Hume reinforces the ideational limit to what can be discovered by any philosophically correct theory. Hume thereby denies his metaphysical antagonists the possibility of contradicting his conclusions about the finite divisibility of extension, on the grounds that they can have no better access to the real properties of extension than

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according to Hume naive realism is false, which makes Hume's realism more difficult, arguably more difficult than necessary, to reconcile with his skeptical idealist methodology. Flew, in his *David Hume: Philosopher of Moral Science*, pp. 29-37, criticizes Hume's efforts to preserve the realist idea-reality distinction even as a matter of natural belief. Broad, p. 168, voices similar objections. Fogelin, in *Hume's Skepticism in the Treatise of Human Nature*, pp. 53-92, maintains that Hume restores commonsense naturalism by explaining the compulsion of imagination-driven belief in an external world despite the skepticism that follows from a methodology of strict empiricism. See also Wilbanks, *Hume's Theory of Imagination.* An earlier version of this interpretation with less emphasis on Hume's skepticism is found in Price's important study, *Hume's Theory of the External World.* The most balanced and historically informed interpretation of the conflict in Hume's philosophy in my opinion is presented by Norton, *David Hume: Common-Sense Moralist and Sceptical Metaphysician*, pp. 192-310.

is disclosed by the content of their finite finitely divisible ideas. Hume anticipates the metaphysical realist objection:

> 'Twill probably be said, that my reasoning makes nothing to the matter in hand, and that I explain only the manner in which objects affect the senses, without endeavouring to account for their real nature and operations... I answer this objection, by pleading guilty, and by confessing that my intention never was to penetrate into the nature of bodies, or explain the secret causes of their operations. For besides that this belongs not to my present purpose, I am afraid, that such an enterprize is beyond the reach of human understanding, and that we can never pretend to know body otherwise than by those external properties, which discover themselves to the senses.[66]

The disclaimer puts Hume and his infinitist critics in the same boat. Hume at least acknowledges the fact that both sides are unable to validate conclusions about the true nature of extension as opposed to the mind's impressions and ideas of extension and extended things. However unsatisfying the explosion of metaphysically more grandiose ambitions in the philosophy of space and time may appear, Hume affirms what he regards as the only acceptable ideational limit, beyond which no methodologically circumspect theory of the divisibility of extension can legitimately aspire. The strategic gain is undoubtedly Hume's. If the correct method in inquiring about the nature of extension is limited for one and all to an investigation of the properties of impressions and ideas of extended objects, then there is no prospect of advancing an experientially transcendent metaphysics of infinite divisibility.

There may yet be a way for Hume to span the epistemological gulf between idea and reality. We might proceed in an indirect Kantian manner by asking what an adequate idea of the divisibility of extension presupposes about the real divisibility of extension. Assumption (6) in the expanded reconstruction of Hume's inkspot argument paraphrases his explicit assertion that 'ideas are adequate representations of the most minute

[66] *Treatise*, pp. 63-64.

parts of extension'. This suggests a way to bridge the gap between the finite divisibility limitations of immediate complex ideas inherited from the finite divisibility of immediately originating sense impressions to the finite divisibility limitations of extension in reality. Hume, in what might otherwise appear to be a careless elision, describes ideas as adequate representations, not merely of impressions of the most minute parts of extension in thought, in experiences of sensation or reflection, but of the most minute parts of extension in external reality.

Hume's proposition raises several problems. How can Hume claim that our ideas are adequate representations of the most minute parts of extension without reasoning in a circle? What entitles Hume to proclaim these ideas adequate unless he already knows or assumes that the limitations of ideas are also limitations of reality? For that matter, even if our ideas are adequate representations of the most minute parts of extension, does it not beg the question from the outset to suppose that extension has most minute parts? If extension is infinitely divisible, then there are no least or most minute parts, but subdivision continues without end through infinitely many subsegments. The subdivision of extension encounters infinitely many extended subsegments, between any two points of which there are by assumption infintely many additional subsegments, even if extension is in some sense divisible in the opposite direction by infinitely many infinitely small Euclidean points. Hume does not consider the difficulty, so any defense of his conclusion must go beyond the text. What can reasonably be said on his behalf?

If we first construe Hume's reference to 'most minute parts' not as designating the indivisibles of his own theory of the extensionless parts of extension, but instead as referring generally and in more neutral fashion either to these or to Euclidean points or infinitesimals, then the last named potential source of circularity is avoided. It would be strange, though obviously not unimaginable, for Hume simply to have assumed that his sensible extensionless indivisibles are the most minute parts of extension at this stage of the argument.

For the classical geometrician, although the division of an extended body continues through infinitely many extended subdivisions, ultimately every extended body is measured by and divisible into infinitely many abstract Euclidean points. There is a difference between top-down reductive analysis and bottom-up constructive constitution. From an analytic or top-down perspective, the classical subdivision of extension is always into infinitely many infinitely divisible line subsegments, while from the synthetic bottom-up perspective, extension must ultimately be built up from infinitely many infinitely small individual Euclidean mathematical points. The fact that the analytic and constructive perspectives do not converge in the classical model provides one source of Hume's criticism of infinitary metaphysics and mathematics, as we have already seen in his Berkeleyan rejection of the possibility of a synthetic construction of the extension of the most minute parts of the tiniest insect from infinitely many mathematical points.[67]

It would not be unreasonable for Hume to intend as 'the most minute parts of extension' in this passage either mathematical points in the theories he criticizes or his own sensible extensionless indivisibles. If successful, the argument proves without simply presupposing its conclusion that the most minute parts of extension are sensible extensionless indivisibles. At the outset, until excluded by the final inference, the most minute parts of extension to which the assumption refers might in principle be ideal abstract mathematical points. We need only charitably read Hume as leaving open for the sake of argument the possibility of any and all contenders under consideration for the most minute parts of extension, until the proof's conclusions in (9) and (10) are reached, and sensible extensionless indivisibles alone remain. In assumption (6) of the inkspot argument, Hume might then intend to remain neutral about the exact kind of most minute parts that constitute extended things from the constructive bottom-up point of view. He holds in either case, whether the parts

[67] Supra, note 13.

in question are mathematical points or sensible extensionless indivisibles, that the mind's ideas of extension are sure to be adequate representations of the atomic constituents of finite extension, whether sensible or insensible. It just turns out that the only candidate for the most minute parts of extension to survive criticism are sensible extensionless indivisibles. Such ambivalence about the most minute parts of extension in its assumptions may save Hume's proof from overt circularity, by permitting him to hold until the inference is complete that from a bottom-up standpoint there could be minimally minute parts of extended things on both classical and phenomenal theories of the divisibility of extension. Hume's reference to the least or most minute parts of extension as such in that case need not presuppose the truth of his own synthetic theory of the constitution of extension by sensible extensionless indivisibles.

Adequate Ideas of Finite Divisibility

As for the adequacy of ideas of extension, Hume in the first paragraph offers an independent criterion for assessing their worth that need not require prior assumptions about their correspondence with the nature of reality. In the sentence preceding the adequacy argument, he begins his discussion with this general epistemological observation:

> Wherever ideas are adequate representations of objects, the relations, contradictions and agreements of the ideas are all applicable to the objects; and this we may in general observe to be the foundation of all human knowledge.[68]

From a Kantian perspective, this solution must seem hopelessly naive. It may even be inconsistent with Hume's philosophical skepticism about the existence and nature of the external world.[69] What Hume proposes is that adequate ideas are those

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[68] *Treatise*, p. 29.

[69] *Enquiry*, p. 165. *Treatise*, p. 13: "Nothing is more requisite for a true philosopher, than to restrain the intemperate desire of searching into causes, and having establish'd any doctrine upon a sufficient number of experiments, rest contented with that, when he sees a farther examination

that agree with their objects. But what access can Hume consistently claim we might have to objects in themselves independently of our impressions and ideas?

One answer might be found in the comparative verisimilitude of immediate sense impressions. It has been so long since I have seen the Tower of London that my idea of the White Tower is of a round building with three cupolas. Is this an adequate idea or not? The best answer is to visit the site again or consult the photographs in a book, and compare the idea in memory with more immediate sense impressions. That is as close as I can get to the object itself, and for Hume, if he is being true to his methodology, the problem surely admits of no other solution. If my imagination then compels my belief that the relations, contradictions, and agreements of the idea in memory apply equally to the object, where this is determined by comparison of the idea with immediate sense impressions, then the idea is adequate. If, on the contrary, my impressions of the Tower reveal it to be a square structure with four cupolas, as is in fact the case, then the first idea must be judged inadequate.

We test for the adequacy of ideas about objects by examining them with respect to relations, contradictions, and agreements ascertained for them. If an idea leads to contradictions, then it cannot be adequate. This explains why Hume devotes such attention to his *reductio* disproofs against the infinite divisibility of extension in the *Treatise*. The arguments in this sequel to his main criticisms in the inkspot experiment encourage Hume to conclude that the idea of infinity harbors internal inconsistencies. If, on the contrary, an idea has not strayed from its originating sense impressions, then on close reexamination of the impressions copied into the idea, as in the inkspot and other experiments with perceptions, and the grain of sand thought experiment, there will be an exact matching or 'agreement' of

would lead him into obscure and uncertain speculations. In that case his enquiry wou'd be much better employ'd in examining the effects than the causes of his principle."

the properties and relations of impression and idea. The idea will then fit easily and correctly into the network of relations and prior beliefs about the object as the best possible direct knowledge of its real properties.

We accordingly check the idea of extension against its originating sense impressions to decide whether it also observes the same finite divisibility limitations. When we make the comparison, we naturally find the ideas adequate by Hume's criterion. There are no relations, contradictions, or agreements of the one that do not also belong to the other. Yet there are incongruities between originating impressions of extension and the mathematician's 'ideas' (or what Berkeley might prefer to call 'notions'[70]) of infinitely divisible extension. By Hume's standard, such ideas are inadequate. This is roughly equivalent, despite occasional ambiguities of expression, to saying that the traditional mathematician's 'idea' of the infinite divisibility of extension is no idea or no real or genuine idea at all.

If the idea of extension inherits the finite divisibility limitations of its originating finitely divisible sense impressions, and if by experiential criteria it is an adequate idea of extension, then imagination according to Hume's naturalism compels the belief that extension in reality is also at most finitely divisible. This is just what it means for the idea to be adequate, with no disagreements between the idea and its object, where each matches the other, feature for feature. There is no higher standard by which to determine the adequacy of an idea than by careful comparison with its originating sense impressions, since, from the standpoint of a strictly empiricist methodology, there is no more accurate or more reliable information about the object to be obtained than is disclosed by its immediate sense impressions. That is why Hume instructs us to perform the inkspot experiment and the grain of sand thought experiment.

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[70] Berkeley, *Principles*, pp. 53, 80, 105-106; *Three Dialogues*, pp. 232-233. Flage, "Berkeley's Notions", pp. 407-425; *Berkeley's Doctrine of Notions: A Reconstruction Based on his Theory of Meaning*. Lee, "What Berkeley's Notions Are", pp. 19-41.

They provide carefully engineered encounters with relevant occurrent experience in sensation and imagination. The experimental method of reasoning provides the only way of testing the adequacy of our ideas of the divisibility of extension by comparing them with their originating impressions. There is no more that we can justifiably say about the nature of reality than what we can learn through empirical inquiry involving repeatable sense experiences and exercises of imagination. Reality is best known through the study of immediate impressions of sensation and reflection and the ideas to which they give rise. The virtues of Hume's methodology might be defended epistemically, even if, contrary to Hume's philosophically skeptical conclusions, belief is naturally compelled to regard reality as distinct from our most vividly convincing ideas of reality.

The Kantian, despairing of actually knowing the thing-in-itself, in this light looks almost more consistently Humean than Hume. The proposal to judge the adequacy of ideas by comparing them with their originating impressions can appear to rationalist demands for certainty as little more metaphysically satisfying than comparing an idea with itself. Hume's polemical reply in the first *Enquiry* is to commit idle philosophical speculation about extraexperiential reality to the flames. In company with other moderns, Hume despises Scholastic speculative metaphysics, and in the Enlightenment spirit of Newton's natural philosophy, rejects pure reason in favor of 'experimental' empirical methods in philosophy.[71]

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[71] *Enquiry*, p. 165: "When we run over libraries, persuaded of these principles, what havoc must we make? If we take in our hand any volume; of divinity or school metaphysics, for instance; let us ask, *Does it contain any abstract reasoning concerning quantity or number?* No. *Does it contain any experimental reasoning concerning matter of fact and existence?* No. Commit it then to the flames: for it can contain nothing but sophistry and illusion." Noxon, *Hume's Philosophical Development: A Study of His Methods*, pp. 37-54, documents the numerous occurrence of hypotheses in Newton's scientific writings, despite Newton's slogan *Hypotheses non fingo* repeated three times in the *Principia*. See also Newton, *Opticks*, p. 404: "For Hypotheses are not to be regarded in experimental Philosophy." Kuypers, *Studies in the Eighteenth Century Background of Hume's Empiricism*, writes, p. 16: "Even Berkeley, an

Bayle's Trilemma and the Metaphysics of Space

The positive part of Hume's critique of infinity in the inkspot experiment is supposed to prove that the *minima sensibilia* revealed by the inkspot experiment are not only sensible and indivisible, but extensionless. Hume must show that *minima sensibilia* comprise the sensible extensionless indivisible constituents of extension. It is only in this way that Hume can avoid Bayle's trilemma by moving beyond the divisibility limitations of the idea of extension to the divisibility limitations of extension in reality.

Hume seems to characterize if not implicitly define extension as a property an object has just in case it is divisible into at least two spatial parts. An extended body is usually said by Hume and the metaphysicians of his day to be capable of division into left and right halves. But as this presupposes a privileged perspective and spatial orientation, it may be better to interpret extension as implying divisibility and leave it at that. If this partial analysis of the idea of extension is correct, then it is obvious that the empirical discovery of indivisible *minima sensibilia* in sense impressions by the inkspot experiment automatically implies the existence of extensionless components of extension. If an extension is capable of division, then indivisibles are necessarily extensionless.

What is more puzzling is how extensionless units can be put together to constitute extension. Hume does not develop the position as he might, but the idea seems to be that when

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outspoken critic of the conceptions of absolute time and space, the infinite divisibility of matter, and the individual existence of the forces as entities, all of which he attributed to Newton, nevertheless recognized Newton's empirical technique and asserted that the latter's mistaken notions on these matters were irrelevant to his real achievements in natural philosophy." Berkeley, *Philosophical Commentaries*, Notebook B, Comments 372-374: "I see no wit in any of ['the Mathematicians'] but Newton, The rest are meer triflers, meer Nihilarians. The folly of the Mathematicians in not judging of sensations by their senses. Reason was given us for nobler uses. Sir Isaac owns his book could have been demonstrated on the supposition of indivisibles."

two or more indivisible extensionless impressions are brought together, the subject in the inkspot experiment will experience a sense impression of what is phenomenally divisible and hence extended. At an appropriate distance, the inkspots by hypothesis are visible but indivisible, and the slightest retreat causes them to vanish from the phenomenal field. When the subject moves back to just that point where indivisibles disappear, but the real objects that cause the impression are juxtaposed or made slightly to overlap, then at that same distance the spot they conjointly comprise should reappear in the subject's field of vision. The spot will then be restored to a sufficient size to be discriminated as an indivisible and hence extensionless but sensible impression. Finally, returning to the point where the original indivisible was detected should enable the subject to see the juxtaposed indivisibles as an extended object. At this distance, if the two inkspots are again dislocated, no longer laid edge to edge or made to overlap, they must appear again as indivisibles.

This is the sense in which Hume's sensible extensionless indivisibles are supposed to constitute the atomic units of extension. Hume writes:

> . . . my senses convey to me only the impressions of colour'd points, dispos'd in a certain manner. If the eye is sensible of any thing farther, I desire it may be pointed out to me. But if it is impossible to shew any thing farther, we may conclude with certainty, that the idea of extension is nothing but a copy of these colour'd points, and of the manner of their appearance.[72]

An idea of the distribution or arrangement of *minima sensibilia*, tiny colored or tangible points that are at once both extensionless and indivisible, is for Hume nothing other than the idea of extension. The traditional conception in geometry is that extension is infinitely divisible into infinitely many extensionless points in infinitely many line subsegments, so that subdivision of a finite extension in principle can continue endlessly.

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[72] *Treatise*, p. 34.

This is precisely the concept that Hume's theory of *minima sensibilia*, sensible extensionless indivisibles, is meant to contradict. Hume's argument is that if every part of extension is extended, then, since to be extended is to be divisible, extension itself is infinitely divisible into infinitely many line segments, each of which in turn contains infinitely many ideal line subsegments, and so on, infinitely, with each subsegment consisting of infinitely many Euclidean mathematical points. The inkspot experiment invalidates the inference by demonstrating the existence of sensible indivisibles, implying that extension is at least not phenomenally or ideationally infinitely divisible. The inkspot experiment thereby proves that some proper parts of extension, the indivisibles, are extensionless.[73]

The startling part of Hume's inkspot argument is the conclusion that extensionless indivisibles as atomic constituents of the idea of extension also have phenomenal properties, in particular color and tangibility. Hume's extensionless indivisibles are not just *minima*, but *minima sensibilia.* The phenomenal appearance of extensionless indivisibles is indispensable to Hume's efforts to solve Bayle's skeptical trilemma. The classical rationalist concept of extension entails the infinite divisibility of extended things viewed from an analytic perspective as in principle endlessly successive subdivisions. From a synthetic standpoint, on the other hand, infinitism cannot easily explain the emergence or construction of extension out of nonsensible extensionless and infinitely divisible ideal abstract Euclidean mathematical

[73] Berkeley originated the phrase '*minima sensibilia*'. But it is usually supposed that Berkeley's sensible indivisibles unlike Hume's are extended rather than extensionless, and that Hume's essential innovation was to have treated these *minima sensibilia* as extensionless. The same position is adopted by Jesseph, *Berkeley's Philosophy of Mathematics*, p. 57. Arguments against this interpretation and for the claim that Berkeley's *minima sensibilia* are already extensionless rather than extended are made by Raynor, "'*Minima Sensibilia*' in Berkeley and Hume". See also Atherton, *Berkeley's Revolution in Vision.* If Berkeley's *minima sensibilia* are extended, then they cannot be indivisible despite being in some sense finitely extensionally minimal. This strikes me as implausible and possibly unintelligible, so I am inclined to agree with Raynor that Berkeley's *minima sensibilia* are extensionless.

points. The constructive synthesis of extension out of its elements is explained in Hume's theory of extension by the perceivability of sensible extensionless indivisibles as the individual building blocks of spatial extension. Sensible extensionless indivisibles, as opposed to Euclidean points, can be experienced by vision and touch. When juxtaposed in aggregates of two or more they constitute extension in the phenomenal field, like a distantly-viewed pointillist canvas. That Hume's extensionless indivisibles as atomic constituents of extension are sensible, colored and tangible, leads Antony Flew to remark: "Anyone familiar with the theories and paintings of Seurat might also mischievously characterize the Hume of this Section as 'the Father of Pointillisme'..."[74]

Hume's Solution to Aristotle's Contact Problem

As a final refinement, Hume offers a solution to a problem about the possibility of contact between indivisibles. Aristotle argues in *Physics* 231^a29-$^b6$:

> ... if that which is continuous is composed of points, these points must be either *continuous* or *in contact* with one another: and the same reasoning applies in the case of all indivisibles. Now for the reason given above they cannot be continuous; and one thing can be in contact with another only if whole is in contact with whole or part with part or part with whole. But since indivisibles have no parts, they must be in contact with one another as whole with whole. And if they are in contact with one another as whole with whole, they will not be continuous; for that which is continuous has distinct parts, and these parts into which it is divisible are different in this way, i.e. spatially separate.[75]

The objection has obvious implications for Hume's positive theory of sensible extensionless indivisibles as the atomic constituents of extension. Hume's sensible extensionless indivisibles are meant to avoid Bayle's trilemma where Euclidean mathe-

[74] Flew, "Infinite Divisibility in Hume's *Treatise*", p. 265.
[75] Barnes edition.

matical points fail. The advantage of Hume's indivisibles is that, unlike ideal abstract extensionless indivisibles, they are not just so many 'nothingnesses' of extension, in Bayle's phrase, but, since each is sensible even if individually extensionless, they can when appropriately juxtaposed collectively add up to spatial extensions that are phenomenally perceivable.

Aristotle claims that indivisibles cannot touch. But if indivisibles are not in direct continuous contact, they cannot constitute continuous extension. If two indivisibles are touching on a line, then the left side of one indivisible touches the right side of the other. Indivisibles by definition, however, do not have left and right sides, since to do so they would have to be divisible into parts. Without mentioning Aristotle, Hume addresses the problem first as an objection to imperceivable mathematical points. He writes:

> A simple and indivisible atom, that touches another, must necessarily penetrate it; for 'tis impossible it can touch it by its external parts, from the very supposition of its perfect simplicity, which excludes all parts. It must therefore touch it intimately, and in its whole essence, *secundum se, tota, & totaliter*; which is the very definition of penetration. But penetration is impossible: Mathematical points are of consequence equally impossible.[76]

We might think of the penetration of one indivisible by another as something like the absorption of two quicksilver droplets into a single drop. Such an analogy is imperfect insofar as it requires that Hume's indivisibles also have parts, unless their penetration is supposed to result in one indivisible being annihilated by the other.[77] Hume rejects such a possibility,

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[76] *Treatise*, p. 41.

[77] Ibid.: "I answer this objection by substituting a juster idea of penetration. Suppose two bodies containing no void within their circumference, to approach each other, and to unite in such a manner that the body, which results from their union, is no more extended than either of them; 'tis this we must mean when we talk of penetration. But 'tis evident this penetration is nothing but the annihilation of one of these bodies, and the preservation of the other, without our being able to distinguish particularly which is preserv'd and which annihilated. Before the approach we have the idea of two

and with it the theory of mathematical points. He contrasts indivisible mathematical points with his own concept of *minima sensibilia*. He sees no difficulty in the idea of direct contact between sensible extensionless indivisibles, since he finds it imaginable that two sensible extensionless indivisibles might be juxtaposed without having left- and right-hand parts, and without either interpenetrating or destroying one another.

> Taking then penetration in this sense, for the annihilation of one body upon its approach to another, I ask any one, if he sees a necessity, that a colour'd or tangible point shou'd be annihilated upon the approach of another colour'd or tangible point? On the contrary, does he not evidently perceive, that from the union of these points there results an object, which is compounded and divisible, and may be distinguish'd into two parts, of which each preserves its existence distinct and separate, notwithstanding its contiguity to the other? Let him aid his fancy by conceiving these points to be of different colours, the better to prevent their coalition and confusion. A blue and a red point may surely lie contiguous without any penetration or annihilation. For if they cannot, what possibly can become of them? Whether will the red or the blue be annihilated? Or if these colours unite into one, what new colour will they produce by their union?[78]

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bodies. After it we have the idea only of one. 'Tis impossible for the mind to preserve any notion of difference betwixt two bodies of the same nature existing in the same place at the same time."

[78] Ibid. See Thijssen, "David Hume and John Keill and the Structure of Continua", pp. 279: "It is obvious that Hume does not really answer [Aristotle's] 'touching' argument. He merely claims that indivisibles can touch, and in so doing he is no longer considering points as the absolute extensionless entities they really were, but is instead concentrating on their *physical* qualities. Hume himself probably did not feel very easy about his answer because he adds that it is "the natural infirmity and unsteadiness both of our imagination and senses" that chiefly give rise to these objections (as, for example, in the 'touching' argument) and at the same time render it so difficult to give a satisfactory answer to them." But I think it is possible for Hume to answer Thijssen's criticism by observing that there is no inconsistency in the absolute extensionlessness of sensible

The contact problem in Aristotle arises only on the assumption that indivisibles are insensible. Where sensible extensionless indivisibles are concerned, Hume has direct experiential evidence in the inkspot experiment to prove that sensible extensionless indivisibles can be seen to touch one another without disappearing. Since by hypothesis Hume's *minima sensibilia* are extensionless but constitute extension in the aggregate through juxtaposition, without touching sides or parts and without annihilation, they provide a better explanation of the constitution of sensible extension. It is only a matter of suppressing our cultural predispositions about the nature of mathematical points to avoid error in understanding the finite divisibility of extension into sensible extensionless indivisibles. After depositing the above rhetorical questions with the reader, Hume offers this inkspot experiment refrain:

> What chiefly gives rise to these objections, and at the same time renders it so difficult to give a satisfactory answer to them, is the natural infirmity and unsteadiness both of our imagination and senses, when employ'd on such minute objects. Put a spot of ink upon paper, and retire to such a distance, that the spot becomes altogether invisible; you will find, that upon your return and nearer approach the spot first becomes visible by short intervals; and afterwards becomes always visible; and afterwards acquires only a new force in its colouring without augmenting its bulk; and afterwards, when it has encreas'd to such a degree as to be really extended, 'tis still difficult for the imagination to break it into its component parts, because of the uneasiness it finds in the conception of such a minute object as a single point.[79]

The argument against infinite divisibility and in support of sensible extensionless individisibles marks the experiential foundation of Hume's theory of extension. Hume's proof begins with phenomenological introspection of phenomenal data in sense perception. The inkspot experiment provides essential

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extensionless indivisibles and the physical properties by virtue of which they are perceptible.

[79] *Treatise*, pp. 41-42.

evidence for both the negative and positive objectives of the two-fold task of Hume's critique of infinity.

The inkspot experiment, if Hume has correctly interpreted it, proves first that the only adequate idea of extension is finitely divisible into adequate ideas of sensible extensionless indivisibles. Hume's empiricist epistemology and philosophy of mind implies that all such ideas are copies of immediate sense impressions or impressions of reflection. Along with the premise that complex ideas of extension are adequate representations of the most minute parts of extended things, the inkspot argument is supposed to entail that spatial extension is at most finitely divisible into sensible extensionless indivisibles. This falsifies the infinite divisibility thesis in classical mathematics and metaphysics, in the negative part of Hume's two-fold task, and upholds his positive conclusion that space according to a more properly humanized conception of extension is a juxtaposition of finitely many sensible extensionless indivisibles.

The soundness of Hume's argument depends on the truth of his theory that every idea derives from a corresponding impression. The principle highlights the importance of Hume's thesis of sense impressions as the source of ideas in the argument, and establishes the strength of his conclusions within an empiricist framework. To overthrow Hume's critique of infinity and doctrine of extensionless sensible indivisibles, it would be necessary to attack the groundwork of Hume's methodology by undercutting its fundamental assumptions about the origin of ideas in immediate experience. This focuses all critical pressure in the assessment of Hume's finitist theory of extension on a single point, and, in an obvious sense, on the viability of the empiricist enterprise as a whole.

Empiricism and the Experience of Spatial Extension

It may be unnecessary to consider general objections to the experiential epistemological foundations of Hume's empiricism. We may concentrate instead on a single counterargument more particularly concerned with Hume's objections to infinite divisibility and his defense of the thesis of sensible extensionless indivisibles. If Hume's empiricism fails, the inkspot argument, which depends essentially on the premise that the only origins of adequate ideas of extension derive from impressions of finitely extended things, or from such ideas as are mediated by memory, imagination, and reason, is invalidated. Hume's finitist metaphysics of extension, and the finite divisibility of extension into sensible extensionless indivisibles, stands or falls with his empiricist theory of knowledge and philosophy of mind.

The criticism strikes at the heart of Hume's methodology. It questions the claim reconstructed as the assumptions of the inkspot argument to the effect that the limitations of impressions cannot be overcome in corresponding ideas that copy them, unless mediated by memory, imagination, or reason. It is tempting to argue from an information theory standpoint that Hume errs in negatively describing the differences between sense impressions and the ideas from which they derive or to which they give rise as 'limitations' or 'imperfections' that are uncorrectable except by the activity of more powerful mental faculties. The very opposite may be true, if on empirical grounds it can be shown that the cognitive capacity of Hume's perceiver in the inkspot experiment or grain of sand thought experiment needs less information storage capacity or processing space for an idea of the infinite divisibility of an extended object than for a finitely extended object finitely divisible into a particular number of constituents.

Hume seems to have made the contrary assumption. He holds that where impressions of an extended thing are not infinitely divisible, the impressions cannot be copied or converted into corresponding ideas containing greater information about the thing's divisibility into more, let alone infinitely many more, parts than the finite impressions from which the ideas derive. This can be seen to follow as a natural consequence of Hume's thesis that ideas are replicas of their originating impressions. We should not expect to find more information in an idea as a copy of an impression than in the impression itself. As the impression fades into an idea, the information it contains about an extended object's articulation into parts is more likely to blur and become indistinct or altogether lost, rather than sharpen into greater distinctness. If so, then the objection indicates that an idea of divisibility based on sense impressions cannot be upgraded in information content *ex nihilo* to reflect the degree of articulation needed to produce an adequate idea of infinite divisibility. Yet, again, the inference is reasonable only if we assume that altering the content of an idea in this way requires an upgrade rather than a diminution of information.

The defense of Hume's argument now takes a new, more complicated turn. If the empiricist assumption in the inkspot argument wrongly presupposes that the idea of infinite divisibility is more complex or contains more information than the idea or impression of finite divisibility, then there is no reason on this way of looking at things to conclude that Hume is on the right track when he asserts that an adequate idea of infinite divisibility cannot derive from a comparatively less perfect, complete, or informative impression of finite divisibility. The problem is internal to Hume's philosophical system, because it arises within the framework of his characteristically empiricist methodology. The objection thereby questions the truth of Hume's implicit assumption that the idea of infinity and infinite divisibility is more complex or requires more information than an originating sense impression of finite divisibility in order to 'perfect' or 'correct' its 'defects'.[80]

The criticism gains momentum from the observation that an adequate idea of infinite divisibility need not contain more information than an idea or impression of finite divisibility just because infinity is greater in cardinality than any finite number. By some standards of information storage and manipulation in cognitive computational science, it can be simpler and informatively more economical to think of, remember, and perform calculations involving extension without further ado

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[80] See Lachman, Lachman, and Butterfield, *Cognitive Psychology and Information Processing: An Introduction*, esp. pp. 130-182, 298-404. Minsky, *Computation: Finite and Infinite Machines*.

simply as infinitely divisible, rather than trying to remember and process more specialized information about its exact degree of finite divisibility. Consider the problem of trying to recall whether an extended thing is articulated into precisely 102,615 or 102,616 finite subdivisions of sensible extensionless indivisibles. For large numbers it may be less work and require less information to think of extension simply as infinitely divisible, for which any finite number of subdivisions more or less makes no difference to the extension's infinite divisibility. This is precisely the popular nonscientific attitude toward overwhelmingly large numbers, like the number or stars or grains of sand on the beach.[81] Hume need not deny that the information required to upgrade a sense impression's finitely divisible articulation to an idea of infinite divisibility might somehow be externally supplied. Perhaps this could be done by borrowing concepts from other impressions or ideas in the way the mind constructs the complex idea of a centaur, even if there must always be a net increase in information in the transition from originating sense impression to idea. He allows that the perfection or improvement of such an idea in principle can occur, but only by the mind's agency of reason, memory, or imagination.

The objection is mistaken in several ways. Hume first of all does not maintain that an adequate idea of finite divisibility requires the mind to process specific information about the articulation of a finite extension or finitely extended object into a particular finite number of parts. He makes just the opposite requirement when he says of his own sensible extensionless indivisibles that: "... the points, which enter into the composition of any line or surface, whether perceiv'd by the sight or touch, are so minute and so confounded with each other, that 'tis utterly impossible for the mind to compute their number..."[82] An adequate Humean idea of a finitely

[81] See below, Conclusion, 'Hume Against the Mathematicians', esp. note 19.

[82] *Treatise*, p. 45; also pp. 46-47.

divisible extension or finitely extended object unarticulated into particular finite numbers of constituents, finite divisibility as an idea in its own right, need be no more information-laden, and no more a cognitive burden to memory and computation routines, as a result, than the hypothetical idea of infinite divisibility.

The criticism remains misguided, even if correct in its assessment of the relative information requirements of ideas of finite versus infinite divisibility. The problem overlooks the fact that Hume's empiricist philosophy of mind is intended primarily to explain the experiential *origin* of ideas from sense impressions. Only if this is lost sight of is it possible to imagine that ideas and impressions can be compared for information content more or less on a par or in the abstract, without consideration of the conditions that must be satisfied in order for ideas to arise from impressions. Hume's position is that if an idea of infinite divisibility could exist, it would be ontically dependent on and so could not exist without an originating impression with at least as much information about the articulation of extension into subdivisions as the idea. If Hume's inkspot argument is correct, then there can be no adequate idea of infinite divisibility in the first place to enter into any such comparison with originating sense impressions of finite divisibility for purposes of judging differences in their respective information levels.

This does not prevent Hume's critic from comparing the information content of an imagined idea of infinite divisibility with that of its originating sense impression, in order to determine whether on Hume's principles the idea could in principle derive from an impression of only finite divisibility. But unless Hume's theory of the origin of ideas is rejected, the objector must find an alternative satisfactory explanation within the resources of an empirical cognitive information science of how an adequate idea of infinite divisibility could possibly be obtained from finite impressions of finitely divisible extension. The evaluation of information content in an impression of finite divisibility with that of a hypothetically preexistent idea of infinite divisibility blatantly ignores Hume's doubts about how an adequate idea of infinity could possibly occur. As such, the computational objection begs the question against the main point of Hume's critique.

Where would the additional information come from to distinguish an adequate idea of infinite divisibility from an originating impression of finite divisibility? What would be the source of extra information, indeed, infinitely more information, if, as Hume asserts, all ideas are derived from impressions and recombinations of impressions and resulting ideas by the mind's idea-building operations? How, within Hume's humanized experiential epistemology, could the mind possibly arrive at an adequate superhuman idea of infinite divisibility? How could an idea of the infinite divisibility of extension enter into the mind's idea factory from finite impressions of finite quantities and finite subdivisions of extension? If these questions cannot be satisfactorily answered, then there is no reason for Hume to consider seriously the possibility that there could be an idea, let alone an adequate idea, of the infinite divisibility of extension with no greater causally unexplained information content than an originating impression of finite divisibility. Within Hume's system there appears no straightforward solution to any of these problems. The challenge to Hume's thesis about the origin of ideas might then need to shift from internal criticisms that are friendly to some form of Hume's empiricism to hostile external criticisms directed against the most fundamental principles of Hume's theory of knowledge and philosophy of mind.

The obvious and perhaps the only vulnerable point in Hume's inkspot argument is the assumption that ideas derive ultimately from immediate impressions which they copy. If this starting place is correct, the balance of Hume's inference in the inkspot argument appears unassailable. This remark focuses all the pressure in the defense of Hume's conclusions on a single apex of the argument, the assumption that ideas are experiential in origin, and that empiricism is the correct theory of the genealogy of ideas. The difficulties here are of such largescale metaphilosophical proportion in continuing perennial disputes between rationalism and empiricism, apriorism and aposteriorism, that they may admit of no final satisfactory resolution. A discussion of the issues can nevertheless serve to concentrate attention on problems of infinite divisibility in classical mathematics and metaphysics as they arise from the particular standpoint of Hume's attempt to refute infinity on empiricist grounds, clarifying the disagreement between divergent ideological and methodological approaches to metaphysics, philosophy of science, and the theory of mind.

If Hume's theory of perceptions and the origin of ideas is accepted, then his finitist conclusion that extension consists of and is at most finitely divisible into sensible extensionless indivisibles becomes difficult if not impossible to dislodge. The data or interpretation of the data of the inkspot experiment might be questioned, as in Broad's contrary phenomenological criticism. Yet even Broad does not contradict so much as repeat Hume's own observations in his closing remarks about the inkspot experiment. The claim that there are perceptual limitations to the divisibility, size, and grain thresholds for visual and tactile properties in the phenomenal field appears true for empiricist and anti-empiricist alike. The inkspot argument seems to require no more than the admission that vision and touch have lower limits. If this much is granted, Hume's explanation of the origins of adequate ideas is sufficiently empowered to sustain his attack on the infinite divisibility of extension, and his corollary defense of the existence of sensible extensionless indivisibles.

As in many criticisms of well-developed philosophical positions, the most damaging objection to Hume's inkspot argument may be the simplest to state — a flat refusal to accept the argument's underlying presuppositions. The deepest criticism of Hume's position questions whether it is true that an adequate idea of infinite divisibility must depend on originating impressions of infinitely divisible extension. If Hume is not granted this vital assumption, his critique of infinite divisibility and positive doctrine of sensible extensionless indivisibles as the atomic constituents of extension cannot hope to convince anyone so completely out of sympathy with its strong if impure empiricism. Why should Hume's theory of the experiential origins of ideas be accepted? Hume seems to believe both that the account is intrinsically plausible and that there can be no satisfactory alternative. He exhibits his theory of human nature in a favorable light as a natural development from obvious facts about the phenomenology of perception, and the role of ideas in memory, reason, and the imagination. Similarly, he proposes to disprove anti-empiricist alternatives in part by criticizing contrary metaphysical commitments to infinity and infinite divisibility, deriving absurdity, outright contradiction and logical inconsistency, from assumptions that imply the possibility of concepts not constrained by his thesis of the experiential origin of ideas.

The defense of Hume's argument from external criticism requires weighing what Hume regards as an essential supplement to the inkspot experiment, the *reductio* proofs he raises against the concept of infinite divisibility. These inferences are not advanced from the standpoint of Hume's own empiricist assumptions, but involve premises adduced for purposes of criticizing his opponents in the ongoing controversy about infinity and the infinite divisibility of extension in mathematics and metaphysics. It is in these disproofs that Hume most effectively upholds his experiential epistemology and philosophy of mind as the only viable alternative to rationalism, exposing the logical absurdities in what are supposed to be its ideas of infinity and infinite divisibility. Hume exposes the folly of opposing views by introducing infinitary principles for the sake of argument, and showing that ironically their authors commit the unpardonable rationalist sin of logical self-contradiction.

PART TWO

REFUTATIONS OF INFINITE DIVISIBILITY

Having denied the infinite divisibility of extension, our author finds himself obliged to refute those mathematical arguments, which have been adduced for it; and these indeed are the only ones of any weight.

— Attributed to Hume
'Abstract of a Book Lately Published...'

No priestly *dogmas*, invented on purpose to tame and subdue the rebellious reason of mankind, ever shocked common sense more than the doctrine of the infinite divisibility of extension, with its consequences; as they are pompously displayed by all geometricians and metaphysicians, with a kind of triumph and exaltation.

— Hume, *An Enquiry Concerning Human Understanding*
Section XII, Part II

CHAPTER 4

HUME'S *REDUCTIO* ARGUMENTS

Incoherence of Infinite Divisibility

To refute the infinite divisibility of extension, Hume argues not only that a correct epistemology precludes the idea, but that the concept is internally inconsistent. For this purpose, he advances four *reductio* disproofs of infinite divisibility, followed by a dilemma concerning the requirements of exact equality and proportion in geometry.

Hume's battery of *reductio* criticisms of the infinite divisibility thesis are important to his enterprise for three reasons. First, the arguments show that there is no alternative but to replace the concepts of infinity and infinite divisibility with a strict finitist theory of extension. Second, the refutations emphasize the inadequacy of infinitist metaphysics by exposing logical contradictions in some of its most cherished principles about space, time, and number. Third, as *reductio* proofs, the arguments add rhetorical force to Hume's critique of infinity and theory of sensible extensionless indivisibles by showing that the infinite divisibility thesis can be defeated out of its own mouth by what Hume sees as a more judicious application of rationalist assumptions. It is a striking irony that Hume locates confusions amounting to logical inconsistencies in the concepts of infinity and infinite divisibility. For while these are regarded by rationalist thinkers like Descartes as the pride of classical mathematics, on examination they are seen by Hume to be anything but the logically circumspect clear and distinct ideas that Descartes among others believes provide an ideal model for philosophy and the sciences.

The importance of Bayle's trilemma to Hume's critique of infinity explains the relation between Hume's central inkspot argument and the *reductio* proofs. Hume's *reductio* proofs, like the inkspot argument, uphold at least the first part of his twofold objective of defeating infinity and defending the existence of sensible extensionless indivisibles at the foundations of an empiricist theory of extension. But while the *reductio* arguments obviously contribute to the negative task of disproving infinite divisibility, they only partially support the positive task of establishing an alternative theory of sensible extensionless indivisibles. By opposing the possibility of infinite divisibility, they imply the contrary thesis that finite extension is at most finitely divisible into finitely many extensionless indivisibles. The *reductio* arguments thereby uphold the inkspot argument in the negative task of refuting the concept of infinite divisibility in Hume's critique. But they are insufficient in themselves for Hume's positive task of proving that space is a tapestry of *sensible* extensionless indivisibles. To avoid the third prong of Bayle's trilemma, Hume needs the phenomenal evidence of the inkspot experiment as well as the *reductio* disproofs of the concept of infinite divisibility.

Implicit in much of Hume's criticism of infinite divisibility in these arguments is a disjunctive syllogism, in which a fundamental opposition between infinite divisibility and finite divisibility into extensionless indivisibles is presupposed. Hume expresses the opposition in the second paragraph of the *Treatise*, Book I, Part II, Section II, when he maintains that: "Every thing capable of being infinitely divided contains an infinite number of parts; otherwise the division would be stopt short by the indivisible parts, which we should immediately arrive at."[1] The disjunctive syllogism based on the opposition between the infinite divisibility of extension and the concept of extensionless indivisibles, in conjunction with the *reductio* disproofs, has this form:

[1] *Treatise*, p. 29.

Hume's Disjunctive Dilemma

1. Space (extension) is either infinitely divisible or finitely divisible into extensionless indivisibles.
   (condensed version of Bayle's trilemma)
2. But space (extension) is not infinitely divisible.
   (inkspot argument and *reductio* disproofs)

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3. Space (extension) is finitely divisible into extensionless indivisibles. (1,2)

Hume rejects the second horn of Bayle's trilemma, and concentrates on the first and third horns. Bayle's second horn postulates extension in space as divisible into a fabric of physical points. But since physical points are extended, the divisibility of extension into physical points on the assumption of the second horn continues infinitely, as the first horn states. If, on the contrary, physical points are extensionless, then we are driven to Bayle's third horn, that spatial extension is finitely divisible into extensionless ideal mathematical points.

Although Hume does not accept any of Bayle's three alternatives, he comes close to admitting a version of the third trilemma horn. Hume's solution to Bayle's trilemma advances a fourth possibility that Bayle does not consider. He modifies Bayle's third horn in a theory of extension as finitely divisible into sensible extensionless indivisibles. The finite divisibility of extension into extensionless indivisibles in Hume's theory avoids the absurdities Bayle attributes to the infinite divisibility model, as reflected in Zeno's paradoxes. By allowing the extensionless indivisibles to be sensible rather than ideal, abstract, or mathematical, on the other hand, enables Hume to account for the constitution of perceivable extension as an arrangement of *minima sensibilia*. In imputing the above implicit disjunctive dilemma to Hume, it has accordingly been appropriate to characterize extensionless indivisibles generically rather than specifically as ideal or abstract mathematical points. The opposition between infinite divisibility and divisibility into extensionless indivisibles leaves open as alternatives that the extensionless indivisibles in question might either be sensible or insensible. Hume agrees with Bayle that extension cannot be built up out of the insensible 'nothingnesses' of extension required by classical infinitary mathematics. The unspecified metaphysical status of extensionless indivisibles in Hume's background reasoning makes it possible for Hume to solve Bayle's trilemma by maintaining that space is composed of sensible extensionless indivisibles, rather than by Euclidean points that are not perceivable in any quantity or any distribution or arrangement.

The *reductio* arguments in Hume's critique help prove the existence of extensionless indivisibles. But only Hume's inkspot experiment establishes extensionless indivisibles more particularly as sensible, phenomenal, or experienceable, rather than ideal, abstract, or mathematical. If the disjunctive syllogism based on an opposition between infinite divisibility and divisibility into extensionless indivisibles is implicit in Hume's reasoning, then his *reductio* arguments and geometry dilemma can be understood not only in isolation as negative disproofs of infinite divisibility, but also as supporting Hume's positive doctrine of spatial extension as finitely divisible more specifically into sensible extensionless indivisibles.

Argument from the Addition of Infinite Parts

The idea of Hume's first *reductio* proof is that if a finitely extended body is infinitely divisible, then it must be possible to add its infinitely many parts one to another so as to constitute an infinitely extended body. Thus, every finitely extended body, if infinitely divisible, is infinitely extended. The contradiction is obvious, but the exact route to inconsistency in Hume's argument has been the subject of dispute.

The inference, as we have seen, is a version of Bayle's refutation of infinite divisibility in the first horn of his trilemma. Hume's formulation appears in the *Treatise*, Book I, Part II, at the beginning of Section II, *Of the infinite divisibility of space and time.* Hume assumes for purposes of indirect proof that a finitely extended thing is infinitely divisible, and rejects the assumption after deducing the contradiction that whatever is finitely extended is also infinitely extended. He begins by observing that:

> Every thing capable of being infinitely divided contains an infinite number of parts; otherwise the division would be stopt short by the indivisible parts, which we should immediately arrive at... But that this latter supposition is absurd I easily convince myself by the consideration of my clear ideas. I first take the least idea I can form of a part of extension, and being certain that there is nothing more minute than this idea, I conclude, that whatever I discover by its means must be a real quality of extension. I then repeat this idea once, twice, thrice, &c. and find the compound idea of extension, arising from its repetition, always to augment, and become double, triple, quadruple, &c. till at last it swells up to a considerable bulk, greater or smaller, in proportion as I repeat more or less the same idea. When I stop in the addition of parts, the idea of extension ceases to augment; and were I to carry on the addition *in infinitum*, I clearly perceive, that the idea of extension must also become infinite. Upon the whole, I conclude, that the idea of an infinite number of parts is individually the same idea with that of an infinite extension; that no finite extension is capable of containing an infinite number of parts; and consequently that no finite extension is infinitely divisible.[2]

The argument is a standard *reductio ad absurdum*. The assumption is that a finitely extended thing may be infinitely divisible or divisible into infinitely many parts, and the absurdity to which the assumption is reduced is that a finitely extended thing is then also infinitely extended. The contradiction reflects back on falsehood of the assumption, permitting Hume to conclude that a finitely extended thing can only be finitely divisible. The *reductio* assumption is indifferent with respect to whether the infinitely many parts into which a finite extension is supposed to be divisible are themselves supposed to be indivisible or extended. The argument is explicitly reconstructed as follows:

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[2] Ibid.

Addition of Infinite Parts

1. Suppose for purposes of indirect proof that some finitely extended thing is infinitely divisible into infinitely many (indivisible or extended) parts.
2. If a finitely extended thing is infinitely divisible into infinitely many (indivisible or extended) parts, then these parts could be added one to another *ad infinitum* to constitute a body of infinite extension.

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3. A finitely extended thing (despite appearances) is infinitely extended, contrary to the first assumption. (1,2)
4. Therefore, no finitely extended thing is infinitely divisible into infinitely many (indivisible or extended) parts. (1,3)
5. Therefore, space (extension) is not infinitely divisible. (4)

The first conclusion in (3) follows directly from (1) and (2) by detachment, *modus ponendo ponens.* Proposition (4) completes the *reductio* by withdrawing the hypothesis in assumption (1) because of the deduction of the contradiction in (3). The final conclusion in (5) follows from (4) on the supposition that if no finite extension is infinitely divisible, then space or extension itself is not infinitely divisible.

Hume claims, reasonably enough, that if an extended body is infinitely divisible, then it must contain an infinite number of discrete parts. If it does not, then the division of the body's extension must terminate at some definite point marked by the finite number of parts into which it can be divided. What follows next is perhaps the most controversial aspect of the proof. Hume asserts that the parts into which an extended body can be infinitely divided must themselves have at least some extension ('quality of extension'). If this is true, and if there are infinitely many subdivisible parts obtainable from the infinite divisibility of a finitely extended body, then, Hume argues, it must be possible in principle to 'add' these parts successively one to another so as to constitute an infinite extension. The disproof makes commonsense application of the principle that if there are infinitely many subdivided parts within a finitely extended object, then combining them all or adding them together must finally constitute an infinite extension. But it is a contradiction for a finitely extended object also to be infinitely extended. The hypothesis that a finite extension is infinitely divisible must therefore be mistaken.

The picture Hume's objection invokes is of a finitely extended object, such as a penny, containing so many parts that juxtaposing one with another would fill every corner of supposedly infinitely extended three-dimensional space. This need not crowd out or leave no room left over for other finitely extended objects in more remote parts of space, because any number of finite and even infinite quantities can be added to and absorbed by an infinity without increasing its cardinality. As we shall see, the conclusion creates difficulties for the occupation of finite bounded parts of space by adjacent finite extensions contained within a finite extension. There is no room in that case for the infinite space required by each finitely extended component of the finitely extended portion of space in which they are supposed to be housed.

Hume's statement of the argument commits him to assuming that the infinitely divisible components of finite extensions are themselves extended. This is the traditional infinitist assumption of Euclidean geometry, by which there is supposed to be at least one point between any two points. If the assumption is true, then there are infinitely many subsegments, and hence infinitely many extended components, on both sides of any subdivided extension. Hume considers the view here only to refute it. Yet even so, the supposition is not really needed for his argument. The same conclusion holds, provided that two indivisibles in juxtaposition constitute a complex entity that is extended by virtue of being spatially divisible. Accordingly, the argument is reconstructed to allow that finite extensions by hypothesis may either be divided into infinitely many extended or indivisible components. There will then equally result an infinite extension from the infinite divisibility of finite extensions into indivisible or extensionless parts, provided that two parts added or juxtaposed together constitute an extension, and that an infinite quantity divided or multiplied by two remains infinite.

The argument from the addition of infinite parts has provoked the most criticism on traditional mathematical grounds. Flew argues that Hume confuses the Aristotelian distinction between actual and potential divisibility, and repeats the objection of many that Hume fails to understand the concept of a limit, while conflating infinite divisibility or endless subdivision with division into an infinite number of components.[3] Flew protests that infinity is not a number. He accuses Hume of supposing that the infinite number of parts an infinitely divisible extension contains is a positive definite quantity of things to be added together, rather than the unlimitedness of possible subdivisions. Flew contends that:

> The second of Hume's fundamentals, though allegedly obvious, is in fact false. He says: 'whatever is capable of being divided *in infinitum*, must consist in an infinite number of parts...' (T 26). This, though it struck him, and has struck many others, as a self-evident truth, is mistaken twice over. First, and less importantly, to say that something is divisible into so many parts is not to say that it consists of — that it is, so to speak, already divided into — that number of parts. A cake may be divisible into many different numbers of equal slices without its thereby consisting in, through already having been divided into, any particular number of such slices. Second, and absolutely crucially, to say that something may be divided *in infinitum* is not to say that it can be divided into an infinite number of parts. It is rather to say that it can be divided, and sub-divided, and sub-sub-divided as often as anyone wishes: infinitely, without limit. That this is so is part of what is meant by the saying: "Infinity is not a number!"[4]

Let us reserve judgment for the moment about whether or not Hume shares the modern concept of infinity. The mathematicians against whom Hume's *reductio* is directed are

[3] Flew, "Infinite Divisibility in Hume's *Treatise*", p. 260.

[4] Ibid., pp. 259-260.

committed to all that seems to be required for the objection. They would surely accept the proposition that every extension (or the measure of every finitely extended body in space) can in principle be divided into an infinite number of ideal abstract Euclidean subsegments (or point-sets).[5] If extension is infinitely divisible, then these subsegments must themselves be infinitely divisible. In that case, each of the infinitely many subsegments into which an extension is infinitely divisible must themselves be extended, which is to say infinitely continuously divisible. But infinitely many extended things in immediate juxtaposition must occupy an infinite extension, even if they appear confined to the space of a penny.

This does not require as Flew holds that infinity itself be regarded as a number. Nor, in contemporary terminology, need it confuse ordinal with cardinal numbers, as the (first) ordinal of infinity with the sequence of cardinal numbers, 1, 2, 3, ... The parts into which an extension is to be divided are such that a finite spatial extension according to the infinite divisibility thesis is not just potentially but actually divided into or constituted by infinitely many parts. This is true provided we understand divisibility from the traditional infinitist mathematician's standpoint as an abstract condition rather than an activity in real time, as Flew's objection seems to require. Baxter provides the most appropriate interpretation of the 'parts' of extension to which Hume's argument refers, when he describes them abstractly as the components of a region of space.[6] If the parts into which a finite extension is divisible are thought of in this way, then it seems appropriate

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[5] Ibid. Flew's criticisms are expanded in *David Hume*, pp. 38-43. Newman, "Hume on Space and Geometry", pp. 1-31; Flew, "'Hume on Space and Geometry': One Reservation", pp. 62-65; Newman, "Hume on Space and Geometry: A Rejoinder to Flew's 'One Reservation'", *Hume Studies*, 8, 1982, pp. 66-69. *Enquiry*, p. 156-158.

[6] Baxter, "Hume on Infinite Divisibility", p. 133: "The parts of a region of space could be called locations, but I will usually use the term 'part' as Hume does. This will serve as a reminder that metaphysical principles applicable to extended objects are to be applicable to regions of space."

to regard finite extension on the infinitist thesis as consisting of infinitely many extended parts, each of which in turn consists of infinitely many extended parts, and so on, infinitely. Yet the infinite subdivision of finite extension is abstract; so it is not supposed to occur in the way in which a concrete physical object like a cake is actually divided into a certain number of slices. If anyone seriously supposed that a cake could be divided into infinitely many integral physical parts (as opposed to spatial parts, regions or parts of regions of space occupied by parts of the cake), then in Flew's terms the cake counterfactually would 'already' have to be infinitely divided or constituted by infinite numbers of infinitely subdivisible extended parts before a knife ever touches the frosting.

The second part of Flew's objection is that divisibility *in infinitum* does not imply divisibility into an infinite number of parts. This is also implausible, and the point has nothing directly to do with the observation that infinity is not a number. Flew understands the infinite divisibility of space as the property of being divided 'as often as anyone wishes'. But this is certainly inadequate as a concept of infinity. The wishes of finite beings in dividing physical things in real finite time cannot approximate the infinite divisibility of extension in the abstract sense to which traditional mathematics is committed. The added clause that these wishes may extend 'without limit' also falls short of infinity, since that description applies as well to indefinite, indeterminate, or inexhaustible, but still finite moments of time or wish-instances, yielding at most indefinite, indeterminate, or inexhaustible but still finite sets and series of mathematical objects. The mathematicians against whom Hume directs the *reductio* proof from the addition of infinite parts would doubtless be dissatisfied with a concept of divisibility of every finitely extended part into anything less than infinitely many continuously finitely extended parts.[7]

[7] Flew, "Infinite Divisibility in Hume's *Treatise*", p. 260: "The contradictions and absurdities, whether real or only apparent, which make the doctrine of infinite divisibility scandalous to Hume spring from precisely

Robert Fogelin raises a somewhat different complaint about what he takes to be Hume's misunderstanding of the mathematical concept of a limit. "It is true," he writes, "that if we take a finite extension (however small) and repeat it *ad infinitum*, we will get an infinite extension. That, however, is quite beside the point, because the proof of infinite divisibility depends upon the possibility of constructing ever smaller finite extensions, as in the sequence [1/2, 1/4, 1/8, etc.] whose sum approaches, but does not exceed, 1."[8] Fogelin's observation recalls the suggestion sometimes made about a method for counting out an infinite string of whole numbers or positive integers in a finite amount of time, even in the space of a single second. The procedure is to count the first number in the first 1/2 second, the second number in the next 1/4 second, the third number in next 1/8 second, the fourth in the next 1/16 second, and so on, in general, the *n*th number in the $1/2n$ second, to infinity.

Hume anticipates and answers this kind of objection in an important note to the text, in which he considers a critic's distinction between *proportional* and *aliquot* fractions. This is the same concept required for Fogelin's method in an imaginary successive subdivision to the limit of infinity. Hume writes:

> It has been objected to me, that infinite divisibility supposes only an infinite number of *proportional* not of *aliquot* parts, and that an infinite number of proportional parts does not form an infinite extension. But this distinction is entirely frivolous.

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this proposition 'that whatever is capable of being divided *in infinitum*, must consist in an infinite number of parts'... Hume, therefore, proposes to start from two fundamental principles, one of which is the very misconception which generates the paradoxes he wishes to remove." This, in one sense, is obviously true. But the 'obnoxious doctrine' to which Flew also refers, is not Hume's, but the assumption of those mathematicians against whom his first *reductio* is directed. Flew also seems to overlook the *reductio* purposes of Hume's discussion of Malezieu's rationalist proof from the unity of existents.

[8] Fogelin, "Hume and Berkeley on the Proofs of Infinite Divisibility", p. 51. Note that the modern concept of limits permits limit values not only to be 'approached' without actual contact, but attained (though obviously not surpassed). Thus, the limit 0 is reached, for example, for certain values of x, in $\lim_{x\to 0} \sin(x)$.

> Whether these parts be call'd *aliquot* or *proportional*, they cannot be inferior to those minute parts we conceive; and therefore cannot form a less extension by their conjunction.[9]

The reply, as Fogelin observes, seems to depend on Hume's prior claim that our ideas are 'adequate representations' of the most minute parts of extension. This is a major premise in Hume's inkspot argument, linking the limitations of ideas of extension to the properties of extension in reality. If the least but still conceivable parts of extension are extended or indivisible, then, if they are infinite in number, taken together they must presumably constitute or add up to an infinite extension, regardless of whether they are exactly equal (*aliquot*) or relatively proportional in size.[10]

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[9] *Treatise*, p. 30, n. 1. Hume's distinction between *proportional* and *aliquot* parts derives from Bayle's *Systême de philosophie*, *Oeuvres diverses*, IV, pp. 292-293. See Thijssen, "David Hume and John Keill and the Structure of Continua", p. 280: "Bayle himself makes it quite clear that he is explaining a very common distinction which actually goes back to the fourteenth century, although the terminology may have changed slightly over the years. Other contemporary authors who used this distinction include Walter Charleton, Isaac Barrow, and Newton."

[10] See Frasca-Spada, *Space and the Self in Hume's Treatise*, pp. 15, 33-38. Frasca-Spada writes, pp. 42-43: "Why does Hume reject this distinction [between aliquot and proportionate parts] as frivolous? If one calls an indivisible point an aliquot part, Hume's treatment of, say, the grain of sand can be transposed into the following terms: we can conceive as many divisions and subdivisions of the grain of sand, as we like; each of these divisions will uncover proportional parts, whose notion depends, on the one hand, on a relation, and, on the other, on the conception of some aliquot part. Aliquot parts would thus have the role of affording the imaginal content of proportional parts. Their conceptual inevitability would express the limitation of human mind in the most basic, and not unattractive, phenomenalistic terms: whatever process of division we perform, we shall as a matter of fact stop somewhere. From this point of view, one may safely suppose that Hume calls the distinction between proportional and aliquot parts frivolous because the two kinds of parts cannot be talked about separately; they always go together, as correlatives, in any conceivable process of division." I doubt, however, that Hume would regard a distinction as frivolous only on the grounds that the objects of the distinction are always

There is a dilemma implicit in Hume's reply to the objection. Hume need not limit the argument to exposing absurd consequences in the infinite divisibility of finite extension into infinitely many extended parts. As indicated above, Hume can draw the same conclusions from the hypothesis that finite extension is divisible into infinitely many extensionless indivisible parts. The dilemma builds on the assumption that the least conceivable parts of finite extension are either extended or indivisible. If the parts are extended, then, even if they are proportional, their addition must result in an infinite extension. If they are not extended but indivisible, then their juxtaposition in groupings of two or more will constitute a spatial extension; for in that case they will be divisible into at least two parts. If there are infinitely many indivisibles, and if two adjacent indivisibles are sufficient to constitute an extension (leaving aside the objection of Aristotle's 'touching argument' against the possibility of contact between indivisibles, which has already been addressed in connection with Hume's inkspot argument), then, since an infinite quantity divided by two is still infinite, it follows that a juxtaposition of infinitely many indivisible least conceivable parts of any finite extension necessarily constitutes an infinite extension.

Fogelin argues that Hume's proof fails unless he can justify the claim that we have an adequate idea of indivisibles as the ultimate parts of extension, or can at least offer a better explanation of what he means by the concept of an adequate idea.[11] An attempt to clarify Hume's position in unpacking his criteria for the adequacy of ideas by comparison more directly

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found together. In that case, for example, there would be no justification for distinguishing between a naturally functioning heart and a naturally functioning brain, since we never encounter one without the other. I offer an alternative explanation of Hume's rejection of the distinction in my exposition of Hume's refutation of infinite divisibility from the addition of infinite parts. Also, Frasca-Spada, "Reality and the Coloured Points in Hume's *Treatise*, Part 2: Reality".

[11] Fogelin, "Hume and Berkeley on the Proofs of Infinite Divisibility", p. 54.

with their objects via originating impressions was made in the defense of the same assumption as it appears in Hume's inkspot argument. Fogelin's objection leaves open the possibility that Hume's proof might be acceptable if his theory of an adequate idea could be satisfactorily explained. Yet Hume's concept of an adequate idea of the indivisible or least conceivable parts of extension has already been described as essential to the inkspot argument. The constituents of extension for Hume are the *minima sensibilia* revealed at the threshold of experienceability, in impressions that support the adequate idea of a point that cannot be further subdivided into separate parts without vanishing from sight or touch. There is no positive adequate idea of anything smaller or less extended than this, Hume maintains, so that the idea of such a point just is the adequate idea of an indivisible or least conceivable part of extension.[12]

It is interesting that, although Hume offers this proof as a *reductio*, he combines his opponents' assumptions with his own thesis about the adequacy limitations of the idea of extension. There is nothing objectionable in principle about this strategy. Hume and his opponents in the infinite divisibility controversy might share many premises while disputing others. Hume has no obligation in advancing his objection against infinite divisibility to make use only of those assumptions he has reason to believe infinite divisibility theorists are likely to accept. It is enough for his purposes that the main assumption of the indirect proof is characteristically accepted by advocates of the infinite divisibility thesis.

The proof is far removed from Hume's analysis of extension. Yet the account of spatial divisibility that emerges hypothetically in this first *reductio* argument is internally consistent and does not trivialize Hume's critique of infinity. Consider that on Hume's theory finite extension is supposed to be composed of finitely many sensible extensionless indivisibles. Contrary to the assumption of infinite divisibility that drives the counterargument, these ultimate constituents of spatial extension cannot

[12] *Treatise*, pp. 38-39.

be further subdivided, but as *minima sensibilia* are also *minima divisibilia.* This is the sense in which the fundamental units of Hume's theory of extension are themselves extensionless, indivisible, and in which their corresponding ideas, derived as Hume believes from sense impressions of finite extension and impressions of reflection in acts of imagination, are supposed to be adequate. If finite extension is infinitely divisible, as the argument assumes for purposes of indirect proof, then it follows, as every gradeschool geometry student is taught, that any subdivision of finite space has extension and is infinitely further subdivisible into infinitely many extended line subsegments, each of which in turn is infinitely divisible into infinitely many Euclidean points. Hume reasons hypothetically from the infinitist's assumption, contrary to his own beliefs, that if a finitely extended thing is infinitely divisible, then there must be infinitely many parts; for otherwise, as he maintains, '... the division would be stopt short by the indivisible parts, which we should immediately arrive at...'

In the argument from the addition of infinite parts, Hume intends not only to cast doubt on the infinite divisibility thesis, but to support his positive strict finitist doctrine of sensible extensionless indivisibles. The proof shows that affirming infinite divisibility and thereby denying the existence of extensionless indivisibles (sensible or not) leads to outright logical contradiction.[13]

Malezieu's Argument from the Unity of Existents

Hume attributes his second *reductio* argument to Nicholas de Malezieu. The inference trades on the assumption that only unitary things can exist, and purports to show that an existent extended object necessarily lacks unity if it is infinitely divisible, and hence paradoxically by the assumption both exists and does not exist. The contradiction in principle might reflect back either on the falsehood of the unity of existents assumption, or on the assumption that an existent extended object is infinitely

[13] Fogelin, *Hume's Skepticism in the Treatise of Human Nature*, pp. 27-32.

divisible. Hume, predictably in this context, upholds the unity thesis and casts blame for the inconsistency on the premise that existent extended objects are infinitely divisible.

The argument occurs immediately after the first *reductio* from the addition of infinite parts. Hume introduces the proof in this way: "I may subjoin another argument propos'd by a noted author..."[14] The absurdity to which the assumption of infinite divisibility is supposed to be reduced is formulated as the proposition that extension itself, the extended object as described by the hypothesis of the argument, is both unitary and infinitely divisible. The proof is compactly and rather abstractly stated by Hume in the following passage:

> 'Tis evident, that existence in itself belongs only to unity, and is never applicable to number, but on account of the unites [*sic*], of which the number is compos'd... 'Tis therefore utterly absurd to suppose any number to exist, and yet deny the existence of unites; and as extension is always a number, according to the common sentiment of metaphysicians, and never resolves itself into any unite or indivisible quantity, it follows, that extension can never at all exist.[15]

Taking into account the essential features of Malezieu's unity of existents assumption, and rounding out the demonstration in more conventional *reductio* style, the reconstruction of Hume's second argument against infinite divisibility has this structure:

Unity of Existents

1. Only unitary things exist.
2. Suppose for purposes of indirect proof that extended things are infinitely divisible.
3. If extended things are infinitely divisible, then necessarily they lack unity.
4. Extended things exist.

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5. Extended things exist and do not exist. (1,2,3,4)

[14] *Treatise*, p. 30. Hume's note 2 reads simply: "Mons. *Malezieu*".
[15] Ibid.

6. Therefore, no extended things are infinitely divisible. (2,5)
7. Therefore, space (extension) is not infinitely divisible. (6)

The deductive structure of Hume's second *reductio* argument, like the first, is relatively straightforward. Malezieu's rationalist assumption about the unity of existents occurs in assumption (1). The infinite divisibility hypothesis made for purposes of indirect proof appears in (2). The existence of extended things is explicitly assumed in assumption (4). The assumption that infinitely divisible extended objects necessarily lack unity in proposition (3) requires further justification.

If all four assumptions are accepted, then the contradiction in (5) follows from the uncontested premise in (4) that extended things exist, and the conclusion of the inference that extended things do not exist, from (1), (2), and (3). These are, respectively, Malezieu's principle that only unitary things exist, the hypothesis that extended things are infinitely divisible, and the premise that if extended things are infinitely divisible, then necessarily they lack unity. Propositions (2) and (3) imply that extended things necessarily lack unity, which, together with (1), entails that extended things do not exist. Conclusion (6) states that no extended things are infinitely divisible, in which assumption (2) is blamed for the contradiction in (5). This, as in Hume's first *reductio* from the addition of infinite parts, supports the final result in (7), that space, extension generally, is not infinitely divisible.

Hume regards the argument as 'very strong and beautiful'.[16] But some of his detractors have seen it as a specious bit of sophism. Flew notes that the argument is 'an oddity':

> ... incongruous with both the general temper and the stated object of [Hume's] 'attempt to introduce the experimental method of reasoning into moral subjects,' and so reminiscent of the sort of positive natural theology for which Hume had least respect, that he surely ought to have asked himself whether its

[16] Ibid.

> aptness here is not an indication that something is going badly wrong; which it is.[17]

The incongruities, if that is what they are, are easily explained on the interpretation according to which Hume's argument is intended as a *reductio* of assumptions and methods he does not really share. If that is what Flew finds odd, then it is odd that he does not say the same about all four of Hume's other *reductio* arguments. Flew is right to observe that Hume's argument is an oddity, however, if by this he means that the inference is more obscure and difficult to interpret than Hume's other proofs.

The proof is meant to show that not even a rationalist should accept the infinite divisibility of extension from within the system of principles supposedly known to pure reason, while the remark that the argument is strong and beautiful might be understood as an instance of Hume's taunting irony.[18] The assumptions that only unitary things exist, and that an extension if infinitely divisible lacks unity, demand more careful explanation. It is unclear first in what sense by 'the common sentiment of metaphysicians' extension is supposed to be a number that 'never resolves itself into any unite [*sic*] or indivisible quantity'. The reconstruction imposes an intelligible order on Hume's statement of the argument, but its faithfulness to the original text is less confident here than in the case of Hume's other *reductio* arguments. Yet if Hume does not intend something like the reconstructed *reductio* inference (which, however, agrees in essentials with interpretations offered by other commentators, including Flew), it is hard to know what he might have meant instead.

An obvious if indecisive objection to Hume's adaptation of Malezieu's *reductio* must be resolved if the argument is to

[17] Flew, "Infinite Divisibility in Hume's *Treatise*", p. 264.

[18] The possibility is offered with appropriate caution, since I share Norton's sentiment, p. 126, n. 31, that: "... since I lack that intuitive grasp of irony so marked among some of Hume's commentators, I am unable to say with authority that Hume is here [second *Enquiry*, p. 294] being only ironical."

have any plausibility. It appears at first as though the unity of any extended object required on the present assumption as a condition of its existence would be threatened by any level or degree of divisibility, and not just specifically by infinite divisibility. What is supposed to be special about the infinite divisibility of extension that is incompatible with its unity as an existent entity? Suppose that extension is only finitely divisible. Does it not equally follow that a finitely divisible extension is not a unity, and hence not existent, because it consists of at least two parts? Hume evidently means something different than this when he argues that 'existence in itself belongs only to unity', and he perceives a different challenge to the unity of an extended thing than the mere possibility of its division into parts. The answers to these questions will shed light on what Hume understands by a unity, and why the infinite divisibility of extension is supposed to be incompatible with the unity and existence of an extended thing.

It would help if Hume had named his specific dialectical opponents among the 'metaphysicians' who are said to believe that extension is always a number, to aid in determining whether and in what sense those under attack might accept this strange proposition.[19] Perhaps no more is meant than that these thinkers treat extension as equivalent to the number obtained in the measurement of an extended body, the metric of extension, rather than an identification of extension with number in neo-Pythagorean fashion. Hume regards it as unsatisfactory to claim that a finitely extended body is itself a unity, because he says that in that same fictional sense any collection of things, twenty random men or the universe itself, might also be considered a unity. He anticipates the objection:

> 'Tis in vain to reply, that any determinate quantity of extension is an unite; but such-a-one as admits of an infinite number of fractions, and is inexhaustible in its sub-divisions. For by the same rule these twenty men *may be consider'd as an unite.*

[19] Hume may specifically have in mind the Cartesian analytical geometers, or the English algebraist Wallis and geometer Barrow.

> The whole globe of the earth, nay the whole universe *may be consider'd as an unite.* That term of unity is merely a fictitious denomination, which the mind may apply to any quantity of objects it collects together; nor can such an unity any more exist alone than number can, as being in reality a true number. But the unity, which can exist alone, and whose existence is necessary to that of all number, is of another kind, and must be perfectly indivisible, and incapable of being resolved into any lesser unity.[20]

The point is that the unity requirement can always be trivially satisfied, so that in the end it comes to nothing, and cannot possibly be used to decide metaphysical questions. This merely reasserts Hume's conclusion; so, on pain of circularity, it may be necessary to interpose additional assumptions.

The infinitely divisible components of finitely extended bodies cannot be said to exist, because, as infinitely successively subdivisible, they do not constitute any definite quantity. The threat to the unity of an extension is not merely that it can be divided, since extension by definition is spatially divisible. The problem is rather that the infinite divisibility of extension is incompatible with its unity, on the grounds that the infinite divisibility of an extension necessarily deprives its subdivisions of any definite magnitude. To restate Hume's conclusion on the present interpretation, there are no units of extension, and therefore no unity, if extension is infinitely divisible. If this is what Hume means by saying that the number metaphysicians equate with extension 'never resolves itself into any unite or indivisible quantity', then perhaps he agrees with Flew in affirming the conception by which infinity is not a number.[21] A circularity nevertheless threatens if Hume's strategy is simply to assume that a set or series with infinite cardinality cannot be definite in quantity or number.

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[20] *Treatise*, pp. 30-31.

[21] Flew, "Infinite Divisibility in Hume's *Treatise*", p. 260. See the discussion of this point above in Chapter 4, *Argument from the Addition of Infinite Parts.*

Hume's argument contradicts contemporary set theory and philosophy of mathematics. These modern conceptions find it perfectly acceptable to treat infinite and even transfinite sets as unitary objects, in something more than the trivial sense in which unity can be imposed on any 'random' collection. Hume's recourse is to refuse appeals to the set theory unity conditions for infinite sets as failing to involve genuine or adequate ideas. He might object that any such defense of infinite divisibility begs the question in criticizing his argument by assuming that there is an adequate set theory idea of unity for infinite sets. Hume can hold fast to the thesis that in order to have an adequate idea of an infinite set as a unity we must *per impossibile* obtain the idea from an originating impression of sensation or reflection. Hume's reasons for rejecting these possibilities have already been considered in discussing the inkspot argument.[22]

Finite Divisibility of Time

Hume's third argument against the infinite divisibility thesis appears on the heels of the Malezieu unity of existents proof. This is a temporal counterpart of the first argument from the addition of infinite parts, involving at least two finitely extended but infinitely divisible objects. Hume introduces the proof as analogous to his previous discussion: "All this reasoning [in the Malezieu argument] takes place with regard to time; along with an additional argument, which it may be proper to take notice of."[23]

Hume asserts by appeal to what he calls 'the nature of motion' that if space or extension were infinitely divisible, then time would be too. He proceeds by parallel argument from the addition of infinitely many moments in the hypothetical infinite

[22] If extension is infinitely divisible, then there cannot exist extensionless indivisibles, because their existence would signify a limit beyond which divisibility could not continue. If there are indivisibles, then for the same reason extension cannot be infinitely divisible, but must reach a finite terminus where divisibility comes to a definite end.

[23] *Treatise*, p. 31.

division of finite time intervals to demonstrate that duration is not infinitely divisible, and concludes that therefore space also cannot be infinitely divisible. He formulates the argument in this way:

> 'Tis a property inseparable from time, and which in a manner constitutes its essence, that each of its parts succeeds another, and that none of them, however contiguous, can ever be co-existent. For the same reason, that the year 1737. cannot concur with the present year 1738. every moment must be distinct from, and posterior or antecedent to another. 'Tis certain then, that time, as it exists, must be compos'd of indivisible moments. For if in time we could never arrive at an end of division, and if each moment, as it succeeds another, were not perfectly single and indivisible, there would be an infinite number of coexistent moments, or parts of time; which I believe will be allow'd an arrant contradiction. The infinite divisibility of space implies that of time, as is evident from the nature of motion. If the latter, therefore, the former must be equally so.[24]

The nature of motion, according to Hume, requires that the infinite divisibility of space entails the infinite divisibility of time. The remaining assumptions are that precisely coexistent moments cannot exist, which Hume in Leibnizian terminology seems to regard as a matter of the (analytic) relation of ideas, together with the hypothesis of the proof that time is infinitely divisible. The proof is this:

Divisibility of Time

1. The nature of motion proves that the infinite divisibility of space implies the infinite divisibility of time.
2. If time were infinitely divisible, then there would be an infinite number of precisely coexistent distinct moments.
3. There cannot be any precisely coexistent distinct moments of time.
4. Suppose for purposes of indirect proof that time is infinitely divisible.

[24] Ibid.

5. There is an infinite number of precisely coexistent distinct moments of time. (2,4)
6. Therefore, time is not infinitely divisible, but divisible at most into finitely many indivisible moments. (3,5)
7. Therefore, space (extension) is not infinitely divisible.(1,6)

The argument differs in structure and content from Hume's previous two *reductio* refutations. The hypothesis that extension is infinitely divisible is not introduced for purposes of indirect proof. Instead, the assumption in (4) that time is infinitely divisible is made the basis for the contradiction in (3) and (5). Along with the assumption in (1) that the infinite divisibility of space implies that of time, the conclusion in (7) that extension is not infinitely divisible follows from the preliminary conclusion in (6) that time is not infinitely divisible. Conclusion (6) is deduced from the intuitively justified assumption in (3) that there cannot exist distinct but precisely coexistent moments, and the preliminary conclusion in (5) that there is an infinite number of precisely coexistent distinct moments, which derives by simple detachment from assumptions (2) and (4). These are the premises, respectively, that if time were infinitely divisible, then there would be an infinite number of precisely coexistent distinct moments, and the hypothesis that time is infinitely divisible.

The analogy between space and time which Hume takes for granted in this proof is criticized by Flew. Flew claims to have found an elementary logical fallacy in Hume's inference: "The first of the two parenthetical paragraphs urging the applicability of 'All this reasoning ... to time,'" Flew states, "is remarkable for the introduction of a fresh fallacy."[25] The fallacy is supposed to be this: "Certainly the conclusion [of the third *reductio*] is contradictory. But it simply does not follow from the premiss."[26] Flew, however, describes the defect in

[25] Flew, "Infinite Divisibility in Hume's *Treatise*", p. 265.
[26] Ibid.

Hume's reasoning as a *petitio principii*, standardly regarded as a rhetorical rather than logical fallacy. Flew writes:

> Possibly Hume was misled into thinking that [the conclusion follows from the assumption in his third *reductio*] because, being stubbornly convinced that finite periods could not be infinitely divisible into shorter periods all of which would either precede or succeed any one of the others, it seemed to him that any sub-division of one of his postulated minimum durationless moments could only be into moments which were at the same time both different and simultaneous; which is indeed absurd. But, since this reconstruction presupposes the conclusion it is supposed to prove, it scarcely provides "an additional argument ... proper to take notice of" (T 31).[27]

The reconstruction Flew offers is indeed circular, but it is not Hume's argument. Moreover, if the argument presupposes its conclusion, then it can hardly be said as Flew also claims that the proof's conclusion does not follow from its premises. The difficulty in that case would not be the argument's logical structure as valid or invalid, but its alleged insignificance or triviality.

In one sense, as Flew remarks, Hume's proof is not 'an additional argument'. The third *reductio* is a variation of Hume's first argument from the addition of infinite parts of extension applied to the hypothesis of the infinite divisibility of time. Hume, it should be remembered, begins this argument with the acknowledgment: 'All this reasoning takes place with regard to time...' The connection between the first and third *reductio* proofs is presented as two sides of the same coin. Yet in another sense, the argument is obviously different, even if, as Flew observes, the third argument presupposes the results of the earlier proof.

By introducing the inference as something Hume may 'possibly' have thought, Flew indicates that his attribution of the argument to Hume is conjectural. But there may be another, noncircular, way to reconstruct the proof. This is

[27] Ibid.

especially true since, as Flew rightly perceives, Hume's essential assumption about the infinite divisibility of time, presented above as proposition (2), implies that there are infinitely many precisely coexistent distinct moments. Flew's objection can be answered by justifying this assumption in Hume's third *reductio* argument without begging the question against the infinite divisibility thesis.

It is best to begin with an interpretation of the context in which Hume's argument about temporal divisibility occurs, in order to gain an understanding of the analogies between space and time that he seems to presuppose. The analogy Hume admits between space and time can then be used to give a noncircular explanation of the reasoning to support his assumption that if time is infinitely divisible, then there are infinitely many distinct yet precisely coexistent moments. References to time in this section of the *Treatise* are few and far between. Several commentators have remarked that, despite its title, Section I of Book I, Part II, *Of the infinite divisibility of our ideas of space and time*, contains not a word about time or the infinite divisibility of time or our ideas about time.[28] Without entering into disputes about the comparative metaphysics of space and time, it may suffice for immediate purposes to observe that Hume's conclusions in the proof are correct if the assumptions are true and the parallel first argument from the addition of infinite spatial parts is valid.

Hume offers a similar inference about time that exactly mirrors the structure of the inkspot argument for the finite divisibility of extension and the theory of sensible extensionless indivisibles. He acknowledges the demonstration to have the same form as his previous argument, and feels entitled thereby to apply the conclusion that the ideas both of space and time are compounded of indivisibles. He maintains: "There is another very decisive argument which establishes the present doctrine concerning our ideas of space and time, and is founded only on that simple principle, *that our ideas of them are compounded*

[28] Ibid., p. 262. Broad, "Hume's Doctrine of Space", p. 161.

of parts, which are indivisible. This argument may be worth the examining."[29] There follows an outline of the proof from the indivisibility of the constituents of the idea of extension, to which Hume adds the claim that our idea of such an indivisible must include a sensible property of color or touch.

> If a point be not consider'd as colour'd or tangible, it can convey to us no idea; and consequently the idea of extension, which is compos'd of the ideas of these points, can never possibly exist. But if the idea of extension really can exist, as we are conscious it does, its parts must also exist; and in order to that, must be consider'd as colour'd or tangible. We have therefore no idea of space of extension, but when we regard it as an object either of our sight or feeling.[30]

The proof for time is complete when from this general thesis about the nature of extension, Hume draws again on the supposed isomorphism of the divisibility of space and time. He concludes: "The same reasoning will prove, that the indivisible moments of time must be fill'd with some real object or existence, whose succession forms the duration, and makes it be conceivable by the mind."[31]

The heart of Hume's argument appears in assumptions (1)-(3). Hume is unclear about why or how 'the nature of motion' proves that the infinite divisibility of space implies the infinite divisibility of time. Here he need have nothing more in mind than the fact that in classical kinematics motion, time, and distance are mathematically interdefinable. If distance or extension is infinitely divisible, and if time is determined by the equations of physics as distance divided by velocity as the metric of extension in space, then the infinite divisibility of time is equally logically implicated.

Hume's second assumption is less easy to justify. He claims that if time were infinitely divisible, then there would obtain an infinite number of precisely coexistent distinct moments, which

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[29] *Treatise*, p. 38.
[30] Ibid., p. 39.
[31] Ibid.

he regards in the third assumption as an evident absurdity. The absurdity of distinct but coexistent moments is not hard to understand, since distinct moments are defined by and in a sense are nothing other than particular positions in a temporal sequence. To conclude that there is more than one distinct precisely coexistent moment is to conclude that there is more than one simultaneous time. This implication plainly contradicts the ordinary concept of succession in time. But to claim that the coexistence of two or more distinct moments of time follows logically from an assumption that time is infinitely divisible is much more contentious.

The proposition might be thought to hold on the grounds that adding or somehow combining together the infinitely divisible temporally extended or indivisible parts of any temporally extended parts of any finite portion of time produces an infinitely large unit. This suggestion reinforces the parallelism with Hume's first *reductio* proof from the addition of infinite spatial parts. The problem is that two or more finite but infinitely divisible segments of extension in space or duration of time must by addition or juxtaposition finally constitute an infinite extent of space or time. If the finite but infinitely divisible segments of space or time that give rise to these monstrosities are contiguous, then they must literally crowd each other out of whatever finite space or time might otherwise be thought to enclose them, by expanding infinitely outward into infinite space or infinite time.

Although Hume does not make this observation in the addition of infinite spatial parts of extension argument, the same conclusion should also hold there. The assumption implies that the infinite divisibility of two contiguous finite spatial extensions must crowd each other out of the space they jointly occupy. They do so not in the sense of leaving no room for each other anywhere in the limitless distances of infinite space. As previously observed, infinite quantity in classical infinitary mathematics can be 'added' to infinite quantity without net increase. Hume's point is rather that within the limited finite space from which the two adjacent segments are chosen, to which both belong and within which both are supposed to fit, both cannot be contained if either one is infinitely divisible, and if, as Hume's argument projects, the addition of infinitely many parts results in an infinitely large extension. For finite extension in space, there must obtain the counterpart of simultaneous coexistent distinct moments of time, as deduced in Hume's third *reductio*. This might then be called the precisely co-present occurrence of distinct regions of space — as evident a conceptual absurdity as in the case of time.

Conceivability of Indivisible Mathematical Points

The strategy in Hume's fourth and final explicitly *reductio* proof is to invoke the characteristically rationalist principle (which, occurring elsewhere in his writings, Hume evidently accepts) that whatever is clearly conceivable is possible.

Hume argues that extensionless indivisibles, perversely designated here as 'mathematical points', are clearly conceivable, and hence, by the present assumption, possible. But if extension is infinitely divisible, then indivisibles are impossible. Therefore, extension is not infinitely divisible. Hume creates unnecessary confusion both in the *Treatise* and first *Enquiry* by his equivocal use of the term 'mathematical points'. He sometimes adopts the phrase to denote the ideal abstract Euclidean points of the classical infinitist theory he is at pains to refute, while in other places, as in this argument, it can be assumed to designate extensionless indivisibles generally, including the sensible extensionless indivisibles of his own phenomenal theory of extension. The context of the proof suggests that in this instance Hume uses the term more specifically to refer to sensible extensionless indivisibles as the conceivable ultimate constituents of extension, to the exclusion of insensible ideal or abstract Euclidean mathematical points.[32]

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[32] Hume in the *Treatise*, p. 32, clearly intends non-Euclidean non-ideal and sensible *punctiforma* by the term 'mathematical points'. Yet on p. 40, he uses the same expression to mean Euclidean abstract ideal mathematical points

The argument, like some of Hume's previous objections to infinity, establishes a connection between both aspects of his two-fold metaphysical task of rejecting the traditional notion of infinite divisibility and replacing it with an alternative empiricist concept of sensible extensionless indivisibles. Hume's argument in this instance does not serve to establish the existence of sensible extensionless indivisibles, but appeals to the phenomenal data of the inkspot experiment for evidence that indivisibles or 'mathematical' points as least parts of finitely extended things are clearly conceivable. This he thinks provides all the leverage needed to overthrow the infinite divisibility thesis.

> 'Tis true, mathematicians are wont to say, that there are here equally strong arguments on the other side of the question, and that the doctrine of indivisible points is also liable to unanswerable objections. Before I examine these arguments and objections in detail, I will here take them in a body, and

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as 'maintain'd in the schools', and on pp. 40-41, he defends 'mathematical points' against a 'second objection'. The term also appears on p. 38, where its exact ideal or non-ideal, abstract or non-abstract meaning is ambiguous, though in context it is probably not meant to designate ideal abstract Euclidean mathematical points. In the *Enquiry*, Hume uses the term 'mathematical points' explicitly to designate ideal Euclidean points, reserving the term 'physical points' for the colored or tangible extensionless indivisibles of his theory of extension. This potentially causes additional confusion with his use of the term 'physical points' for extended material particles in the Bayle trilemma discussion of the *Treatise*, p. 40. Fogelin complains of these discrepancies in Hume's use of the term 'mathematical points' in *Hume's Skepticism in the Treatise of Human Nature*, p. 31. Kemp Smith, in *Philosophy of David Hume*, pp. 286-287, regards the terminological shift in referring to extensionless indivisibles as 'physical' instead of 'mathematical' points from the *Treatise* to the *Enquiry* as an improvement. Flew, in "Infinite Divisibility in Hume's *Treatise*", pp. 268-269, disputes this; but he confuses Hume's indivisibles with ideal entities, when he maintains, p. 269: "For the points, or spots, which Hume has in mind are ideal and not physical; and, although they are the constitutive elements of the idea of extension, they are — unlike physical points — specifically not supposed to be themselves extended." Hume's indivisibles are neither extended nor ideal, but occupy space despite being unextended and indivisible.

> endeavour by a short and decisive reason to prove at once, that 'tis utterly impossible they can have any just foundation. 'Tis an establish'd maxim in metaphysics, *That whatever the mind clearly conceives includes the idea of possible existence*, or in other words, *that nothing we imagine is absolutely impossible*... Now 'tis certain we have an idea of extension; for otherwise why do we talk and reason concerning it? 'Tis likewise certain, that this idea, as conceiv'd by the imagination, tho' divisible into parts or inferior ideas, is not infinitely divisible, nor consists of an infinite number of parts: For that exceeds the comprehension of our limited capacities. Here then is an idea of extension, which consists of parts or inferior ideas, that are perfectly indivisible: consequently this idea implies no contradiction: consequently 'tis possible for extension really to exist conformable to it: and consequently all the arguments employ'd against the possibility of mathematical points are mere scholastick quibbles, and unworthy of our attention. These consequences we may carry one step farther, and conclude that all the pretended demonstrations for the infinite divisibility of extension are equally sophistical; since 'tis certain these demonstrations cannot be just without proving the impossibility of mathematical points; which 'tis an evident absurdity to pretend to.[33]

The proof blunts aprioristic objections against the possibility of indivisible units of extension. It does this by showing that the mind can clearly conceive of indivisibles, on the assumption that whatever is clearly conceivable is logically possible. Then it seeks to overturn efforts to prove that extension is infinitely divisible, on the grounds that none of the arguments can hope to succeed without demonstrating the impossibility of mathematical points or extensionless indivisibles, the futility of which is already guaranteed.

The criticism as Hume explains it lends itself to this reconstruction in terms of the conceivability criterion:

[33] *Treatise*, pp. 32-33.

Conceivability of Indivisibles

1. Whatever is clearly conceivable is possible.
2. Indivisible (but not Euclidean ideal) 'mathematical' points as the least parts of extended things are clearly conceivable.
3. If extension is infinitely divisible, then indivisible (but not Euclidean ideal) 'mathematical' points as the least parts of extended things are impossible.
4. Suppose for purposes of indirect proof that extension is infinitely divisible.

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5. Indivisible (but not Euclidean ideal) 'mathematical' points as least parts of extended things are possible. (1,2)
6. Indivisible (but not Euclidean ideal) 'mathematical' points as least parts of extended things are impossible. (3,4)
7. Indivisible (but not Euclidean ideal) 'mathematical' points as least parts of extended things are both possible and impossible. (5,6)
8. Therefore, space (extension) is not infinitely divisible.(4,7)

The assumptions of Hume's argument in this formulation are: (1) the conceivability criterion of logical possibility; (2) a statement of the phenomenal evidence of the inkspot experiment that (sensible) indivisible 'mathematical' points as the least parts of extended objects are clearly conceivable; (3) the opposition between the infinite divisibility and extensionless indivisibles theses; the claim that not both principles can be true, but if one is true the other is impossible; if extension is infinitely divisible, then there cannot be extensionless indivisibles, and if there are extensionless indivisibles, then extension cannot be infinitely divisible; (4) the hypothesis offered for purposes of indirect proof that extension is infinitely divisible.

From these assumptions, the conclusions in (5)-(8) are easily deduced. The first preliminary conclusion in (5) follows directly from (1) and (2). If, as the inkspot experiment is supposed to show, Hume's indivisible 'mathematical' points are clearly conceivable, and if that which is clearly conceivable is possible, then the extensionless indivisibles of Hume's theory are possible. Conclusion (6), that the indivisible 'mathematical' points are impossible, follows from the hypothesis in (4), and the opposition between the infinite divisibility thesis and the thesis of extensionless indivisibles in (3). The contradiction in (5) and (6) is made explicit by the conjunction of these propositions in conclusion (7), which reflects negatively on the infinite divisibility hypothesis made for purposes of indirect proof in (4). This entails the final conclusion in (8) that space and extension are not infinitely divisible.

The premise that conceivability entails possibility is often a rationalist principle.[34] So it may be surprising that Hume should have endorsed it both within and outside of *reductio* contexts.[35] Hume's largely empiricist methodology obviously does not commit him to rejecting every worthy rationalist thesis. He selectively incorporates many such principles more typically associated with rationalist philosophy, such as Leibniz's distinction between matters of fact and relations of ideas. The situation is analogous to Descartes's use of empirical data to advance the sciences once he has given them a rationalist foundation by proving the existence of a veracious God. The analogy is reflected in mirror-image from the predominantly empiricist side when Hume in the present argument and elsewhere accepts a version of the rationalist conceivability criterion in order to extend knowledge in drawing conclusions about the contents and workings of the mind in relation to logical possibilities.

The standard objections to conceivability or imagination as a test for possibility are those involving *a priori*, and especially mathematical, ideas. In a version of the criticism originally owing to Thomas Reid and revived by Saul A. Kripke, it is supposed to be possible to conceive or imagine both that Goldbach's unproven conjecture that every even

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[34] Descartes, *Meditations*, VI, p. 190. Leibniz, *Discourse on Metaphysics*, Article XXV.

[35] *Treatise*, pp. 32, 89, 236, 250, 650-654. *Enquiry*, p. 25.

number greater than 2 is the sum of two primes is true, and alternatively that the conjecture is false. All that is needed is to imagine both that the generalization holds for every such even number, and that somewhere on the distant reaches of the number line there is an unknown even number that is not the sum of two primes. Yet, since presumably either Goldbach's conjecture or its negation is impossible, either the conjecture or its negation is conceivable but not possible. It seems to follow that conceivability or imaginability, especially in mathematical and other synthetic *a priori* matters, is a faulty criterion of possibility.[36]

Hume need not be bothered by such examples. To have a clear idea of the truth or falsehood of Goldbach's conjecture, in Hume's sense, it would be necessary to have a corresponding impression, or, in the case of a complex idea, multiple impressions of sensation or reflection, to provide the raw material that may then be reworked by memory, imagination, and reason into the appropriate mediated idea. It is doubtful whether ideas of the kind required for Kripke's counterexample could be experientially derived from impressions of sensation or reflection. Hume can avoid the objection by adopting a version of intuitionism in the philosophy of mathematics, that typically goes along with a weaker version of strict conceptual finitism. If Hume denies the realist *a priori* claim that either Goldbach's conjecture or its negation must be true, and if true, necessarily true, then Kripke's realist counterexample to the conceivability criterion of logical possibility is forestalled. Ironically, the more typically rationalist 'maxim in metaphysics' that whatever is conceivable is logically possible might best be defended by an

[36] Kripke uses the example more specifically to demonstrate that not all necessary truths are known *a priori*. Kripke, *Naming and Necessity*, pp. 36-38. The objection is a variation originally deriving from Reid, *Essays on the Intellectual Powers of Man*, p. 431. See Casullo, "Reid and Mill on Hume's Maxim of Conceivability", pp. 212-219, and "Conceivability and Possibility", pp. 118-121. Wright, *The Sceptical Realism of David Hume*, pp. 92-97.

empiricist rejection of platonic realism and apriorism in the philosophy of mathematics.[37]

The importance of the argument for Hume is indicated by the fact that, alone among the *reductio* proofs, he recapitulates its essentials in the first paragraph of Section IV, when he tries to impose a tighter organization on the preceding thirteen pages of relatively disconnected reflections about space and time, parts of which have already been cited:

> Our system concerning space and time consists of two parts, which are intimately connected together. The first depends on this chain of reasoning. The capacity of the mind is not infinite; consequently no idea of extension or duration consists of an infinite number of parts or inferior ideas, but of a finite number, and these simple and indivisible: 'Tis therefore possible for space and time to exist conformable to this idea: And if it be possible, 'tis certain they actually do exist conformable to it; since their infinite divisibility is utterly impossible and contradictory.[38]

The fact that Hume accepts the conceivability criterion, and does not merely assume it hypothetically for purposes of indirect proof, has further implications in assessing the merits of the conceivability argument, which is in some ways the weakest weapon in Hume's arsenal. Here as elsewhere in Hume's indirect proofs, there is a combination of rationalist and empiricist premises. But because Hume is unequivocally committed to the conceivability criterion, the conceivability of indivisibles argument cannot simply be regarded as a *reductio ad absurdum* of traditional infinitism; it is simultaneously a defense

[37] The status of infinity in intuitionistic logic and mathematics is complex. Many intuitionists accept Aristotle's concept of potential infinity while denying the possibility of actual infinity. Wittgenstein like Berkeley and Hume seems to have accepted a strict finitism both in the *Tractatus Logico-Philosophicus* and *Remarks on the Foundations of Mathematics*. See Kielkopf, *Strict Finitism: An Examination of Ludwig Wittgenstein's Remarks on the Foundations of Mathematics*.

[38] *Treatise*, p. 39.

of Hume's doctrine of sensible extensionless indivisibles to which he is philosophically bound.

Conceivability as a criterion of possibility is a flimsy reed to lean on because of counterexamples like Kripke's problem about Goldbach's conjecture. As suggested, there may be a way to avoid the difficulty, but only if Hume is willing to adopt a version of intuitionism about the indeterminacy of mathematical propositions for the metaphysics and applied mathematics of space and time. There is no direct positive evidence to indicate that Hume would be willing to do this, and even some considerations to suggest on the contrary that he could not adopt any standard version of intuitionism.[39]

Hume relies on the inkspot experiment to uphold the premise that sensible extensionless indivisibles are clearly conceivable. Infinitists typically say the same about abstract Euclidean points and line segments, and about the infinitesimals of the calculus. Classical mathematicians often claim to have clear and precise conceptions of infinitary theoretical entities, so that on the conceivability criterion, each should have equal title to proving that the atomic components of their respective non-Humean theories of extension are logically possible. By the opposition between infinite divisibility and indivisibility, these theories are jointly incompatible — if either one is true, the other is logically impossible. This makes Kripke's counterexample to the conceivability criterion as it applies to mathematical possibility in Goldbach's conjecture and its negation all the more relevant.

At this stage of his discussion, Hume can reasonably deny that the infinitist clearly conceives of infinitely divisible line segments. He can point out putative inconsistencies in the concept of infinite divisibility implied by the inkspot argument and the previous three *reductio* proofs. If the infinitist truly

[39] Hume's assertions about either positive or negative propositions being true in 'the sciences, properly so-called' might imply his unwillingness to accept an intuitionistic denial of semantic realism or bivalence in logic and mathematics.

has a clear conception of mathematical points as components of infinitely divisible extension, then either the contradictions uncovered by Hume's indirect proofs could never arise, or they would admit of a straightforward solution. Hume in any event believes that the infinitist has no satisfactory way to answer these difficulties.

Second, Hume can counter that insofar as the infinitist claims to have a clear idea of something like an atomic constituent of extension, it cannot be the idea of an abstract mathematical Euclidean point. These are the 'nothingnesses of extension' that Bayle denounces as unable to explain the constructive constitution of extension. The only adequate idea of the ultimate components of extension for Hume is that of the sensible extensionless indivisibles of his own finitist theory. The infinitist might reply that the concepts are entirely different, and that the clear concept of a Euclidean mathematical point is an abstract general rather than concrete empirical or phenomenal idea. Against this claim, following Berkeley, Hume is prepared to reject the existence of aprioristic abstract general ideas, and to reinterpret what sometimes passes for abstract thinking in terms of particular ideas deputized to represent others belonging to the same relevant category. In *Treatise*, Book I, Part II, Section III, *Of the other qualities of our ideas of space and time*, just two pages after concluding the conceivability argument, Hume recapitulates his earlier pronouncements in Part I, Section VII. He argues: "All abstract ideas are really nothing but particular ones, consider'd in a certain light; but being annexed to general terms, they are able to represent a vast variety, and to comprehend objects, which, as they are alike in some particulars, are in others vastly wide of each other."[40] Hume's thesis of the experiential origin of ideas in impressions of sensation and reflection does not allow the mind to entertain abstract general ideas in any other than this harmless sense. Berkeleyan representative generality is too weak to support a general idea of the ultimate components of extension different

[40] *Treatise*, p. 34. See also pp. 17-25; *Enquiry*, pp. 154-155.

from those afforded by phenomenal experience, as described in the inkspot experiment. On that view, an 'abstract' idea of the atomic constituents of extension is nothing but a particular idea of Hume's own sensible extensionless indivisibles delegated to represent others in the same category. Hume believes that the mind can clearly and distinctly conceive of these, and in the conceivability argument takes their conceivability as proof that extension cannot be infinitely divisible.

Appealing explicitly to the conceivability criterion in connection with the 'very material' question 'concerning *abstract* or *general* ideas, *whether they be general or particular in the mind's conception of them*', Hume concludes:

> ... nothing of which we can form a clear and distinct idea is absurd and impossible... Now as 'tis impossible to form an idea of an object, that is possest of quantity and quality, and yet is possest of no precise degree of either; it follows, that there is an equal impossibility of forming an idea, that is not limited and confin'd in both these particulars. Abstract ideas are therefore in themselves individual, however they may become general in their representation. The image in the mind is only that of a particular object, tho' the application of it in our reasoning be the same, as if it were universal.[41]

[41] *Treatise*, pp. 19-20. Hume makes even more important applications of Berkeley's rejection and empiricist reinterpretation of apriorist abstract ideas in the 'hint' mentioned in the note to the first *Enquiry*, paragraph 125, pp. 157-158. In his refutation of abstract ideas, Hume seeks to prove what he calls, *Treatise*, p. 18: "the first proposition, *that the mind cannot form any notion of quantity or quality without forming a precise notion of the degrees of each.*" Hume's concept of an 'adequate idea' makes its first appearance in this context, when he writes several pages later, pp. 22-23: "First then I observe, that when we mention any great number, such as a thousand, the mind has generally no adequate idea of it, but only a power of producing such an idea, but its adequate idea of the decimals, under which the number is comprehended. This imperfection, however, in our ideas, is never felt in our reasonings; which seems to be an instance parallel to the present one of universal ideas." But this does not show that Hume in the *Treatise* already makes explicit use of the Berkeleyan 'hint' rejecting any idea of infinity, because the argument there does not preclude the possibility of producing

It appears that Hume has several resources for evading the objections of infinitists who claim to be able to clearly conceive abstract general ideas of Euclidean mathematical points in the infinite divisibility of finite extension. The conceivability argument in Hume's critique of infinity provides a fourth reason for seeking an alternative to the infinite divisibility of extension, which Hume advances in his positive doctrine of sensible extensionless indivisibles. The four *reductio* arguments in Hume's refutation undermine the conceivability of infinite divisibility by uncovering contradictions in the concept. If infinite divisibility is not conceivable, then there is no reason to suppose it is logically possible. If Hume's argument is correct, on the other hand, it is conceivable and therefore possible for extension to be constituted by sensible extensionless indivisibles in the best and perhaps the only alternative theory of space.

The Geometry Dilemma

The final *Treatise* argument against infinite divisibility is what I shall call the geometry dilemma. It is not explicitly *reductio* in form, but is easily reformulated as such. The argument is a nested disjunctive syllogism in structure, which Hume presents as a dilemma, arising from two embarrassing questions for infinitist mathematicians. There are three occurrences of the argument in the text, in Book I, Part I, Section II, in the 'Appendix', and in the 'Abstract'. Like the *reductio* proofs, the geometry dilemma complements Hume's inkspot argument by refuting the concept of infinite divisibility from non-Humean assumptions.

Hume lays out two paths for the infinitist metaphysician or mathematician to follow in accounting for the idea of exact equality and proportion in geometry, each of which leads to reasons for rejecting the infinite divisibility thesis. The dilemma allows that the idea of equality and proportion in geometrical thinking either depends or does not depend on the

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an idea of infinity from the mind's 'adequate idea of the decimals', if this were to permit an adequate idea of an infinite series of decimals.

existence of indivisibles. If the idea of equality and proportion depends on indivisibles, which Hume refers to as the 'exact' but 'impractical' standard of equality and proportion, then, by the opposition between infinite divisibility and the theory of indivisibles, extension is not infinitely divisible. If the idea of equality does not require indivisibles, then its standard of equality and proportion is too coarse, imprecise, or inexact to support conclusions about the fine-grained subdivisions required by the classical theorems of geometry. The contrary infinite divisibility thesis is supposed to be inadequate to uphold the propositions of geometry because it entails that any line segment has the same infinite number of points as any other.

This is undoubtedly the longest, most extensive argument in Hume's criticisms of the infinite divisibility thesis. It would be tedious to reproduce the passage in its entirety, but enough must be included to judge the tenor of Hume's argument, as a basis for informal reconstruction and evaluation. Hume writes:

> ... [I] maintain, that none of these demonstrations can have sufficient weight to establish such a principle, as this of infinite divisibility; and that because with regard to such minute objects, they are not properly demonstrations, being built on ideas, which are not exact, and maxims, which are not precisely true. When geometry decides any thing concerning the proportions of quantity, we ought not to look for the utmost *precision* and exactness. None of its proofs extends so far... I first ask mathematicians, what they mean when they say one line or surface is EQUAL to, or GREATER, or LESS than another? Let any of them give an answer, to whatever sect he belongs, and whether he maintains the composition of extension by indivisible points, or by quantities divisible *in infinitum*. This question will embarrass both of them... As to those, who imagine, that extension is divisible *in infinitum*, 'tis impossible they can make use of this answer [exact geometrical magnitude determined by exact number of constituent indivisibles], or fix the equality of any line or surface by numeration of its component parts. For since, according to their hypothesis, the least as well as greatest figures contain an infinite number of parts; and since infinite numbers, properly speaking, can neither

> be equal nor unequal with respect to each other; the equality or inequality of any portions of space can never depend on any proportion in the number of their parts. 'Tis true, it may be said, that the inequality of an ell and a yard consists in the different numbers of the feet, of which they are compos'd; and that of a foot and a yard in the number of the inches. But as that quantity we call an inch in the one is suppos'd equal to what we call an inch in the other, and as 'tis impossible for the mind to find this equality by proceeding *in infinitum* with these references to inferior quantities; 'tis evident, that at last we must fix some standard of equality different from an enumeration of the parts.[42]
>
> [There follows an eight-page discussion of alternative ways in which geometry might try to regain the concepts of equality and proportion by practical measures or without appeal to empirical observation.]

In the 'Abstract' to the *Treatise*, Hume indicates that the following material is to be added to the remarks quoted above:

> *To be inserted in* Book I. page 52. line 17. *after these words* (practicable or imaginable.) *beginning a new paragraph.* To whatever side mathematicians turn, this dilemma still meets them. If they judge of equality, or any other proportion, by the accurate and exact standard, *viz.* the enumeration of the minute indivisible parts, they both employ a standard, which is useless in practice, and actually establish the indivisibility of extension, which they endeavour to explode. Or if they employ, as is usual, the inaccurate standard, deriv'd from a comparison of objects, upon their general appearance, corrected by measuring and juxta position [*sic*]; their first principles, tho' certain and infallible, are too coarse to afford any such subtile [*sic*] inferences as they commonly draw from them. The first principles are founded on the imagination and senses: The conclusion, therefore, can never go beyond, much less contradict these faculties...[43]

Later, in the 'Appendix' to the *Treatise*, Hume adds:

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[42] *Treatise*, pp. 44-46.
[43] Ibid., 'Abstract', p. 638.

> [The author refutes the infinite divisibility of extension] by denying Geometry to be a science exact enough to admit of conclusions so subtile as those which regard infinite divisibility. All Geometry is founded on the notions of equality and inequality, and therefore according as we have or have not an exact standard of those relations, the science itself will or will not admit of great exactness. Now there is an exact standard of equality, if we suppose that quantity is composed of indivisible points... But tho' this standard be exact, 'tis useless; since we can never compute the number of points in a line. It is besides founded on the supposition of finite divisibility, and therefore can never depend on any conclusion against it. If we reject this standard of equality, we have none that has any pretensions to exactness.[44]

The argument is reconstructed as a dilemma. Its basis is excluded middle, in the proposition that the concept of exact equality in extension, which appears necessary to any adequate geometry, either rests or does not rest on the existence of indivisibles. There is a nested subdilemma which considers two possibilities to account for the idea of exact equality of extension. It occurs in the second horn, where the assumption is exact equality is not based on indivisibles. The idea might be explained by the infinite divisibility of extension into infinitely many ideal Euclidean mathematical points or line segments, or by the comparison and measurement of extension in impressions of sensation and reflection. With the elimination of both subdilemma alternatives, Hume concludes that extension is not infinitely divisible. The inference offers several alternatives leading to the same conclusion, refuting the infinite divisibility of extension:

Geometry Dilemma

1. The idea of exact equality in extension is either based on the existence of indivisibles or not.
2. If the idea of exact equality in extension is based on the existence of indivisibles, then (because of the opposition

[44] Ibid., 'Appendix', pp. 658-659.

between the infinite divisibility and extensionless indivisibles theses) extension is not infinitely divisible.

3. If the idea of exact equality in extension is not based on the existence of indivisibles, then it can only be based on the infinite divisibility of extension into infinitely many ideal Euclidean extended line segments, or on the comparison and measurement of extended objects, and therefore on sense impressions.
4. But the idea of exact equality in extension cannot be based on the infinite divisibility of extension into infinitely many ideal Euclidean extended line subsegments, because infinitely divisible unit standards of any arbitrarily chosen extension by hypothesis contain precisely the same infinite number of ideal Euclidean extended line subsegments.
5. Nor can the idea of exact equality in extension be based on sense impressions in the comparison and measurement of extended objects, for then it will be too coarse, imprecise, or inexact to establish the formal theorems of geometry.

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6. Therefore, in either case, space (extension) is not infinitely divisible. (1,2,3-5)

The proof in its original statement runs over ten pages in Hume's otherwise compact exposition. This may be why Hume sought as an afterthought to condense its content both in the 'Appendix' addition, and in the restatement of the anonymously published 'Abstract'. The first presentation partially loses sight of the dilemma, and bogs down in considerations of various ways in which geometry might achieve precision by more practical use or entirely without the benefit of sense perception.

Hume's dilemma begins with the assumption in (1) that the idea of exact equality in extension is either based or not based on the existence of indivisibles. Proposition (2) frames half of the opposition between the infinite divisibility and extensionless indivisibles theses. Proposition (3) contains the basis for the proof's subdilemma. It offers two choices for an account of the idea of exact equality and proportion. The idea of equality of extension in principle might be explained in terms of the divisibility of extension into infinitely many ideal Euclidean extended line segments, or in terms of the comparison and measurement of extended objects in perception. Hume's reason for rejecting the possibility that an adequate idea of exact equality could depend on the infinite divisibility thesis is condensed in assumption (4). Assumption (5) similarly refutes the possibility that an idea of exact equality adequate for the purposes of mathematics could be based on imprecise judgments of sensation in the comparison and measurement of extension.

From the first hypothesis in assumption (1), that the idea of equality in extension is based on the existence of indivisibles, it follows by the opposition thesis in (2) that space and extension are not infinitely divisible. From the second hypothesis in (1), that the idea of equality in extension is not based on the existence of indivisibles, it follows from the assumption in (3) that the idea of extension can only be based either on the infinite divisibility of extension into infinitely many ideal Euclidean extended line subsegments, or on the experiential comparison and measurement of extended objects, and therefore on sense impressions. Assumption (4) rules out the option that extension could be infinitely divided into ideal Euclidean extended line segments. This leaves only the possibility that the idea of equality in extension is based on the experiential comparison and measurement of extended objects, rejected by assumption (5) as too coarse or imprecise to support the exact formal theorems of geometry. Conclusion (6) follows by disjunctive dilemma, collecting the results of the proof's two horns. By either hypothesis, space and extension are not infinitely divisible but constituted by indivisibles.

There is a sense in which Hume's geometry dilemma is a *reductio ad absurdum*. Of course, any deductive argument can be reconstructed as a *reductio*. But here the interpretation seems particularly natural and unstrained. Hume's geometry dilemma reduces to absurdity the implicit metatheory of classical Euclidean geometry. It reveals the inconsistency whereby on the one hand geometry affirms the infinite divisibility of extension, while on the other presupposes ideas of exact equality and proportion that are adequate to its theorems only if extension is not infinitely divisible. To reconstruct the proof in *reductio* form, the hypothesis formulates something like the claim that any geometry incorporating the infinite divisibility of extension thesis provides an adequate mathematics of spatial extension. The contradiction that is then derived reflects back on the infinite divisibility thesis.[45]

The key to Hume's rejection of infinite divisibility in an adequate idea of equal extension is given in assumption (4). Hume argues that no standard of measurement chosen for purposes of determining exact equality and proportion of geometrical magnitude can be based on the infinite divisibility of extension. If there were such a standard, he reasons, its exact measure could not depend on the number of extended (nonindivisible) components it contains. The infinite divisibility thesis implies that there is precisely the same (infinite) number of ideal Euclidean extended line subsegments in any chosen standard unit as in any extension whatsoever. It is only by supposing that finite extension is divisible into finitely many extensionless indivisibles that we can make sense of the idea of a unit standard like a centimeter as being larger than half a centimeter and smaller than two centimeters. In that case, a centimeter contains precisely twice the finite number of finitely many indivisibles as half a centimeter, and precisely half the finite number of finitely many indivisibles as two centimeters. These are real differences in geometrical magnitude based

[45] The geometry dilemma as noted is presented in at least two other forms. Both are versions of what has come to be known as 'Hume's Fork', a general dilemma based on the Leibnizian distinction between relations of ideas and matters of fact, which Hume accepts. See ibid., pp. 44, 53. Flew, *David Hume*, pp. 43-48; *Hume's Philosophy of Belief: A Study of his First Inquiry*, pp. 53-55. An alternative interpretation of these passages is offered by Waxman, *Hume's Theory of Consciousness*, pp. 115-127.

on real differences in definite numbers of components. The infinite divisibility thesis by contrast implies that the number of ideal Euclidean extended line segment components by which a centimeter, half a centimeter, two centimeters, or an arbitrary extension is constituted are indistinguishable in number. Thus, Hume's geometry dilemma is a continuation of the reasoning in his second *reductio* refutation of infinite divisibility, in his version of Malezieu's argument from the unity of existents.

The first horn of the geometry dilemma is unproblematic. If the exactness of the idea of equality in geometry requires the existence of indivisibles, then extension (in the sense of geometrical magnitude) cannot be infinitely divisible, but must be constituted by extensionless indivisibles. In the second part of the dilemma, Hume shows that if the idea of exact equality of extension is not based on the existence of indivisibles, then it can only be grounded on the infinite divisibility of extension into ideal Euclidean extended line segments. Such an idea, he argues, must be derived experientially from the comparison and measurement of extended objects, and hence ultimately from sense impressions. It might be objected that it does not follow from the mere fact that an idea of equality based on comparison and measurement of finite extension is inadequate to support the formal theorems of geometry that extension itself is not actually infinitely divisible, nor that extensionless indivisibles are the constituents of extension, but only that these claims cannot satisfactorily be proved on the basis of such a limited idea of equality. Hume, in some places at least, claims not to draw inferences about the 'nature of body' or reality beyond the impressions and ideas of reality itself, although we have also seen that he offers conclusions about the finite divisibility limitations of extension in reality from the limitations of impressions and ideas of spatial extension. So, in one sense, perhaps, he could be indifferent to the criticism. Yet there is a more powerful reply to made in defense of Hume's geometry dilemma.

The second half of the proof can be interpreted as a chain of existence inferences. The chain begins with the requirements of precision in geometry, to which it introduces considerations about the origins of the idea of equality in extension, and concludes with the existence of extensionless indivisibles. Hume has no quarrel with the standards of exactness geometry demands of adequate ideas of equality of extension. The requirements of exact equality presupposed by classical geometry on the contrary constitutes the basis for Hume's criticism of infinitary mathematics. Hume does not propose a nonclassical discrete geometry such as Berkeley suggests in the notebooks of his *Philosophical Commentaries*. If geometry is to be the exact science it purports to be, then Hume believes it must have an adequate idea of the exact equality of geometrical dimensions. This requirement in turn he thinks can only be satisfied if extension is constituted by and divisible into finitely many sensible extensionless indivisibles. Sense impressions by themselves as they occur in the practical comparison and measurement of extended things are, in Hume's words, 'too coarse to afford any such subtile inferences as [mathematicians] commonly draw from [their first principles]'.

That Hume's conclusion concerns both idea and reality is indicated by the fact that he disavows experience of sensible extensionless indivisibles as providing an adequate standard of exact equality in geometry, but insists that indivisibles must exist in order to provide the mind with an adequate idea of the exact equality of extension. He acknowledges that 'the enumeration of the minute indivisible parts... is useless in practice', but claims that without indivisibles there can be no experiential foundation for an adequate idea of exact equality, required by the applied mathematics of natural philosophy. "There are few or no mathematicians," Hume states, "who defend the hypothesis of indivisible points; and yet these have the readiest and justest answer to the present question. They need only reply, that lines or surfaces are equal, when the numbers of points in each are equal; and that as the proportion of the numbers varies, the proportion of the lines and surfaces is also vary'd."[46] For Hume, this is what makes reference to 'indivisible points' the 'just' though 'useless' reply to the question what is meant by the concepts of equality and greater or less than in classical geometry. Remarkably, Hume in this argument steals the fire from traditional infinitist mathematicians, who insist that an empiricist methodology is inadequate for the exact standards of proof required by the ideal properties of mathematical objects. Hume turns the tables on such detractors by admitting that experience does not provide an adequate measure of exact equality of extension in practice. But Hume, unlike his infinitist adversaries, does not propose to substitute abstract general ideas of infinite divisibility, including infinite Euclidean points or infinitesimals, for judgment in experience. Instead, he argues that without an adequate idea of exact equality, the precision required of geometrical demonstrations can never be attained, and that an adequate idea of exact equality is logically incompatible with the infinite divisibility of extension.

Fogelin disputes the second subinference beginning with assumption (4) of the reconstruction. He suggests a different model of geometrical demonstration that downplays reliance on empirical observation in geometry. He argues: "... in geometrical proofs, equalities are *stipulated* rather than discovered by observation. In geometry, lines are *set* equal to each other."[47] What Fogelin says is true, but it is doubtful that Hume would need to be reminded of this. His main point is not about the way in which equalities are introduced or used in geometrical proofs. He is rather concerned with the deeper and more basic characteristically Humean question about the *origin of the idea* of equality, and whether the correct account of its (experiential) source is compatible with the infinite divisibility thesis. Regardless of how geometry *uses* the ideas of equality or proportion, whether by empirical observation and measurement or stipu-

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[46] *Treatise*, p. 45.

[47] Fogelin, "Hume and Berkeley on the Proofs of Infinite Divisibility", p. 57.

lation, the mathematician must first *possess* the idea of equality and proportion in order to begin, and this is where Hume's dilemma gets its hold. If we do not possess the idea of equality in the first place, then there is no conceptual foundation for stipulations about particular lengths in particular geometrical figures being equal.

The idea of equality is taken for granted by infinitist mathematicians and metaphysicians. Yet the concept of equality under Hume's scrutiny in the geometry dilemma shows itself incapable of supporting the infinite divisibility thesis. Far from believing, as Fogelin charges, that we are supposed to determine exact equality of extension by sensory comparison and measurement, Hume clearly states that such a standard is 'useless in practice'. He claims only that sense impressions of finitely divisible extension are the source and 'ultimate standard' of the adequate idea of exact equality in geometry:

> As the ultimate standard of these figures is derived from nothing but the senses and imagination, 'tis absurd to talk of any perfection beyond what these faculties can judge of; since the true perfection of any thing consists in its conformity to its standard.[48]

This raises an interesting question for the interpretation and assessment of Hume's geometry dilemma. It appears that Hume considers only to refute the possibility of obtaining the idea of exact equality and proportion in geometry from sensations of comparison and measurement. Having dispensed with the infinite divisibility thesis as unable to explain the idea, the second dilemma horn is thwarted, so that the only possibility is that the idea of exact equality derives from the experience of the finite divisibility of extension into sensible extensionless indivisibles. Hume wisely maintains that his indivisibles confer the idea of equality only for purposes of reasoning, and do not provide a practical basis for determinations of geometrical equality and proportion. By rejecting infinite divisibility and sensory comparison and

[48] *Treatise*, p. 51.

measurement as incapable of explaining the idea of exact equality, Hume inherits the same problems Berkeley tries to untangle in exploring the difficulties and advantages of a nonclassical discrete geometry.

If finite extension is finitely divisible into only finitely many sensible extensionless indivisibles, then a given finitely extended line segment must either contain an even or odd number of indivisibles. Let us think first only of the problems of divisibility of such a line segment. We can contrast the *bipartition* of a line segment with its *bisection* by another line, line segment, or curve. The bipartition of a line segment occurs when it is broken exactly in half, and the two subsegments are separated in space. The bisection of a line segment occurs when another line, line segment, or curve, crosses the line segment at its exact midpoint. The problem is that in either sense of the division of a line segment, not all line segments according to the exact idea of geometrical equality implied by Hume's indivisibles thesis are exactly divisible. A line segment consisting of an even number of indivisibles can be exactly bipartitioned into two exactly equal subsegments, but has no exact midpoint at which to be bisected. A line segment consisting of an odd number of indivisibles cannot be bipartitioned into two exactly equal subsegments, but at most into two subsegments of some even number and some odd (the even number plus or minus 1) indivisibles; yet it has an exact bisection midpoint.

It is a theorem of the classical geometry of the continuum that every line segment can be exactly bipartitioned and has an exact bisection midpoint. The theorem is guaranteed in classical geometry because there is said to be a point between any two points, and hence neither an even nor odd number of points, and because two halves of a line segment contain precisely the same infinite number of points. By denying this, Hume strays from classical continuous geometry in the direction of a Berkeleyan discrete geometry, which he nowhere acknowledges or tries to develop. Hume must then regard classical geometry as at best a matter of practical convenience, since, as he says, counting indivisibles is out of the question.

Yet his idea of the exactness of equality and proportion in geometry implies that every finite line segment actually consists of a finite number of sensible extensionless indivisibles. From the standpoint of Hume's critique of infinity, the theorems of classical continuous geometry are only approximately true. It is Hume's demand for an adequate idea of exactness in classical geometry that motivates his geometry dilemma in the first place.

Hume does not address the problem, but he might have answered the objection in the following way. Hume can reasonably claim that having an adequate idea of the exact equality and proportion of extension is enough for classical geometry. If we can acquire the concept of finitely divisible extension from experience, say, in the inkspot experiment, then we will have an adequate idea of exact equality or proportion of extension for theoretical purposes in classical geometry, without accepting its paradoxical consequences for the infinite divisibility of our adequate ideas of extension or of real spatial extension. Applied geometry in natural science and engineering involving real spatial extension is always a relatively roughshod practical matter, where one indivisible extensionless unit of extension more or less is not going to make much of a difference anyway. Hume is not obligated to trace the necessary truth in the demonstrations of classical geometry to particular matters of fact about the ultimate constitution of extension, but only to relations among our adequate ideas thereof, including whatever experientially originating ideas we may have of the exact equality and proportion of extension. He writes: "... the necessity, which makes two times two equal to four, or three angles of a triangle equal to two right ones, lies only in the act of the understanding..."[49]

[49] Ibid., p. 166.

CHAPTER 5

ANTITHESIS IN KANT'S SECOND ANTINOMY

Kant's Anti-Divisibility Argument

The divisibility of extension ultimately into indivisibles is a longstanding concept of metaphysics. Hume, as we have seen, champions a version of this position as the conclusion of the geometry dilemma. Kant, in the first *Critique*, A434-35/B462-63, presents three antinomies or paradoxes of reason, the second of which, the 'Second Conflict of the Transcendental Ideas', is supposed to demonstrate *a priori* that there must and that there cannot possibly exist indivisibles or simple atomic constituents of extension. To understand the implications of the *reductio* proofs in Hume's critique of infinity we shall now compare Kant's *reductio* with Hume's conclusions about the finite divisibility of extension.

Kant's antinomy consists of two propositions, thesis and antithesis. *Thesis*: "Every composite substance in the world is made up of simple parts, and nothing anywhere exists save the simple or what is composed of the simple." *Antithesis*: "No composite thing in the world is made up of simple parts, and there nowhere exists in the world anything simple."[50] Whatever the status of the thesis, the antithesis in Kant's second antinomy, that there cannot exist simples or indivisible atomic parts of composites, appears unsound. We gain a better sense of the strength of Hume's position, and especially of the force of his *reductio* disproofs of infinite divisibility, by

[50] *Critique of Pure Reason*, A435/B463.

remarking its avoidance of Kant's *reductio* disproofs of the indivisibility antithesis. Hume's idea of sensible extensionless indivisibles is so different from Kant's that the antithesis in Kant's second antinomy does not apply to its fundamentally different assumptions about the metaphysics of space.

The antithesis of Kant's antinomy is proved by a complex *reductio* argument in several steps. Almost twice as long as his proof of the thesis that every composite is composed of simple parts, the antithesis is best understood as a progression of subarguments. Kant first seeks to establish that every part of a composite must occupy a distinct part of space:

> Assume that a composite thing (as substance) is made up of simple parts. Since all external relation, and therefore all composition of substances, is possible only in space, a space must be made up of as many parts as are contained in the composite which occupies it. Space, however, is not made up of simple parts, but of spaces. Every part of the composite must therefore occupy a space.[51]

The assumption is made only for purposes of indirect proof, and later rejected in the antithesis conclusion. To follow Kant's argument, it may be worthwhile to import a few terms for the sake of clarity and emphasis that are not explicit in the original statement. Where Kant speaks of composite things, he evidently means composite extended things, as opposed to composite ideas or abstract ideal entities. Kant later qualifies the term by referring in this connection not simply to composite things, but to 'everything real', by which he clearly intends spatially extended objects. The hypothetical spatial division of composite extended things and the spaces in which they are contained has no intuitive plausibility unless it is assumed that the parts and spaces in question are distinct. The textual evidence in support of this interpolation is that later in the proof Kant uses the similarly suggestive phrase, 'constituents external to one another'. Kant's reference to 'simples' and 'simple parts' is qualified here by the synonymous term

[51] Ibid.

'indivisibles', to facilitate comparison with Hume's theory of sensible extensionless indivisibles.

With this preparation, the first part of Kant's proof for the antithesis of the second antinomy can now be reconstructed. The subargument for Kant's preliminary conclusion that every distinct part of a composite extended entity must occupy a distinct part of space has this logical structure:

Kant's Argument for Distinct Parts of Space

1. There exists a composite extended thing made up of distinct indivisible parts.
2. Compositions of extended things are possible only in space, so that the space containing a composite extended thing must consist of just as many distinct parts as the composite extended thing it contains.
3. Space is not made up of indivisible parts, but of spaces.

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4. Every distinct part of a composite extended thing must occupy a distinct part of space. (1,2,3)

The second stage of Kant's proof tries to derive from the result in (4) above that the indivisible parts of a composite extended object must occupy distinct spaces or parts of space. Kant continues:

> But the absolutely first parts of every composite are simple. The simple therefore occupies a space.[52]

The argument is straightforward. The premise in (5) seems to be a disguised tautology or truism of some sort, on the most obvious interpretation of Kant's reference to the 'absolutely first parts' of a composite.

Simple Parts of Composites Occupy Distinct Spaces

5. The absolutely first distinct parts of every composite are simple.

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[52] Ibid.

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6. Every simple indivisible part of a composite occupies a distinct space or part of space. (4,5)

Kant concludes the argument with an explicit contradiction, as required by its *reductio* strategy. He appeals to the assumption that the only possible candidates for the spatially distinct ('external to one another') component parts of spatially extended things belong to the Aristotelian-Scholastic metaphysical categories of substance and accident. Kant argues:

> Now since everything real, which occupies a space, contains in itself a manifold of constituents external to one another, and is therefore composite; and since a real composite is not made up of accidents (for accidents could not exist outside one another, in the absence of substance) but of substances, it follows that the simple would be a composite of substances — which is self-contradictory.[53]

The proof of the antinomy in its final phase is reconstructed by formulating Kant's suppressed premise about the categories of substance and accident to which spatially distinct component parts of spatially extended things are supposed to be limited. The assumption is stated more explicitly as proposition (8) below.

Together with a consequence of the distinction between substance and accident, and the concept of accident as ontically dependent on its inherence in substance, the first conclusion of this part of Kant's argument in (10), that accidents cannot be spatially distinct or external to one another, follows directly from the Aristotelian-Scholastic premise formulated in (9). The next proposition in (11), that therefore only substances can constitute the spatially distinct components of spatially extended things, is derived by disjunctive syllogism from (8) and (10). This leads to the conclusion in (12) that the indivisible parts of spatially extended things are divisible composites of substances. The contradiction reflects back on assumption (1),

[53] Ibid.

introduced for purposes of indirect proof, that there exists a composite extended thing made up of simple parts. The contradiction supports the opposite conclusion of the antithesis that there exists no composite extended thing made up of simple indivisible parts.

7. Every spatially extended thing contains a manifold of distinct parts, so that every spatially extended thing is composite.
8. The only possible metaphysical candidates for the spatially distinct component parts of spatially extended things are substances and accidents.
9. Accidents cannot exist independently of substances.

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10. Accidents cannot be spatially distinct or external to one another. (9)
11. Only substances can possibly constitute the spatially distinct component parts of spatially extended things. (8,10)
12. Every simple indivisible part of a spatially extended thing is a divisible composite of substances. (7,11)
13. There exists no composite extended thing made up of simple indivisible parts. (1,2)

The thrust of the antithesis is that the parts of composite extended things must occupy space, so that a correspondence holds between the distinct component parts of spatially extended things and distinct parts of space. This implies that even the supposedly indivisible components of spatially extended things must occupy distinct parts of space. Kant concludes that if every extended thing is a composite of distinct parts, and if only substances can constitute the spatially distinct parts of spatially extended things, then paradoxically every indivisible part of an extended thing is a divisible composite of substances.

Proposition (7) may appear to beg the question by presupposing that no composite is constituted by indivisibles. Yet Kant's argument is subtler than this. The proof would lack interest altogether if it were so blatantly circular as merely to assume that the distinct parts into which spatially extended things are divisible must themselves be spatially extended, and hence divisible *ad infinitum.* Kant posits instead that every spatially extended thing is a composite of distinct (not necessarily extended) parts. From this he infers the infinite divisibility of extended things, drawing on the premise that the indivisibles into which extended things hypothetically are divisible must occupy distinct spaces, assuming that whatever occupies space must itself be extended in space.

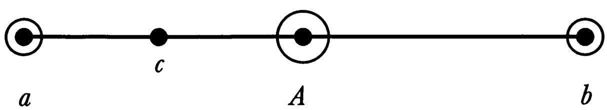
Kant attempts to solve the problem of the divisibility of extension in his 1756 treatise, *Metaphysicae cum geometria iunctae usus in philosophia naturalia, cuius specimen I. continet monadologiam physicam,* and his later rejection or abandonment of the solution.[54] Kant's physical monadology distinguishes between physical and spatial indivisibility, and allows that, while physical monads cannot be decomposed into smaller components, the space or 'spheres of activity', including the part they occupy, remain infinitely divisible. In his 1786 *Metaphysical Foundations of Natural Science,* Kant explictly renounces the proposal as a solution to the paradoxes of the divisibility of extension, which he refers to as an *Ausflucht,* or evasion or subterfuge. Indeed, Kant must have come to regard his physical monadology as unsatisfactory before 1781, and arguably by as early as 1766, or he could not consistently have offered the antithesis of the *Critique*'s second antinomy alongside the thesis as equally implied by the deliberations of pure reason.[55]

Kant explains his reasons for abandoning his previous physical monad theory in the Second Chapter on the 'Metaphysical Foundations of Dynamics', Proposition 4, Observation 1. He notes that from the infinite divisibility of space the infinite divisibility of matter does not follow, unless it is shown that every part of space contains material substance. This he tries to establish by demonstrating that it is impossible for physical points to

[54] *Kants gesammelte Schriften,* pp. 473-487.

[55] Kant, *Metaphysical Foundations of Natural Science,* pp. 49-56.

exist 'in a filled space' and remain mobile within their spheres of activity by 'mere repulsive force', provided that they do not consist in or are not decomposable into other physical points. The reason, according to Kant, is that: "... there can be no point that does not itself on all sides repel in the same way as it is repelled."[56] Kant explains:

> In order to make this fact and thereby also the proof of the preceding proposition intuitable, let it be assumed that *A* is the place of a monad in space, that *ab* is the diameter of the sphere
>
> 
>
>
> of its repulsive force, and hence that *aA* is the radius of this sphere. Thus between *a*, where the penetration of an external monad into the space occupied by the sphere in question is resisted, and *A*, the center of the sphere, a point *c* can be specified (according to the infinite divisibility of space). Now, if *A* resists whatever endeavors to penetrate into *a*, then *c* must resist both the points *A* and *a*, for if this were not so, they would approach each other unimpeded; consequently, *A* and *a* would meet in the point *c*, i.e., the space would be penetrated. Therefore, there must be something in *c* that resists the penetration of *A* and *a*, and thus repels the monad *A* as much as this something is repelled by the monad. Now, since repulsion is a motion, *c* is something movable in space, i.e., matter; and the space between *A* and *a* could not be filled by the sphere of the activity of a single monad, neither could the space between *c* and *A*, and so on to infinity.[57]

Hume's Indivisibles and the Solution to Kant's Antinomy

Kant's second antinomy and Hume's theory of sensible extensionless indivisibles are directly related. The atomistic thesis that every composite substance is constituted by simples in Kant's disproof reformulates Hume's phenomenal reduction of

[56] Ibid., p. 50.
[57] Ibid., p. 51.

extension to experienceable extensionless indivisibles. The contrary anti-atomistic antithesis that no composite thing is composed of simples (since there are none), in turn restates the position of traditional infinitary mathematics and metaphysics that substance and extension or extended things are infinitely divisible. Finally, this appears to contradict Hume's strict finitist doctrine of the divisibility of extension into sensible extensionless indivisibles.

Kant's references to the 'Absolute completeness in the *Division* of a given whole in the [field of] appearance' are usefully compared with Hume's theory of sensible extensionless indivisibles to determine whether or not it escapes Kant's objections. Kant's argument in the antithesis of the second antinomy may have important implications for non-Humean metaphysical atomisms, especially those that are also subject to the second or third prongs of Bayle's trilemma. But the antithesis of Kant's second antinomy leaves Hume's conclusions about the existence of sensible extensionless indivisibles unscathed. Hume would not necessarily accept Kant's crucial inference in (4) of the antithesis argument, that distinct component parts of composite extended things must occupy distinct parts of space. If this implies, as Kant's argument requires, that every part of every extended thing is itself extended, then this is precisely what Hume's theory of sensible extensionless indivisibles is designed to deny. Hume's indivisibles are individually extensionless, and as such are supposed to constitute finite extension only in the aggregate. They are *located in* and *occupy* space, but are decidedly *not extended in* space.

This disagreement between Hume's and Kant's metaphysics of space ripples through Kant's proof of the antithesis of the second antinomy. It is seen in Kant's conclusion (6), that every indivisible part of a composite occupies a distinct space or part of space (an inference Hume rejects); it extends to the derivation of logical inconsistency in proposition (12); and finally expressed in the *reductio* conclusion in (13). Kant on the strength of the contradiction in (12) derives the negation of the assumption that there exists a composite extended thing made up of simple indivisible parts. The problem stands out most clearly in the inference from (11) to (12) in the reconstruction of Kant's proof, where in (11) it is said that only substances can constitute the spatially distinct parts of spatially extended things, and in (12), that every simple indivisible part of a spatially extended thing must be a divisible composite of substances. The inference is invalid unless it is presupposed that only what is extended can occupy space. Proposition (7) must also be understood in this light. When Kant maintains that every spatially extended thing contains a manifold of distinct constituents, and that every spatially extended thing is composite, he evidently intends the principle to apply specifically to the (extended) distinct parts of those composite extended things referred to in premise (4). Otherwise, there is no logical connection between his conclusion in (11), that the distinct parts of spatially extended things are substances, and the paradoxical result in (12), that the indivisible parts of spatially extended things are themselves composed of substances. Hume can agree that the extensionless occupation of space by (indivisible) component parts of composite extended things implies that the constituents of spatially extended things are themselves substances, since he believes them to be sensible, colored and tangible. But from this concession he need not accept Kant's inference that the substantiality of indivisibles entails that the ultimate constituents of spatially extended things are therefore divisible composites of substances.

The conclusion holds only if, as Kant assumes, only extended things can occupy distinct parts of space. This is a proposition, however, which Hume emphatically denies. It might be said that when Kant puts forward assumption (1), he must intend all along to entertain the importantly different proposition that there exists a composite extended thing made up of indivisible *extended* components. This is a premise which, given the Aristotelian categories and related Scholastic apparatus of the argument, undoubtedly proves, as Kant maintains, that such a concept leads to absurdity. Hume's theory of indivisibles bypasses the (1)-(4) subinference in Kant's *reductio*, because the argument has force only if assumption (1) is taken to imply that composite extended things are made up of indivisible extended parts. Such an interpretation of indivisibles is no part of Hume's doctrine of sensible extensionless indivisibles, and is one which he is at pains explicitly to refute. From Hume's standpoint, the concept of extension implies divisibility, and conversely, so that the concept of an extended indivisible is unintelligible. It is no wonder then that Kant, whose starting place in the proof on this interpretation requires a self-contradictory concept of extended indivisibility, ends up deducing an antinomy and advancing a *reductio* disproof of the constitution of extension by indivisibles.

The Object of Kant's Antithesis

The Kantian reply to this Humean objection might take the form of a criticism of the psychological basis of Hume's argument in its appeal to Berkeleyan *minima sensibilia*. Kant's proof of the antithesis of the second antinomy by contrast concerns physical objects and their spatial properties, inviting the argument that Hume's position is sustained only by an invalid inference from perceptual-psychological limitations in the sensory experience and imagination of extended things to the objective limitations of extended things themselves.

For Kant, what are called physical things as the objects of empirical investigation are phenomenal, not noumenal entities. As such, physical things present themselves to perception within the same humanized subjective psychological limitations that Hume is concerned to emphasize. Indeed, the standard way of understanding the antinomies is as delimiting the pretensions of pure reason to arrive at significant synthetic *a priori* metaphysical truths, in this case about the divisibility of extension. Kant must finally agree that empirical evidence of the sort Hume offers in the inkspot experiment is the best and only permissible testimony as to the real properties of phenomenal physical objects. This is especially true where the properties of the objects of sensation reflect psychological limitations in the experience of space and time, which for Kant are subjective pure forms of intuition that do not belong to the noumenal thing-in-itself. Hume raises multiple conceptual arguments against infinite divisibility, which collectively have the effect of disallowing the infinitist alternative. Together with Bayle's skeptical conclusions about the possibility of understanding the divisibility of extension into extended physical points, Hume's theory of extensionless sensible indivisibles gains support not only as an inference from perceptual limitations, but as the best nonskeptical account of the real divisibility of extension.

Further difficulties might be raised against Hume's doctrine of sensible extensionless indivisibles by applying Kant's proof of the antithesis of the second antinomy. Thus, it might be asked how Hume's *minima sensibilia* can appear colored if they are also extensionless. Color sensations are produced by absorption and reflection of lightwaves from an object, and presumably no object can emit lightwaves or reflect back rays of light if it has no spatial extension. The conclusion is unwarranted unless it is presupposed in question-begging fashion against Hume's position that in order to occupy space something colored must also be extended, which again is precisely what Hume denies in charting a fourth alternative to Bayle's trilemma. The inkspot experiment indicates that a point is reached at which a colored object appears just at the threshold of invisibility, such that, if it were any smaller, it could not be seen at all. The units into which extension is divisible for Hume are sensible, extensionless, and indivisible. If two or more of Hume's sensible extensionless indivisibles are placed in juxtaposition, their composite is divisible, which is to say extended. This is not an obviously defective basis for a phenomenal theory of extension in space, and with it Hume is able to avoid the negative consequences of the antithesis of Kant's second antinomy.

The antithesis of Kant's second antinomy is by no means directed against Hume's theory of sensible extensionless indivisibles. Kant in fact seems oblivious of Hume's doctrine, restricting his references to Hume's ideas in the *Critique* to the empiricist analysis of causation, the denial of *a priori* knowledge (which Kant obviously did not share), and skepticism concerning the participation of reason in the justification and acceptance of religious belief.[58] In his 'Observation on the Second Antinomy', Kant explicitly identifies the antithesis as targeting philosophical positions quite different from Hume's:

> Against the doctrine of the infinite divisibility of matter, the proof of which is purely mathematical, objections have been raised by the monadists. These objections, however, at once lay the monadists open to suspicion. For however evident mathematical proofs may be, they decline to recognize that the proofs are based upon insight into the constitution of space, in so far as space is in actual fact the formal condition of the possibility of all matter.[59]

Kant does not seem to intend the term 'monadist' as referring generically to atomists of every persuasion, but more specifically to followers of Leibniz's metaphysics. In his commentary on the thesis of the antinomy, Kant asserts: "The word *monas*, in the strict sense in which it is employed by Leibniz, should refer only to the simple which is *immediately* given as simple substance... and not to an element of the composite."[60]

The sense of the term that emerges in Kant's discussion is not sufficiently general to include Hume's refutations of infinite divisibility. Unless he radically misunderstands Hume's position, Kant cannot have Hume in mind as one of the monadists who raise objections to the mathematical arguments for infinite divisibility. Kant identifies the monadist's indivisibles first as mathematical points in the ideal Euclidean sense, and second as *non*sensible 'physical' points, akin to Bayle's

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[58] *Critique of Pure Reason*, B5, B19-20, A760/B788-A761/B789; B127-128; A745/B773-A747/B774.

[59] Ibid., A439/B467. See Al-Azm, *The Origins of Kant's Arguments in the Antinomies*.

[60] *Critique of Pure Reason*, A442/B470.

disparagement of the 'nothingnesses of extension'.[61] Kant explains:

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[61] It is widely held that Leibniz accepted the existence of actual infinities. Leibniz, *Theoria motus abstracti*, *Die philosophischen Schriften von Gottfried Wilhelm Leibniz*, Vol. 4, p. 228: "*Dantur actur partes in continuo ... eaeque infinitae actu.*" See Capek, "Leibniz on Matter and Memory", in *The Philosophy of Leibniz and the Modern World*, edited by Leclerc, p. 89: "In accepting the mathematical continuity of space, time, and motion, Leibniz — again like Hobbes — began to grapple with the basic problem of the infinitesimal calculus — that is, with the nature of the infinitely small elements of space, time, and motion. Leibniz definitely accepted actual infinity; in the first two paragraphs of [*Theoria motus abstracti*], he explicitly stated that there are actual parts in a continuum and that they are actually infinite in number." For further evidence, see Leibniz, Letter to Arnauld, Göttingen, 30 April 1687, *Discourse on Metaphysics, Correspondence With Arnauld, Monadology*, p. 192: "I can conceive of properties in the substance which cannot be explained by extension, by form and by motion, quite apart from the fact that there is no exact and definite *form* in bodies because of the actual subdivision of the continuum to infinity..." Russell, in *A Critical Exposition of the Philosophy of Leibniz*, Chapter IX, 'The Labyrinth of the Continuum', pp. 109-110, claims that despite this evidence: "... Leibniz denied infinite *number*, and supported this denial by very solid arguments... 'The true infinite,' he says, 'exists, strictly speaking, only in the *Absolute*, which is anterior to all composition, and is not formed by the addition of parts'... We must agree, therefore, that Leibniz's views as to infinity are by no means so simple or so naive as is often supposed." Russell's quotation is from Leibniz's *New Essays on Human Understanding*, where Leibniz in dialogue form has his apparent spokesman Theophilus maintain, here in the translation edited by Remnant and Bennett, §158: "It is perfectly correct to say that there is an infinity of things, i.e. that there are always more of them than one can specify. But it is easy to demonstrate that there is no infinite number, nor any infinite line or other infinite quantity, if these are taken to be genuine wholes... The true infinite, strictly speaking, is only in the *absolute*, which precedes all composition and is not formed by the addition of parts... But it would be a mistake to try to suppose an absolute space which is an infinite whole made up of parts. There is no such thing: it is a notion which implies a contradiction; and these infinite wholes, and their opposites the infinitesimals, have no place except in geometrical calculations, just like the use of imaginary roots in algebra." Russell begins this section by quoting Leibniz, *Die philosophischen Schriften*, Vol. 1, p. 416. Leibniz writes, p. 109: "I am so much in favour of the actual infinite, that instead of admitting that nature abhors it, as is commonly said, I hold that nature affects it everywhere, in order the better to mark the perfections of its

> Were we to give heed to [the monadists], then beside the mathematical point, which, while simple, is not a part but only the limit of a space, we should have to conceive physical points as being likewise simple, and yet as having the distinguishing characteristic of being able, as parts of space, to fill space through their mere aggregation.[62]

The concept Kant rejects is absurd if, but not only if, the indivisibles of spatial extension are supersensibly ideal. Then no assemblage of indivisible points could conceivably constitute sensible extension in space.

The point is already made by Bayle and Hume. Hume deliberately tries to avoid the problem in his theory by allowing that sensible extensionless indivisibles collectively comprise the simple irreducible constituents of sensible spatial extension. Kant concludes: "... when philosophy plays tricks with mathematics, it does so because it forgets that in this discussion we are concerned only with *appearances* and their condition."[63] Kant takes refuge in the distinction of the Transcendental Aesthetic between appearance and noumenal thing-in-itself, conceding, though without compromising the negative conclusion of the antithesis in the second antinomy, that: "The argument of the monadists would indeed be valid if bodies were things in themselves."[64]

Kant's repudiation in the *Critique* A165-66/B206-7 of "... idle objections ... that objects of the senses may not conform to such rules of construction in space as that of the infinite divisibility of lines or angles...", which he further castigates as "the chicanery of a falsely instructed reason", has sometimes

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author. So I believe that there is no part of matter which is not, I do not say divisible, but actually divided; and consequently the least particle must be regarded as a world full of an infinity of different creatures." Concerning Leibniz's invention of the calculus of infinitesimals, see "*Historia et Origo Calculi Differentialis*", *The Early Mathematical Manuscripts of Leibniz*, pp. 22-58. Aiton, *Leibniz: A Biography*, pp. 40-70.

[62] *Critique of Pure Reason*, A439/B467.

[63] Ibid., A441/B469.

[64] Ibid.

been understood as a reference to Hume's finitism. Kant disallows such a metaphysics of space as incompatible with Euclidean geometry in the description of appearances, which for Kant is *a priori* indisputable (hence, his abusive polemics against the finitist). But physical monads are unrelated to Hume's sensible extensionless indivisibles, for two reasons. First, Kant, unlike Hume, does not require physical monads to be sensible. Second, Hume, unlike Kant, requires that sensible extensionless indivisibles are not just physically but spatially indivisible or unextended, and does not permit the spaces they occupy to be infinitely spatially divisible even within what Kant calls their respective 'spheres of activity'.

Whether or not Kant accepts Hume's theory, sensible extensionless indivisibles provide the basis for refuting the antithesis of Kant's second antinomy. As such, they deserve to be taken into consideration by Kant in this context, even if Hume's doctrine is somehow conceptually incoherent. Obviously, both the thesis and antithesis of any Kantian antinomy must in some sense be conceptually incoherent, since each contradicts the other, and each is supposed to be implied by pure reason. If Kant regards Hume's theory of space as unworthy of consideration in the second antinomy, that is further evidence, seen also in Kant's argument against physical monads in the *Metaphysical Foundations of Natural Science*, of his entrenchment in rationalist infinitary Euclidean dogma. Hume, it should be clear by now, observes no such constraints in his empiricist metaphysics of space.

Was Hume's Theory of Space Unknown to Kant?

Was Kant then unaware (and if so, why?) that Hume's metaphysics of sensible extensionless indivisibles provides an analysis of simples that eludes the conclusion of the antithesis in the second antinomy?

Hume's doctrine of sensible extensionless indivisibles neatly avoids the objection to (nonsensible, extended) indivisibles in the antithesis of Kant's second antinomy, just as it side-steps similar skeptical criticisms raised by Bayle's trilemma. Kant's philosophy is known to have been influenced by Hume's writings on metaphysics, but surprisingly Kant evinces no appreciation of the implications of Hume's sensible extensionless indivisibles in forestalling the antithesis of the second antinomy.

That Kant's criticism of atomistic metaphysics in the antithesis is not specifically directed against Hume makes it all the more interesting that Hume's doctrine escapes its negative consequences. The fact suggests that, despite Hume's influence on Kant's ideas, Kant was not sufficiently familiar with Hume's theory of spatial extension to realize its immediate implications for the antithesis in the *Critique*'s second antinomy. This would not be an unreasonable or unprecedented historical hypothesis, nor any complaint against Kant's reading and understanding of Hume, judged in historical context. Kant probably did not intend the antithesis of the second antinomy as a criticism of Hume's theory of sensible extensionless indivisibles. But Kant's purposes are irrelevant to the failure of his argument for the second antinomy, in light of the unanticipated possibility presented by Hume's metaphysics of space. An uncritical acceptance of the rationalist proposition that spatially extended things alone can occupy a location in space may be one dogmatic slumber from which Hume's 'Academic' metaphysical skepticism was unable to awaken Kant.[65]

[65] Kant, *Prolegomena*, p. 5: "I openly confess that my remembering David Hume was the very thing which many years ago first interrupted my dogmatic slumber and gave my investigations in the field of speculative philosophy a quite new direction."

CHAPTER 6

CLASSICAL MATHEMATICS AND HUME'S REFUTATION OF INFINITE DIVISIBILITY

Reason and Experience

The fact that Hume is able to marshall four distinct *reductio* disproofs together with the geometry dilemma against infinite divisibility, all of which also at least indirectly support his doctrine of sensible extensionless indivisibles, makes an impressive inductive appeal for the truth of his conclusions.

The arguments are so different in assumptions and structure that it is unlikely though obviously not impossible for there to be a single pervasive common error. The assumptions, mostly borrowed from rationalist metaphysics and epistemology for purposes of indirect proof, are sufficiently unlike Hume's empiricist stance to lend powerful independent collateral support to his central refutation of infinite divisibility and justification of the theory of sensible extensionless indivisibles in the inkspot argument. The wide variety of assumptions in Hume's four *reductio* arguments and geometry dilemma advertise the limitations of armchair methods of pure reason and conceptual analysis, and indicate that even from the rationalist's turf there can be no adequate logically coherent idea of infinite divisibility. This further diminishes the likelihood that there are common inferential fallacies invalidating Hume's six *Treatise* arguments, including the inkspot argument, four *reductio* disproofs of infinite divisibility, and geometry dilemma. Nor is it probable, though again obviously not impossible, that a philosopher as astute as Hume would have put forward six distinct arguments attacking infinity and upholding sensible extensionless indivisibles, each containing distinct false premises other than those introduced for *reductio* purposes, or distinct logical invalidities, causing each argument to converge as if by accident on the same false conclusions about the divisibility of extension.

There is a rhetorical dimension to the fact that Hume presents five arguments against distinct infinitist assumptions in the *reductio* arguments and geometry dilemma, but has only one key argument to give from his own experiential standpoint in the inkspot experiment. It is possible to see in this opposition an anticipation of Kant's criticisms of rationalism in the *Prolegomena to Any Future Metaphysics*. There rationalism earns Kant's contempt as unscientific in method because its results are not repeatable by multiple practitioners all claiming to follow the dictates of pure reason. Kant's criticism is substantiated by the factionalism among celebrated rationalists of the seventeenth century, who in essentials profess to be practicing the same method, but who reach dramatically different conclusions.[66] The limitations of rationalist methods in the exercise of pure logic and definition of terms or conceptual analysis enable it to produce grandiose internally consistent but free-floating metaphysical systems that carry no guarantee of connection with reality. This was a source of common dissatisfaction with rationalism by Hume's time. The poverty of rationalism led to various attempts by philosophers to replace it with empirical methods that could offer not only logical consistency in science and metaphysics, but also establish truth by anchoring philosophical thought to reality through the causal interaction of world and mind in perception. Kant's criticism of rationalism as leading to too many internally coherent but mutually contradictory systems, which Hume undoubtedly shares at least in spirit, is reflected in the contrast just observed between Hume's single wholeheartedly phenomenal argument against infinite divisibil-

[66] Kant, *Critique of Pure Reason*, Aix-x/Bxx; *Prolegomena*, pp. 1-9. See Jacquette, "The Uniqueness Problem in Kant's Transcendental Doctrine of Method", pp. 425-438.

ity and in support of extensionless indivisibles in the inkspot experiment versus five distinct *reductio* proofs intended to discredit theories of infinitely divisible extension in the *Treatise*. The implication is that empiricism not only leads to a more unitary metaphysics of extension, but that a single empiricist argument is sufficient to hold sway against five distinct rationalist theories based on five different rationalist assumptions. Hume's *reductio* arguments are particularly pointed because their indirect proof structures call attention to unnoticed contradictions, inconsistencies, and absurdities in metaphysics that are supposed to depend fundamentally on logic, pure reason and conceptual analysis, uncontaminated by empirical uncertainties. What deeper indictment of rationalist methodology could there be than the discovery of logical inconsistencies within its implications?

An empirical metajustification for Hume's critique, which Hume himself might find agreeable if indecisive, is still no substitute for the painstaking critical examination and appraisal of each of Hume's six proofs. These have already appeared in preceding discussions of the arguments, all of which can be plausibly reconstructed as valid *reductio* inferences. It remains only to draw general conclusions about the nature of Hume's *reductio* proofs, to place them in perspective alongside Hume's inkspot argument, compare them with the strengths and weaknesses of Hume's positive theory of sensible extensionless indivisibles, and the philosophical problems to which the negative and positive parts of Hume's two-fold task in his critique of infinity are addressed, in order to judge their effectiveness.

The premises Hume introduces for indirect proof against infinitist mathematics and metaphysics include the hypothesis that extension in space or time is infinitely divisible, that only unitary things exist, that whatever is clearly conceivable is possible, and, in the case of the geometry dilemma reconstructed as a *reductio*, that a classical infinitist Euclidean geometry implying the infinite divisibility of extension provides an adequate idea of exact equality and proportion. These do not exhaust the range of assumptions relevant to the problem of infinity, but represent an interesting choice of starting places. It is difficult to see how the traditional rationalisms with which Hume was familiar and against which he may have intended the *reductio* arguments to be directed could derive and maintain their philosophical systems without the characteristic assumption that whatever is clearly conceivable is possible. This alone in Hume's argument from the conceivability of indivisibles leads to outright absurdity reflecting back on the falsehood of the assumption that extension in space is infinitely divisible. The proposition that only unitary things exist in Malezieu's argument from the unity of existents is not quite so central to rationalist methodology. Yet it is also aprioristic in its presupposition that the concept of an existent thing implies its unity. Hume shows that even this innocent-appearing premise leads to logical contradiction. The exclusion from Hume's *reductio* arguments of any characteristic empiricist assumptions provides additional evidence that the indirect proofs are aimed specifically at nonempiricist infinitisms. This consideration further upholds Hume's skeptical position that a properly experiential epistemology and humanized philosophy of mind cannot possibly support an adequate metaphysics of infinity or infinite divisibility.[67]

To complete the evaluation of Hume's *reductio* proofs in the *Treatise*, it is important to weigh objections to his positive theory of sensible extensionless indivisibles. The opposition between infinite divisibility and extensionless indivisibles implicit in Hume's critique makes it necessary to answer questions about

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[67] Leibniz also accepts the unity of existents thesis. He repeats the argument of Bayle's trilemma, including Bayle's skeptical conclusion, but offers another solution. *Die philosophischen Schriften*, II, p. 96 (translation in Russell, *A Critical Exposition of the Philosophy of Leibniz*, p. 103): "Where there are only beings by aggregation, there are not even real beings. For every being by aggregation presupposes beings endowed with a true unity... [If we admit aggregates] we must either come to mathematical points ... or to the atoms of Epicurus ... or we must avow that there is no reality in bodies, or, finally, we must recognize in them some substances which have a true unity." See also Russell, pp. 150, 239-242.

his concept of space in order to sustain even his negative arguments against infinite divisibility. If there are difficulties in Hume's doctrine of sensible extensionless indivisibles, then extension may be infinitely divisible after all, in which case there must be logical flaws in the inkspot argument and in the four *reductio* proofs and geometry dilemma.

Criticisms of Hume's Arguments

The first criticism is that extensionless entities cannot collectively constitute extension. This problem has already been dealt with in discussing Broad's phenomenological criticisms of the inkspot experiment, in the analysis of Hume's sensible extensionless indivisibles as a solution to Bayle's trilemma, and in examining the antithesis in Kant's second antinomy. Bayle's complaint that ideal Euclidean mathematical points are just so many 'nothingnesses of extension' that cannot constitute extension individually or in the aggregate is fully acknowledged by Hume. The significant difference in his theory of extensionless indivisibles is that the atomic components of spatial extension are not ideal, but sensible, despite Hume's occasionally misusing the expression 'mathematical points' to refer to sensible indivisibles.

Hume's sensible indivisibles are experienced by vision and touch. He says they are colored and tangible, even though individually each is extensionless. The indivisibles are discovered at the very threshold of experience of miniature objects in the phenomenal field. Hume's indivisibles have both the property of being sensible, and of being such that their further subdivision and continued existence as sensible things is impossible. The *minima sensibilia* in Hume's theory of extension avoid all three prongs of Bayle's trilemma by virtue of being sensible, extensionless, and indivisible. That Hume's indivisibles are sensible rather than ideal protects his doctrine from Bayle's objection that ideal abstract Euclidean mathematical points are mere 'nothingnesses of extension' that can never be strung together so as collectively to constitute real sensible extension. The visual and tactile properties of Hume's sensible indivisibles imply that the juxtaposition of two or more constitutes extension. It is expressly to avoid the objection that ideal extensionless indivisibles cannot produce spatial extension that Hume requires his indivisibles to be sensible rather than ideal or abstract. Were it not for this innovation, Hume's doctrine would also fall prey to Bayle's *ex nihilo nihil fit* objection to infinite divisibility in the classical theory of extension. If Hume's solution is correct, it deftly escapes Bayle's trilemma by identifying a fourth unanticipated way between Bayle's three horns.

These distinctions indicate a way to soften if not repel a related objection that Hume's doctrine of sensible extensionless indivisibles is subjective, and that the sensible extensionless indivisibles the theory postulates are subjectively relative. The presumption that motivates most philosophical criticisms of subjectivism is that subjective theories cannot provide adequate foundations for objective science. But even if Hume's doctrine of sensible extensionless indivisibles is subjective, and even if extension in space is in some sense objective rather than subjective, it need not follow that Hume's humanized methodology is incapable of supporting a scientifically respectable objective theory of extension. Objectivity despite subjective humanization of the theory of extension is preserved if the underlying structural features of the subjective experience of space which Hume reduces ultimately to sensible extensionless indivisibles are shared by all minds. The spectre of subjectivity is usually the fear that a theory might entail that individual subjects are somehow able to determine reality by an exercise of will, and that different subjects might create for themselves different realities. That is the ambiguity inherent in Protagoras's dictum that man is the measure of all things. There is no reason to suppose that a subjective reading of Hume's doctrine of sensible extensionless indivisibles contradicts the objectivity of external space or extension in space, or somehow potentially fragments the unitary objective reality into as many realities as percipient subjects. Hume bypasses the indictment of subjectivism altogether in the theory of sensible extensionless indivisibles. If he so chooses, he can therefore avoid any philosophically objectionable subjectivisms that may contradict objectivity in the metaphysics of space. By humanizing epistemology and philosophy of mind, and considering the implications of his revisionary philosophy for the mathematical theory of the divisibility of spatial extension, Hume is not committed to an individual or willfully efficacious world-determining subjectivism.

There is finally a theoretical advantage in the subjectivist interpretation of Hume's indivisibles. The appeal to the subjectivity of indivisibles may make it more easily intelligible to speculate that for Hume the divisibility of finitely extended bodies is finite but inexhaustible rather than infinite in cardinality. In that case, not only is there no predetermined terminus, but also no predeterminable limit even in principle to the subjective finite subdivision of finite extension. For then there simply are no *minima sensibilia absolutiva*, but only (subjectively) *minima sensibilia relativa*. The distinction suggests another important sense in which Hume's *Treatise* may have sought to introduce the Newtonian experimental method of reasoning into moral subjects.[68]

The divided core of Hume's skeptical philosophy and psychology of doxastic involuntarism suggests another direction for mediating the subjectivity of extensionless indivisibles and the objectivity of extension in space. David Fate Norton's explanation of the interplay between metaphysical skepticism and commonsense natural belief in Hume's thought makes it attractive to consider the following possibility. Hume might agree that the objectivity of extension in space, and more particularly the exact size of indivisibles, is something for which sentiment disposes human nature toward such an almost irresistible natural belief, that it may appear self-evident, even unquestionable, that the dimensions and numbers of the sensible extensionless indivisible units of extension must be

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[68] Cotes, "Preface" to the Second Edition of Newton's *Philosophiae Naturalis Principia Mathematica, Sir Isaac Newton's Mathematical Principles of Natural Philosophy and his System of the World*, pp. xx-xxvi. See Jacquette, "Aesthetics and Natural Law in Newton's Methodology", pp. 659-666.

objective. But in reflective skeptical reasoning, Hume might also see philosophy as over-riding natural belief, contradicting natural attitude in a skeptical philosophical stance by which the proper criteria for identifying and judging the size of indivisibles can only be subjectively relative.

Norton reinforces Hume's distinction between natural belief in the objective existence and permanence of the external world, when he states:

> ... belief in the objective existence and permanence of the external world is natural and, if we take [Hume] literally, quite unavoidable: " 'tis in vain to ask," he says, "*Whether there be body or not?* That is a point, which we must take for granted in all our reasonings." If this is so, then there is no real question about the existence of body, there is no possibility of pursuing in any significant fashion one traditional metaphysical issue.[69]

Things are not this simple because of the conflict in Hume's dual commitment to both natural belief governed by passion and metaphysical skepticism guided by reason. Reason can cast doubt on that which commonsense sentiment takes for granted, and in which it may for all its conviction be mistaken. Natural belief in the mind-independent objectivity of space and extension in space may therefore be checked by the skeptical conclusions of reason in a correct metaphysics. "Hume thought," Norton continues, "that many principles of nature are hidden from view, and for this reason also he wished to encourage a critical, yet speculative, philosophy. The errors of our natural attitude are not mere blunders, but are, rather, systematic shortcomings due to the depth and complexity of phenomena."[70] This interaction allows Hume to hold that natural belief in the mind-independent objectivity of extension may yet be contradicted by reason in a true metaphysics,

[69] Norton, *David Hume: Common-Sense Moralist, Sceptical Metaphysician*, p. 216. Norton refers to *Treatise*, p. 187.

[70] Norton, p. 218.

possibly of the sort Kant proposes in the Transcendental Aesthetic.[71]

The apparent subjectivity in Hume's inkspot experiment criterion for the existence and dimensions of sensible extensionless indivisibles might belong exclusively to the mind's idea of the elements of extension rather than to indivisibles themselves. Hume in his moments of Academic skepticism might prefer to remain silent about such matters of speculative metaphysics concerning the exact nature of these entities, having satisfied himself that extension must ultimately be constituted by sensible extensionless indivisibles. The same distinction holds according to many commentators for Hume's celebrated three-part analysis, not of causation itself, but of the idea and psychological origins of the idea of causation and necessary causal connection, into ideas of temporal order, spatial propinquity, and regular succession. Hume's account on inspection does not deny the necessity of causal connection, but shows that there is no adequate philosophical proof of causal necessity, attributes the necessity of causal connection to the psychology of the mind's understanding, and explains the sources of natural belief in necessary causal connection via human instincts and human psychological faculties.

Hume knows that he cannot rely on the *reductio* proofs alone to defeat the infinite divisibility thesis and uphold the doctrine of sensible extensionless indivisibles. This might otherwise be regarded as a plausible alternative, since the *reductio* arguments are in some ways less problematic than the inkspot argument. The inkspot argument is subject to criticisms like those raised by Broad, problems of subjective relativity in demands that the theory determine *minima sensibilia absolutiva* rather than *relativa* by the phenomenal inkspot criterion, and the like. Yet Hume cannot give up the inkspot argument in favor of his less controversial indirect proofs, because at most these arguments

[71] Norton, p. 223. On the problem whether Hume accepted the reality of causation see Strawson, *The Secret Connexion: Causation, Realism, and David Hume.*

demonstrate that infinite divisibility is absurd, and that there are extensionless indivisibles. But Hume's *reductio* proofs by themselves do not entail the specific kind of extensionless indivisibles, sensible or insensible, ideal and abstract, into which extension is finitely divisible. Hume needs sensible as opposed to insensible extensionless indivisibles in order to avoid the third prong of Bayle's trilemma, for this is his only way to answer the problem of how sensible extension can be constituted out of insensible indivisible extensionless ideal Euclidean mathematical points as 'nothingnesses of extension'. The condition that extensionless indivisibles are sensible or experienceable can only be supplied by the phenomenal data made possible by something like the equivalent of the inkspot experiment. Hume's *reductio* arguments, due to their essentially negative direction in overturning the infinite divisibility thesis in classical metaphysics, may help to prove that there are extensionless indivisibles, but by themselves do not entail Hume's positive empiricist doctrine in its entirety that extensionless indivisibles are sensible, and as such capable of constituting extension in the aggregate.

Imagism in Hume's Philosophy of Mind

If we follow Hume's exposition from the *reductio* disproofs against infinite divisibility back to the inkspot argument, we can appreciate their logical interrelation. The inkspot argument depends essentially on the claim that there can be no adequate idea of infinity or of the infinite divisibility of extension. The *reductio* arguments and geometry dilemma depend on Hume's claim that we have no adequate idea of infinity or of the infinite divisibility of extension, a position reinforced by contradictions from assumptions that presuppose the existence or adequacy of such ideas. Hume's most fundamental objection to the possibility of the mind's possessing adequate ideas of infinity and infinite divisibility appears to rest on a questionable theory of imagism in the philosophy of mind. A sound objection to Hume's alleged imagism would therefore constitute one of the potentially most serious criticisms of both the inkspot argument and *reductio* proofs.

Flew interprets Hume's first *Treatise reductio* from the addition of infinite parts as unreasonably faulting the idea of infinite divisibility because of the mind's inability to picture it. Flew regards Hume's commitment to mental imagism generally as invalidating the refutation of infinite divisibility. He argues:

> [Hume] equates conceiving with imagining, and mistakes it that to imagine — or at any rate to be able to imagine — is always to form — or at any rate to be able to form — the appropriate mental image or images.[72]
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> Thus Hume tells us: " 'Tis an establish'd maxim in metaphysics, *That whatever the mind clearly conceives includes the idea of possible existence*, or in other words, *that nothing we imagine is absolutely impossible*. We can form the idea of a golden mountain, and from thence conclude that such a mountain may actually exist" (T 32; emphasis added). Earlier he had spoken of "one image or idea" (T 22) in a way which makes it clear that for him "image" and "idea" refer here to the same thing; and he will later desire "our mathematician to form ... the ideas of a circle and a right line; and ... ask, if upon the conception of their contact he can conceive them as touching in a mathematical point, or if he must necessarily imagine them to concur for some space" (T 53). Since conceiving is thus identified with imagining, and since both are always thought of in terms of forming mental images, to "attain a full and adequate conception of infinity" would be to have a clear and distinct idea, or image, of infinity.[73]

Flew's objections to Hume's imagism are made specifically with reference to Hume's appeal to the limitations of mental images in the *reductio* argument from the addition of infinite parts. But similar criticisms might be leveled wherever Hume seems to rely on the properties of clear and distinct ideas of extension or the mind's ability or inability to depict the divisibility of extension. This makes it appropriate to consider the problem

[72] Flew, "Infinite Divisibility in Hume's *Treatise*", p. 259.

[73] Ibid.

of Hume's imagism more generally than in application to any particular *reductio.*

The attribution of mental imagism to Hume is problematic. Certainly Hume nowhere explicitly states that adequate ideas must always or can only be represented or manifested psychologically by mental images. If Hume is committed to this or another strong version of imagism to which philosophical objections are often directed, the testimony must be indirect and inferential. Flew's evidence that Hume accepted some formulation of imagism in particular is equivocal and inconclusive. All it shows in the first instance is that Hume endorses the principle that conceivability implies possibility. We have already seen this quasi-rationalist thesis at work in the reconstruction of Hume's argument from the conceivability of indivisible mathematical points. But the principle by itself does not say what conceivability consists in; in particular, it does not say that conceivability requires mental images, nor of what kind or what role if any mental images are supposed to play in the conception of ideas.[74] Nor should we suppose that Hume regards the inference in the addition of infinite parts *reductio* as implying the mental image of infinitely many extended components of a finite extension being added infinitely one to another paradoxically to produce infinite extension. Hume's strict finitism manifestly excludes the possibility of any such mental image as an impression of reflection in the imagination. The conclusion of Hume's *reductio* is not absurd because it cannot be mentally pictured, but because the same hypothetically infinitely divisible extension cannot be both finite and infinite in length.

Flew's efforts to assimilate Hume's references to 'clear and distinct ideas' with mental images are also disputable, as is his undocumented identification of Hume's 'conceiving with imagining'. Even if Hume equates these concepts, this by itself would not show that he regards imagining as necessarily involving occurrent mental images, that he equates imagining with imaging, despite the potentially philosophically misleading

[74] See note supra 37.

etymology. Flew claims that Hume's use in the *Treatise* of the phrase 'image or idea' convicts him of imagism, but the context makes it equally plausible to understand the 'or' as the 'or' of logical alternation rather than explication or equivalent alternative paraphrase. When Hume says, "That we may fix the meaning of the word, figure, we may revolve in our mind the ideas of circles, squares... and may not rest on one image or idea,"[75] he can easily if not more naturally be read as meaning that the definition of 'figure' need not be fixed with several different images, and, alternatively, need not be fixed by several different ideas, rather than as equating images and ideas, or even images and adequate ideas. Later, Flew acknowledges a similar ambiguity in Hume's reference to 'images' and 'ideas':

> "Nothing can be more minute," [Hume] tells us, "than some ideas, which we form in the fancy; and images, which appear to the sense; since there are ideas and images perfectly simple and indivisible." ("Images" is being used here, presumably for stylistic reasons, as a synonym for "impressions.")[76]

Yet there is no recognition by Flew of the possibility that Hume might also intend 'image' in the sense of an impression or visual image rather than mental image in the passages Flew regards as evidence for Hume's imagism. Even if Hume means mental images in these places, it does not follow that he equates image with idea or concept. His philosophy of mind might be rich enough to allow distinct irreducible categories of ideas or concepts and mental images, interrelated in complex ways.

The same is true of Hume's invocation to mathematicians to try to 'form' 'the idea of a circle and a right line' in judging whether 'if upon the conception of their contact he can conceive them as touching ... or if he must necessarily imagine them to concur for some space'. Hume's remarks imply mental imagism only if forming an idea, conceiving, or imagining something requires an occurrent mental image. But non-imagists and even anti-imagists frequently use this kind of

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[75] *Treatise*, p. 22.

[76] Flew, "Infinite Divisibility in Hume's *Treatise*", p. 259.

mentalistic language without thereby condoning the existence of accompanying mental images in psychological processes. They do not give up talking about forming ideas, conceiving, or imagining, but offer an alternative non-imagistic explanation of what it means to form an idea, or conceive of or imagine something. From the equivocal statements above, it appears that Hume if pressed could do the same.

It has seemed natural to more commentators than Flew to charge Hume's phenomenal psychology with some kind of mental imagism. John Bricke, in his study of Hume's philosophy of mind, concludes: "Were Hume not a thorough-going imagist he would have provided no theory of thinking at all."[77] Bricke faults Hume's 'thorough-going' imagism among other things for being unable to explain the exercise of concepts without the distinction between imaging and recognizing what is imaged. But the evidence Bricke offers of Hume's commitment to a strong or thorough-going imagism is no more complete or convincing than Flew's.[78] As hard as it may be to pin down Hume's position, it is excessive to deny that he accepts any form of mental imagism. He often suggests that when the mind is called upon to justify a concept it must be able to consult the contents of corresponding mental images in order to verify the properties attributed to certain ideas. This is evidently what Hume intends when he asks us to imagine the number of possible divisions of a barley corn or grain of sand, or to think of the inkspot on paper. But does Hume regard all imagining as mental imaging? If Hume's thought experiments are supposed to involve actual mental images, does he regard the occurrence of mental images as essential to his arguments or as a mere aid to reflection? It is worthwhile in trying to understand Hume's reliance on imagination and introspection to review some of the criticisms that have been raised against various types of mental imagism, and then to establish the specific

[77] Bricke, *Hume's Philosophy of Mind*, p. 108.
[78] Ibid., pp. 109-112.

sort of imagism Hume's 'phenomenological' arguments against infinite divisibility seem to require.

There are several kinds of mental imagism, not all of which are philosophically objectionable. Imagism has become an unpopular theory in the philosophy of mind. Yet many researchers in psychology and the brain sciences speak without embarrassment of the mind's ability to entertain nonoptical mental pictures, including dreams, memories of faces seen in early childhood, mental maps of routes followed in goal-oriented behavior, *Gestalts* of various basic types, and the like. Scientific experiments have been designed to test differences in the time required for subjects to accomplish problem solving tasks based on differences in the kinds of mental images they are instructed to call forth, examine, and apply.[79] The introspective data for the existence of mental images is overwhelming. They are widely attested by subjective reports of psychological experience that anti-imagists have yet to satisfactorily explain away.[80] The same phenomenological evidence equally reveals that thinking does not always or

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[79] Hannay, *Mental Images: A Defense*, p. 19: "It would be an exaggeration to say there was a conspiracy against mental images. But 'campaign' would not be too strong a word. The author of a recent account [Dennett, below, p. 141] admits, with disarming candour, that to be able to dispose of mental images would be 'a clear case of good riddance'. The same sentiment is conveyed, less openly but just as unequivocally, in most major recent philosophical writings on imagination. I refer principally to the work of Ryle, Shorter, and Sartre, but not only to them. 'Away with the image!' is a call that finds a welcoming response among many contemporary philosophers of mind, whether the thesis is a plain denial of the existence of mental images, or some more qualified denial, for example that there are no such objects, or that when we see in the mind's eye it is not a mental image that we see." The existence and theoretical import of mental images is disputed by Dennett, *Content and Consciousness*.

[80] The debate is represented by the positions of contributors to Block, ed., *Imagery*, especially in the papers by Dennett and Fodor. See Richardson, *Mental Imagery and Human Memory*. Hampson, Marks, and Richardson, *Imagery: Current Developments*. Horowitz, *Image Formation and Cognition*. Tye, *The Imagery Debate*. See Franklin, "Achievements and Fallacies in Hume's Account of Infinite Divisibility", pp. 89-90.

necessarily occur in terms or by means of mental images. A moderately critical attitude toward mental imagism should therefore adjudicate selectively against strong formulations of the theory that attribute the entertainment of mental images to all thinking. But it need not take issue with the more reasonable view that thinking sometimes takes place or can take place in terms or by means of mental images. If there are sound philosophical criticisms of the thesis that thinking can sometimes make use of mental images, they have yet to be established, and must meet a rigorous explanatory burden while contradicting the evidence of everyday experience.[81]

Hume's inkspot and *reductio* arguments against infinite divisibility might presuppose some version of mental imagism. But since Hume's pronouncements about the phenomenology of adequate ideas of the divisiblity of extension are ambiguous, the best method of interpretation in the absence of conclusive textual evidence is to impute to Hume's theory only the weakest minimal formulation of imaginism needed to uphold his arguments against infinite divisibility and in support of his doctrine of sensible extensionless indivisibles. There are several alternative versions of mental imagism available to Hume. Hume in principle might believe that:

(1) All (adequate) ideas of the divisibility of extension are or require corresponding mental images.
(2) Some (adequate) ideas of the divisibility of extension are or require corresponding mental images.
(3) Not all (adequate) ideas of the divisibility of extension are or require corresponding mental images, but there can exist such mental images which the mind can consult in order to verify the properties of corresponding (adequate) ideas.
(4) Not all (adequate) ideas of the divisibility of extension are or require corresponding mental images, but there definitely exist such mental images which the mind must

[81] For criticisms of Ryle's 'qualified' rejection of mental images, see also Lawrie, "The Existence of Mental Images", pp. 253-257.

be able to consult in order to verify the properties of corresponding ideas, and without which ideas cannot be regarded as adequate.

Of these alternatives, (1) is clearly too strong. It is refuted by phenomenological counterevidence against the extreme claim that all thinking requires mental images. It is also a stronger formulation of imagism than Hume needs to uphold his critique of infinite divisibility. The claim that Hume's arguments can dispense with the strong imagism of (1) is most easily seen in comparison with the remaining possibilities.

By contrast, (2) is too weak. It says merely that some ideas or adequate ideas are or require corresponding mental images. This is not enough to assure that particular mental images are or need to be available for corresponding adequate ideas of the divisiblity of extension as invoked by any of Hume's arguments. The third choice in (3) is unsatisfactory for much the same reason. It allows that not all ideas or adequate ideas of the divisibility of extension are or require corresponding mental images; it indicates only that there may exist mental images that might be consulted to determine the properties of corresponding adequate ideas. Such a method might be helpful in establishing the nature of extension if and when the images are available, but (3) does not rule out the possibility that an adequate idea of extension might have properties other than those belonging to its mental image, such as the property relevant to Hume's argument of being infinitely divisible.

Alternative (4) alone seems to describe a mental imagism with just the right amount of strength to support Hume's conclusions against infinite divisibility. It accounts for his description of the mind's entertaining images of divisibility. But it avoids commitment to the extreme sort of universal or necessary imagism that invites philosophical objections. Although Hume nowhere claims that thinking essentially requires mental images, he sometimes implies that the properties of adequate ideas can only be known or reliably known by examining corresponding clear and distinct images.[82] Whether or not all thought is imagistic, mental images may need to be examined by the mind in order to verify certain features of its adequate ideas of extension. Applications of mental images can belong to two categories, negative and positive. The introspection of mental images confirms that our idea of extension is not infinitely divisible, that it is not a clear and distinct idea of extension as infinitely divisible, and that we do have the idea of *minima sensibilia* or sensible extensionless indivisibles. If the images are not available to support the infinite divisibility thesis, but the images that obtain on the contrary uphold the opposite conclusion that finite extension is at most finitely divisible into sensible extensionless indivisibles (filling in the details as in the inkspot argument and *reductio* proofs), then Hume's critique of infinity is sustained by the content and limitations of mental images in determining the properties of adequate ideas about the divisibility of extension.

Even if not all thinking takes place in terms or by means of mental images, some philosophical questions, especially those concerning the content or adequacy of ideas about perceivable phenomena, such as the properties of extension in space, can only or can best be decided by consulting corresponding mental images. When disputes arise, clear and distinct mental images may provide the only criterion of whether an idea about the divisibility of extension is adequate or inadequate. This moderate version of mental imagism is comparatively philosophically unobjectionable. It implies only that controversies about the adequacy of ideas can be settled introspectively or phenomenologically by bringing forward wherever possible and examining corresponding images to decide the properties of adequate ideas of such concepts as extension. This is a challenge infinitist mathematicians and metaphysicians cannot afford to ignore. If they do, they will have placed themselves in the awkward position of claiming to have adequate *ideas* of something for which they have no clear

[82] See below, note 108.

and distinct mental representations. This in turn leaves them open to the Humean criticism that infinitists merely pronounce terms associated with the 'infinity' vocabulary, and perhaps manipulate the symbols of what purports to be a calculus of infinity or higher infinities, but to which no genuine concepts attach, and of which they have no genuine understanding.

This, interestingly, is Flew's retort. He replies to Hume's assertion "that the capacity of the mind is limited, and can never attain a full and adequate conception of infinity,"[83] by remarking: "... there is no insuperable difficulty about learning the ordinary uses of the words 'infinite' and 'infinity', and we can perfectly well understand what is meant by talk of a series' being infinite or going on to infinity."[84] Yet what is learned by Flew's account is at most how to exchange one set of words for another set of words, though without necessarily understanding at any point what any combination denotes or is supposed to denote. That this is not genuine understanding is indicated by the fact that intuitively we do not regard a machine as understanding a concept when it mechanically exchanges words for other words according to programmed instructions for storage and retrieval of coded messages. We would not say that a machine understands the concept of infinity merely on the basis of its being able to print out the word 'endless', for example, when we type in the question 'What do you mean by 'infinite'?'[85] Hume would want to know what the machine means by the word 'endless', for nothing less is required by his

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[83] *Treatise*, p. 26.

[84] Flew, "Infinite Divisibility in Hume's *Treatise*", p. 259.

[85] Block, "Troubles with Functionalism", in Savage, ed., *Perception and Cognition: Issues in the Foundations of Psychology*; reprinted in Block, ed., *Readings in Philosophy of Psychology*, Vol. 1, pp. 268-305. This is the source of what has come to be known as the problem of Block's jukebox, which achieves a mere mechanical retrieval of sentence for sentence or syntax output for syntax input. The problem is a precursor of Searle's Chinese Room counterexample to the Turing test, 'strong' or mentalistic artificial intelligence, and the functionalism-cognitivism-computationalism family of theories in mechanist philosophy of mind. Searle, "Minds, Brains and Programs", pp. 417-424; *Minds, Brains and Science*.

insistence that the finite mind cannot 'attain a full and adequate conception of infinity'. If the regress does not terminate finally in a clear and distinct idea, a concept or image of infinity, then eventually the word game Flew describes must lead back to the word 'infinite' to 'explain' what 'endless' means. The circularity of trying to achieve understanding at the superficial level of vocabulary networks and word intersubstitutions in that case is only too apparent.

Mental Imagery, Imagination, and Adequate Mathematical Ideas

We may then share Alice's puzzlement in *Through the Looking-Glass*, when she reads the nonsense poem 'Jabberwocky', holding it up to the mirror to decipher the reversed handwriting. This parodies the confusion that results in what we are told by traditional mathematicians concerning the concepts of infinity and infinite divisibility in the absence of clear and distinct ideas or corresponding mental representations:

> 'It seems very pretty,' she said when she had finished it, 'but it's *rather* hard to understand!' (You see, she didn't like to confess even to herself, that she couldn't make it out at all). 'Somehow it seems to fill my head with ideas — only I don't exactly know what they are![86]

As a more relevant literary indictment of the problem of understanding the concept of infinity, consider an episode from Jan Potocki's *The Manuscript Found in Saragossa*. The mathematician in one of his branching series of tales distinguishes having an idea of infinity and infinite divisibility as opposed to developing syntactical facility with the symbols that are supposed to represent the ideas. He admits:

> When I want to indicate the infinitely great, I write a sideways '8' over '1'. When I want to indicate the infinitely small, I write a '1' which I divide by the symbol for infinity. These symbols which I use give me no idea at all of what I am expressing. The infinitely great is the number of fixed stars multiplied

[86] Carroll, *Through the Looking-Glass*, *The Philosopher's Alice*, introduction and notes by Heath, p. 139.

> *ad infinitum*; the infinitely small an infinite subdivision of the smallest of atoms. I can therefore indicate the infinite, but I cannot comprehend it.[87]

The moderate imagism attributed to Hume is sufficient to maintain his criticism of the concept of infinity and the infinite divisibility of extension. It reveals the weakness in the infinist's attempts to use words that are unsupported by clear and distinct ideas. This, as Descartes's theory shows most trenchantly, is an especially inconvenient acknowledgment for rationalists in the philosophy of mathematics.[88]

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[87] Potocki, *The Manuscript Found in Saragossa*, pp. 407-408. See also pp. 423-424, concerning the role of abstraction in arriving at an adequate mathematical idea: "I must confess to you that I do not like this new doctrine. Abstraction seems to me to be no more than a subtraction. To abstract you must remove something. If I mentally take away from my room everything that encloses it, even to the point of subtracting air, I have pure space. If I remove from a length of time its beginning and its end I have eternity. If from an intelligent being I take away the body I have the idea of an angel. If from lines I mentally take away their width, only to be left with their length and the two-dimensional figures that they enclose, I have the elements of Euclid. If I take away an eye from a man and I add to his height I am left with the figure of a Cyclops. All of these are images received by the senses. If these new thinkers can provide me with a single abstraction which I cannot reduce to a subtraction I shall declare myself their disciple. Until then I'll stick to old Aristotle." Even in these supposed cases of abstraction as mental subtraction, thinking away certain features of a sense impression does not produce a positive abstract idea, but results at most in an idea with diminished content relative to its originating sense impression. To think of a road without its width is not to think of a road but of the length or distance of the road; but no such procedure according to Hume will enable the mind to think of an infinite length by mentally subtracting width from the sense impression or idea of a finite stretch of road.

[88] See Descartes, *Rules for the Direction of the Mind*, *Works*, Vol. I, Rule II, p. 3: "*Only those objects should engage our attention, to the sure and indubitable knowledge of which our mental powers seem to be adequate.*" Rule III, p. 5: "*In the subjects we propose to investigate, our inquiries should be directed, not to what others have thought, nor to what we ourselves conjecture, but to what we can clearly and perspicuously behold and with certainty deduce; for knowledge is not won in any other way.*" Commenting in detail on Rule II, Descartes states, p. 5: "This

Hume's imagism implies that, when asked to justify their infinitist terminology, these opponents are seen as fitting words together without understanding their meaning beyond the substitution of one set of terms for another. They make use of an empty calculus, the key terms of which by Hume's critique have no definite meaning. It seems correct to conclude, even if the most moderate mental imagism that can be extended to Hume's philosophy of mind is false, that his arguments put traditional infinitism on the defensive by forcing it to justify its concepts of infinity and infinite divisibility in a non-imagistic way. If this demand involves more than mere word substitution games that can also take place mechanically without understanding, then it is hard to see what could be offered instead. Hume's theory of meaning requires more; it expects a definite connection between words and ideas linked to and ultimately derived from experience in impressions of sensation or reflection, of which clear corresponding mental images are the best sign and most reliable criterion.

If not all adequate ideas are confirmed by mental images, how are the claims made about the properties of concepts to be determined? Imaging is used extensively to guide inquiry and solve problems in many disciplines, including conventional mathematical investigations.[89] Hume's argument interpreted as implying a moderate imagism minimally shifts the burden

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furnishes us with an evident explanation of the great superiority in certitude of Arithmetic and Geometry to other sciences. The former alone deal with an object so pure and uncomplicated, that they need make no assumptions at all which experience renders uncertain, but wholly consist in the rational deduction of consequences. They are on that account the easiest and clearest of all, and possess an object such as we require, for in them it is scarce humanly possible for anyone to err except by inadvertence."

[89] See Segal, *Imagery: Current Cognitive Approaches*. Shepherd and Cooper, with chapters co-authored by Farrell, et al., *Mental Images and Their Transformations*. Rollins, *Mental Imagery: On the Limits of Cognitive Science*. Cognitive experimentation on mental imagery typically involves mathematical problem-solving. For a contrary, 'unconscious' and largely anti-imagistic account of mathematical discovery in the spirit of Henri Poincaré, see also Hadamard, *The Psychology of Invention in the Mathematical Field*.

of proof to the infinitist to offer an alternative method of confirming the validity of concepts of infinity and infinite divisibility without the benefit of clear and distinct mental images. It is one thing to puncture naive global or essentialistic mental imagism in the philosophy of mind, but quite another to advance a satisfactory epistemic alternative to Hume's introspection, phenomenology, or moderate mental imagism in evaluating the adequacy of ideas. Surely, infinitists cannot uncritically accept whatever claims are made about ideas of the infinite. What then is to arbitrate between competing conceptions, if not mental images or clear and distinct ideas? Logical consistency or absence of contradiction alone does not seem sufficient to uphold infinitism, especially if Hume's *reductio* refutations and geometry dilemma are sound. Even if all of Hume's objections fail, mere logical consistency cannot be enough to support infinitism, since, whatever its overall adequacies or inadequacies, a strict finitist mathematics and metaphysics of spatial extension can also be logically consistent. The infinitist will find it difficult to replace Hume's method with another that positively confirms the adequacy of any idea of infinite divisibility. We have seen that the standard definitions of infinite divisibility require a prior understanding of infinity, such as a stipulatively postulated infinite set or series to correlate with infinite subdivisions of an extension, or the infinite moments of time for a process of subdivision to continue 'always' or 'endlessly', or as a process occurring in God's infinite mind, for which there is no noncircular 'abstract' explanation or recursive method of generating infinite quantities. Hume's moderate mental imagism is sufficient in the context of the arguments he deploys to defeat infinite divisibility and support his own theory of sensible extensionless indivisibles, and his conclusions are independently reinforced by indirect proofs against infinite divisibility that do not explicitly rely on mental imagism.[90]

[90] It is doubtful to what extent Hume's *reductio* proofs against infinite divisibility and in support of sensible extensionless indivisibles must rely on

If Hume's arguments are correct, then he discovers the conceptual foundations for an alternative metaphysics of extension, space and time, that stands in opposition to classical infinitary metaphysics and mathematics. This is no small achievement for empiricism, especially in comparison with the well-entrenched infinitism of pure and applied mathematics that still dominates contemporary philosophy and the formal and natural sciences. The impact of Hume's finitism in mathematics and related disciplines is likely to appear liberating or regressive, depending on one's philosophical temperament. The opportunity it affords of taking a critical look and new approach to longstanding problems about infinity and infinite divisibility that extend through philosophy and mathematics from their ancient history predating Zeno's paradoxes to the ontology of infinitesimals, Cantor's hierarchy of transfinite ordinals, and the unsolvable status of the Continuum Hypothesis in standard set theory, should not be ignored. These topics are examined in detail in the Conclusion, 'Hume Against the Mathematicians'.

Hume's critique of infinite divisibility and theory of sensible extensionless indivisibles holds out the promise of humanizing the mathematics and metaphysics of extension, and bringing them back down to earth, restoring the foundations of mathematics to what is psychologically conceivable, as intuitionists, cognitivists, and constructivists in philosophy of mathematics have proposed in different but related ways. The implications of Hume's finitism and positive doctrine of sensible extension-

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mental imagism for their conclusions. Hume's argument from the addition of infinite parts, which is Flew's target in this part of his criticism, does not explicitly appeal to the existence of corresponding mental images to uphold its rejection of the infinite divisibility thesis. The arguments from the unity of existents, from the relation between space and time, and from the conceivability of indivisibles, are even less likely candidates for requiring mental images. This is particularly true since the *reductio* disproofs are directed *ad hominem* at rationalists who think of infinity in terms of abstract ideas for which mental images are unnecessary. The inkspot experiment in Hume's central argument seems to require only the existence of visual rather than mental images, since the experiment refers only to what subjects occurrently perceive while actually looking at a distant inkspot on paper.

less indivisibles have yet to be appreciated and seriously applied to the wealth of problems in the philosophical foundations of mathematics and science. This accomplishment, carrying forward the banner of strict finitism with unwavering rigor, cannot be denied Hume, even by those who from an antagonistic methodology maintain that his conclusions should be rejected and his theory of sensible extensionless indivisibles set aside as a historical curiosity. The higher pinnacle from which Hume's arguments can confidently be dismissed as obviously mistaken is attained only by rejecting Hume's very reasonable demand that human theorizing be limited to what human beings can actually know and ideas human beings can actually consider.

CHAPTER 7

INFINITE DIVISIBILITY IN HUME'S FIRST *ENQUIRY*

Skepticism and Natural Belief

The relation between Hume's philosophy in the *Treatise* and *An Enquiry Concerning Human Understanding* is controversial. Hume later claims to disown the *Treatise*, but it is unclear whether by this he means to distance himself from the substance or only the style and mode of argument of his early system. Interestingly, in the *Enquiry*, Hume combines some of the *Treatise* criticisms of infinite divisibility, which he develops in a similar direction, but with somewhat different emphasis. To understand Hume's critique of infinite divisibility and the theory of extensionless indivisibles in the *Enquiry*, it is necessary to reconstruct its two arguments against infinity, and place them in the context of his preoccupation with the nature of philosophical skepticism in the later work. Then we will be in a position to compare the evolution of his theory of space from the *Treatise* to the first *Enquiry*.

Hume's final thoughts on the divisibility of extension are conveyed almost as asides in notes to paragraphs 124 and 125 of the *Enquiry*. Here Hume reaffirms his early stance against infinite divisibility. The argument in *Enquiry* 124 integrates features of the *Treatise* inkspot experiment and the *Treatise* *reductio* argument from the addition of infinite parts. The second argument appears in Hume's tantalizing partially developed Berkeleyan 'hint' about the refutation of abstract general ideas, in the long note to his discussion of the divisibility problem as an objection to 'all *abstract* reasonings' at the end of paragraph

125. Hume asserts that: "... all the ideas of quantity, upon which mathematicians reason, are nothing but particular ... and consequently, cannot be infinitely divisible. It is sufficient to have dropped this hint at present, without prosecuting it any farther."[91]

The Berkeleyan hint in the note on infinity in *Enquiry* 125 is in some ways the most interesting indication of Hume's later ideas about infinite divisibility and the doctrine of sensible extensionless indivisibles. This is partly a result of the fact that it is only a hint; its incomplete statement requires an interpretive effort in imagining how Hume might have wanted to complete the argument. Hume's Berkeleyan hint in the *Enquiry* replaces the *Treatise* inkspot argument as a basis for concluding that there can be no adequate idea of infinite divisibility. Although Hume continues to accept the theory of the experiential origin of ideas in the first *Enquiry*, it is worth noting that in the later writing he allows the refutation of the idea of infinite divisibility and positive theory of sensible extensionless indivisibles to be upheld without appeal to the copy principle by an alternative route suggested by Berkeley's repudiation of abstract general ideas. Thus, as might be expected, there are continuities and discontinuities in Hume's thought from the *Treatise* to the *Enquiry* in his early and later critique of infinity.

If Hume devotes less space to the problem of infinity in the *Enquiry* as opposed to the *Treatise*, it need not be because he

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[91] *Enquiry*, p. 158, n. 1. Waxman in private communication has advised me that Hume's 'hint' does not appear in the first two editions of the *Enquiry*, but replaces the original assertions: "In general, we may pronounce that the ideas of 'greater', 'less', or 'equal', which are the chief objects of geometry, are from being being so exact or determinate as to be the foundation of such extraordinary inferences. Ask a mathematician what he means when he pronounces two quantities to be equal, and he must say that the idea of 'equality' is one of those which cannot be defined, and that it is sufficient to place two equal quantities before anyone, in order to suggest it. Now this is an appeal to the general appearances of objects to the imagination or senses, and consequently can never afford conclusions so directly contrary to these faculties." The argument, as Waxman rightly observes, is directly related to Hume's observations in *Treatise*, pp. 45-52, and 'Abstract', pp. 658-659.

lost interest in the problem or confidence in his early analysis. On the contrary, Hume in the interim might be regarded as having discovered two more compact, less contentious or more convincing arguments that lend themselves to a simplified presentation. The fact that his new arguments appear in notes rather than being fully developed in the main body of the *Enquiry* may be significant, however, and the contents of Hume's remarks suggest several different explanations of how his early (formulation of his) philosophy may be related to the later.[92]

The arguments against infinite divisibility in the notes to *Enquiry* 124 and 125 are offered as 'skeptical' results about the limitations of reason. The metaphysics of space and the problem of infinite divisibility are introduced as particular though representative problems among several Hume discusses. Hume first writes:

> The chief objection against all *abstract* reasonings is derived from the ideas of space and time; ideas, which, in common life and to a careless view, are very clear and intelligible, but when they pass through the scrutiny of the profound sciences (and they are the chief object of these sciences) afford principles, which seem full of absurdity and contradiction.[93]

There follows an intuitive appeal to the apparent incoherence of the consequences of infinite divisibility in the geometry of space and measurement of time. Hume enlists natural belief against the infinite divisibility thesis in the mathematics of extension:

> But what renders the matter more extraordinary, is, that these seemingly absurd opinions are supported by a chain of reasoning, the clearest and most natural; nor is it possible for us to allow the premises without admitting the consequences. Nothing can be more convincing and satisfactory than all the conclusions concerning the properties of circles and triangles; and yet, when these are once received, how can we deny, that

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[92] Nidditch, "Introduction" to *David Hume: Enquiries*, pp. vii-xxxi.

[93] *Enquiry*, p. 156.

> the angle of contact between a circle and its tangent is infinitely less than any rectilinear angle, that as you may increase the diameter of the circle *in infinitum*, this angle of contact becomes still less, even *in infinitum*, and that the angle of contact between other curves and their tangents may be infinitely less than those between any circle and its tangent, and so on, *in infinitum*? The demonstration of these principles seems as unexceptionable as that which proves the three angles of a triangle to be equal to two right ones, though the latter opinion be natural and easy, and the former big with contradiction and absurdity.[94]

The implied sense of contradiction is then extended by Hume to the problem of infinite divisibility in time. He continues:

> The absurdity of these bold determinations of the abstract sciences seems to become, if possible, still more palpable with regard to time than extension. An infinite number of real parts of time, passing in succession, and exhausted one after another, appears so evident a contradiction, that no man, one should think, whose judgment is not corrupted, instead of being improved, by the sciences, would ever be able to admit of it.[95]

The contradictions entailed by the infinitist concept of extension define certain limits of reason. But Hume's answer is more complex than merely surrendering to skepticism over the prospects of thought achieving understanding about the nature of space and time. He tries to show that what he calls 'Pyrrhonian' skepticism is ultimately self-defeating in its application of reason against reason. In place of a negative conclusion, he offers a solution to the paradoxes of infinite divisibility.

What is remarkable about Hume's pronouncements against infinite divisibility, that may signal a change in his methodology if not his overall position on divisibility in the *Enquiry* from that of the *Treatise*, is his straightforward claim that belief rebels against the concept of infinite divisibility. As in the *Treatise*, Hume in the *Enquiry* pits natural disbelief in infinite divisibility against abstract metaphysical reason. But in the *Enquiry*, Hume

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[94] Ibid., pp. 156-157.

[95] Ibid., p. 157.

does not invoke reason merely to correct the excesses of natural belief in its acceptance of the necessity of causal connection or uniformity of nature, or the existence of mind-independent continuants. Hume here appears instead to invoke natural disbelief in infinite divisibility as a weapon against what he regards as the immoderate and unjustified deliberations of speculative reason in traditional mathematics and metaphysics.

Hume holds that natural belief and metaphysical reason may coincide. Such agreements can occur when reason is exercised in philosophical inquiry, and the examination of arguments for and against a position produces a true metaphysics tempered by instinct and intuition. In the *Treatise*, with respect to the natural beliefs about which Hume prefers to remain skeptical, the appeal to pretheoretical natural belief is no deterent to or substitute for painstaking metaphysical investigations. But in the first *Enquiry*, Hume seems content to set aside doubts about natural beliefs that run counter to the infinite divisibility of extension and time where his own previous reasonings in the *Treatise* happen to agree. That this is actually an incorrect assessment of Hume's procedure in the *Enquiry* appears only when it is observed that the conflict between natural belief and philosophical reason is a two-edged sword that cuts equally against natural belief and metaphysical reason to reinforce the skeptical attitude whenever they collide. Hume's purpose in these brief notes in the *Enquiry* is not simply to offer a naive intuitive bulwark against infinite divisibility that uncritically takes natural disbelief in infinite divisibility as sufficient leverage to overturn abstract metaphysical commitment to infinity. This would violate Hume's practice in the *Treatise* of using metaphysics to correct the extravagances of natural belief. Hume's objective is rather to illustrate the general thesis that skepticism arises whenever natural belief conflicts with metaphysical reason.

This important difference can be seen in the fact that Hume's arguments against infinite divisibility are presented in asides to the main body of the text. By contrast with the *Treatise*, Book I, Part II, the *Enquiry* format indicates that

Hume's aim in the chapter is not primarily to refute the infinite divisibility thesis, but appears instead in the guise of addressing more general questions posed at the beginning of the *Enquiry*, Section XII, 'Of the Academical or Sceptical Philosophy'. These are the problems, in Hume's words, of "What is meant by a sceptic? And how far is it possible to push these philosophical principles of doubt and uncertainty?"[96] The *Treatise*, by comparison, does not introduce the topic of skepticism at length until 112 pages after the discussion of infinite divisibility, in Book I, Part IV, *Of the skeptical and other systems of philosophy.*

Hume distinguishes several kinds and categories of skepticism. He assesses the philosophical strengths and weaknesses of each, and arrives at last at the statement to which the note containing his second argument against infinite divisibility in *Enquiry* paragraph 125 is attached. Here he declares:

> Yet still reason must remain restless, and unquiet, even with regard to that scepticism, to which she is driven by these seeming absurdities and contradictions. How any clear, distinct idea can contain circumstances, contradictory to itself, or to any other clear, distinct idea, is absolutely incomprehensible, as absurd as any proposition, which can be formed. So that nothing can be more sceptical, or more full of doubt and hesitation, than this scepticism itself, which arises from some of the paradoxical conclusions of geometry or the science of quantity.[97]

Hume first observes the conflict between natural disbelief in infinite divisibility and the conclusions that follow unavoidably from the apparently innocent premises that serve as starting places, and logically circumspect chains of inference in abstract reasoning in mathematics and metaphysics. This by itself does not constitute an argument against infinite divisibility, nor does Hume offer it as such. His point is that skepticism is self-defeating when it tries to use reason to undermine reason.

[96] Ibid., p. 149.
[97] Ibid., pp. 157-158.

Hume begins Part II of the section with the statement that: "It may seem a very extravagant attempt of the sceptics to destroy *reason* by argument and ratiocination; yet is this the grand scope of all their enquiries and disputes."[98] Skeptics try to do this, he continues, whenever "[t]hey endeavour to find objections, both to our abstract reasonings, and to those which regard matter of fact and existence."[99]

Hume then introduces the criticism quoted above, that: 'The chief objection against all *abstract* reasonings is derived from the ideas of space and time...'. His diagnosis is that a paradox occurs when skeptics try to confute abstract reasoning by a skeptical *reductio*, using well-reasoned arguments involving apparently irreproachable premises and logically valid inference chains to deduce the self-contradictory conclusions that extension and time are infinitely divisible. The rationale for regarding the conclusion as absurd or self-contradictory is not given, beyond what amounts to Hume's assertion that these results are inconsistent with natural belief about the divisibility of space and time, and the *reductio* argument discussed below in the note to paragraph 124. The skeptic, in observing the absurdity of the consequences of abstract reasoning in the metaphysics of infinitely divisible space and time, indiscriminately casts doubt on reason in general, on the grounds that the conflict between natural and philosophical reason indicates that neither form is entirely to be trusted.

The note to *Enquiry* 124 contains Hume's first criticism of infinite divisibility. It is a *reductio* offered primarily as an example of how skepticism seeks to undermine abstract reasoning. Hume is unequivocal in the argument he gives to support the skeptic's claim in the passage to which the note is attached. He maintains:

> A real quantity, infinitely less than any finite quantity, containing quantities infinitely less than itself, and so on *in infinitum*; this is an edifice so bold and prodigious, that it is too weighty for

[98] Ibid., p. 155.
[99] Ibid., p. 156.

> any pretended demonstration to support, because it shocks the clearest and most natural principles of human reason.[100]

The skeptic's objection to abstract reasoning is interpreted by Hume significantly as the complaint that it implies a contradiction with 'human reason'. This may seem to elevate ordinary human thought to the position of a standard by which the intelligibility of abstract metaphysical reasoning can be criticized. But Hume explains the goal of skeptics more generally as the attempt to 'destroy *reason* by argument and ratiocination', or to destroy reason by means of reason. Hume regards global skepticism about these metaphysical matters as self-defeating and self-refuting. The view is further suggested in his previously noted pronouncement that 'nothing can be more sceptical, or more full of doubt and hesitation, than this scepticism itself, which arises from some of the paradoxical conclusions of geometry or the science of quantity'.

That skepticism in Hume's view is hoist by its own petard in trying to refute abstract reason as part of a misconceived or 'extravagant' program to refute reason in general, appealing unavoidably to reason, 'human reason' or 'argument and ratiocination', is further indicated by the proofs in Hume's two notes about infinite divisibility. Here Hume by no means accepts the 'Pyrrhonian' skeptic's conclusion that human reason cannot rise above skeptical dilemmas about divisibility, but takes the opportunity instead to deliver two reasonable and sincerely intended arguments against infinite divisibility.[101]

This at once rescues reason from the Pyrrhonian skeptic's clutches. Hume's argument does not refute reason in general, a project he rightly regards as absurd, but attacks in particular the reasoning of classical infinitary mathematics and meta-

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[100] Ibid.

[101] Norton, *David Hume*, pp. 239-310. See also Popkin, *The High Road to Pyrrhonism*. Popkin, "David Hume: His Pyrrhonism and his Critique of Pyrrhonism", pp. 53-98. Passmore, *Hume's Intentions*, pp. 132-151. Wright, *The Sceptical Realism of David Hume*, pp. 92-97. Livingston, *Hume's Philosophy of Common Life*. Stroud, *Hume*.

physics. Hume thereby reaffirms the empiricist rejection of infinite divisibility and theory of sensible extensionless indivisibles in the *Treatise.* Yet the argument should not be understood simply as trying to show that the infinite divisibility thesis contradicts natural belief as the final arbiter of metaphysical truth. That kind of argument would no more satisfy Hume in the *Enquiry* than in the *Treatise*, where philosophical reason is frequently called upon to correct the errors of natural belief. The appeal to natural reason or belief in the form of that 'human reason' which contradicts the conclusions of abstract reasonings about space and time is left by Hume to the folly of the 'Pyrrhonian' skeptic, entangled in the paradox of trying to refute reason by means of reason.[102]

Hume's Reductio Argument in Enquiry 124

The argument against infinite divisibility in the note to *Enquiry* 124 grafts, without mentioning its source, the data of the *Treatise* inkspot experiment onto the first *Treatise reductio* from the addition of infinite parts. Hume's inference takes as its starting place the assumption that spatial extension is reducible to sensible extensionless indivisible 'physical points', which he distinguishes from ideal or abstract 'mathematical points'.[103]

The proposition that extension is infinitely divisible, which Hume assumes for purposes of indirect proof, is shown to be false by implying the contradiction that a finite extension is also infinitely extended. The conclusion is reached as in the *Treatise reductio* by considering the consequences of adding infinitely many indivisible physical points of a finitely extended body together to produce an infinitely extended body. The problem is sufficient to reject the infinite divisibility thesis on the hypothesis of physical points, and is only made worse on the infinitist's assumption that the infinite constituents of extension are not indivisible, but infinitely divisible into mathematical points or infinitely divisible extended line subsegments.

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[102] Fogelin, *Hume's Skepticism in the Treatise of Human Nature*, pp. 149-151.
[103] See Part Two, note 30.

The disproof of infinite divisibility in the note to *Enquiry* 124 upholds reason against Pyrrhonian skepticism. It asserts that 'nothing appears more certain to reason' than that infinitely many physical points must constitute an infinitely extended thing. Hume formulates the argument in this way:

> Whatever disputes there may be about mathematical points, we must allow that there are physical points; that is, parts of extension, which cannot be divided or lessened, either by the eye or imagination. These images, then, which are present to the fancy or senses, are absolutely indivisible, and consequently must be allowed by mathematicians to be infinitely less than any real part of extension; and yet nothing appears more certain to reason, than that an infinite number of them composes an infinite extension. How much more an infinite number of those infinitely small parts of extension, which are still supposed infinitely divisible.[104]

The physical points beyond which neither 'eye nor imagination' can discriminate recall the sensible extensionless indivisibles revealed by the *Treatise* inkspot experiment and grain of sand thought experiment. This concept in the *Enquiry* 124 argument is incorporated into a *reductio* refutation of infinite divisibility that is similar in form to the *Treatise* argument from the addition of infinite parts.

Hume maintains that physical points are 'absolutely indivisible'. This appears at first to contradict the divisibility or indivisibility of sensible ideas relative to subject and circumstance, the distance from subject to what is judged a physical point, acuity of eye, and patience, memory, and focus of individual imagination. Perhaps by holding that the physical points are absolutely indivisible Hume does not mean that phenomenal divisibility is absolute, but only that physical points at the visibility or imaginability threshold, are indivisible, as he says, to the 'fancy or senses', in that beyond the threshold they vanish from sensation or imagination. That, in any case, is all the argument seems to require.

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[104] *Enquiry*, p. 156, n. 1.

To close the jaws of the *reductio* on 'the mathematician' it might be expected that Hume would resort at this stage of the argument to the claim that physical points discriminated by finite vision or imagination in finite time cannot be infinite in number. This would bring the argument closer in spirit to the inkspot experiment argument of the *Treatise.* But he leaves the matter open, no doubt in keeping with his argumentative pose of neutrality on 'whatever disputes there may be about mathematical points'. Instead, Hume frames another, different, absurdity. He concludes that the infinitist's assumptions lead inexorably to the contradiction that a finitely extended body composed of infinitely many physical points must be infinitely extended, on the grounds that 'nothing appears more certain to reason, than that an infinite number of [physical points] composes an infinite extension'. The problem is only made worse, though it is already plainly bad enough, if, as the mathematicians Hume criticizes are supposed to believe, extension is supposed to be infinitely divisible into infinitely many mathematical subsegments rather than physical points, or into infinitely small extended parts of extension, each of which in turn is infinitely divisible. The result in either case reflects back negatively on the hypothesis that extension is infinitely divisible.

The inference has the following indirect proof structure. It begins, like the *Treatise reductio*, with premises Hume does not accept, and others about which he remains neutral for the sake of argument, such as the existence of mathematical points.

1. There are physical points, sensible extensionless indivisible parts of extension.
2. Physical points are absolutely indivisible, in that if divided (separated) they would cease to be be present to sensation or imagination.
3. Suppose for purposes of indirect proof that extension is infinitely divisible.

4. Physical points are infinitely less or infinitely smaller in extent than any real (extended) part of extension. (1,2)
5. Finitely extended things are divisible into infinitely many physical points. (3,4)
6. Infinitely many physical points are needed to constitute a finitely extended thing. (5)
7. Finitely extended things are also infinitely extended. (The problem is only compounded if extension is infinitely divisible into infinitely smaller infinitely divisible parts.) (2,6)
8. Therefore, space (extension) is not infinitely divisible. (7)

The argument requires a bit more reconstruction than most of Hume's *reductio* proofs in the *Treatise.* It appears appropriate to refer to 'physical' points in Hume's argument as the sensible extensionless indivisible parts of extension. The reason is that he describes them in the text as 'these *images*', which seems to imply their perceivability in sense experience or imagination, as well as their ideality.

The assumption requires phenomenal evidence for the experienceable extensionless indivisibles established by the inkspot experiment. This may indicate alternatively that Hume presupposes the empirical conclusions of the experiment, or that he now finds the proposition so obvious as not to require justification. The parenthetical remark in step (7) alludes to the *reductio* argument from the addition of infinite parts in the *Treatise.* The proof is refined somewhat to leave open the question whether there are infinitely divisible mathematical points, and derives contradiction even from the more moderate assumption that there are sensible extensionless physical points, thereby concentrating attention more pointedly in particular on the offending assumption that extension is infinitely divisible.

The conclusions in (4) and (5) require additional explanation. That physical points are infinitely less or infinitely smaller in extent than any real or extended part of extension follows directly from assumptions (1) and (2). If physical points were not infinitely smaller than any extended part of extension, they would need to be at least the same size as the smallest extended parts of extension. Every extended thing is divisible, so that, contrary to the definition in assumption (2), physical points would not be extensionless or indivisible unless they were infinitely smaller than the smallest extended parts of extension. This brings us to conclusion (5), that finitely extended things are divisible into infinitely many physical points.

The similarity between this condensed indirect proof against infinite divisibility and Hume's first *Treatise reductio* from the addition of infinite parts should not overshadow the fact that the *Enquiry* 124 *reductio* represents a substantial improvement over its predecessor. It has already been observed that the latter demonstration claims an advantage in its neutrality about the existence of 'mathematical' points. This allows the argument to discover the absurdity of infinite divisibility into indivisible physical points, which must be thought infinitely smaller than any least real parts of extension. The aburdity can then be extended with proportionately greater force to the infinitist assumption that subsegments composed of infinitely many mathematical points, infinitely minute parts of extension, are themselves still infinitely divisible.

The *Enquiry* argument also has the edge over the *Treatise* proofs in its commonsense appeal to the existence of 'physical' points, which are evidently the sensible extensionless indivisibles of the earlier theory. The *Treatise* finds it necessary to introduce indivisibles by the complicated apparatus of the inkspot experiment, with its contentious empiricist assumption about the experiential origin of adequate ideas in impressions of sensation. Here Hume says simply 'we must allow' that there are sensible extensionless physical points, a proposition he seems to believe even his infinitist adversary will accept. The point is not that the *Enquiry* 124 argument is superior to the *Treatise reductio* from the addition of infinite parts because it is always better to assume what may be difficult or controversial to prove. Rather, Hume in the *Enquiry* seems to have found an argument that he regards as more economical but every bit as effective as the *Treatise* inkspot experiment.

The same result is achieved without proving the existence of sensible extensionless indivisibles in the context of advancing a version of the *reductio* from the addition of infinite parts. Hume in the *Enquiry* notices that whether or not the infinitist accepts the infinite divisibility of extension into extensionless mathematical points or extended line subsegments, the argument from the addition of infinite parts goes through on the basis of the infinitist's presumed acceptance of the existence of physical points that cannot be diminished by sensation or imagination. For that matter, the inkspot experiment in the *Treatise* might also be construed not so much as a rigorous proof of the existence of sensible extensionless indivisibles, but as a heuristic visual aid to help explain and call attention to what Hume means by the concept. If seriously challenged about the existence of physical points, Hume in the later argument might still want to invoke something like the data of the inkspot experiment to defend the assumption from objections, instead of relying here as he seems to do on prephilosophical opinion. But the fact that he does not do so explicitly is significant, since it indicates that he regards the argument as complete without further elaboration, and as a proof, moreover, that ought to convince even the classical infinitist, without denying from the outset the possibility or conceivability that extension is infinitely divisible into infinitely many subsegments.[105]

Abstract General Ideas and Hume's Berkeleyan Refutation of Infinity

The same attitude prevails in Hume's second *Enquiry* argument against infinite divisibility in the note to paragraph 125. This is not a *reductio*-style proof, but appears to offer a supplementary if not substitute inference for Hume's *Treatise* argument based on the inkspot experiment. The argument is also dependent in a less direct way on sensation and the empirical data of phenomenal experience of space, but derives more immediately

[105] Hume avoids circularity in the attempt to disprove infinite divisibility by assuming that there cannot be mathematical points in the traditional abstract Euclidean sense.

from Berkeley's wholesale refutation of abstract general ideas. Hume offers the deduction in what looks curiously to be uncharacteristically tentative hypothetical language:

> It seems to me not impossible to avoid these absurdities and contradictions, if it be admitted, that there is no such thing as abstract or general ideas, properly speaking; but that all general ideas are, in reality, particular ones, attached to a general term, which recalls, upon occasion, other particular ones, that resemble, in certain circumstances, the idea, present to the mind.[106]

Hume advances the argument not primarily to dispute the infinite divisibility thesis, but chooses it almost randomly as an example to challenge the Pyrrhonian skeptic's global skepticism toward reason. The infinite divisibility thesis provides an opportunity for the skeptic to raise difficulties about abstract reasoning in the metaphysics of space and time.

Hume in a single master stroke disables both Pyrrhonian skepticism and the infinite divisibility of extension thesis. The proof contains Hume's famous 'hint' against infinite divisibility, and, implicitly, in support of his doctrine of sensible extensionless indivisibles. Although the argument does not appeal to anything like the inkspot experiment for empirical data about the limitations of divisibility or the experienceability of extensionless indivisibles, it rests ultimately on a kind of thought experiment against the existence of abstract general ideas which Hume attributes to Berkeley.

Previously, in *Enquiry* Section XII, Part I, Hume had portrayed Berkeley as a skeptic about Locke's distinction between primary and secondary qualities. Hume agrees that this distinction can only be defended against skeptical objections if abstract ideas exist. "Nothing can save us from this conclusion," he writes, "[that primary like secondary qualities are 'perceptions of the mind, without any external archetype or model, which they represent'], but the asserting, that the ideas of those primary qualities are attained by *Abstraction*, an opinion, which, if

[106] *Enquiry*, p. 158, n. 1.

we examine it accurately, we shall find to be unintelligible, and even absurd."[107] Hume challenges the reader, much as Philonous challenges Hylas in Berkeley's *Three Dialogues Between Hylas and Philonous*, to conceive of an abstract general idea, of 'triangle' or a triangle in general or in the abstract, of no particular size or type, with lines of no particular color.[108] Like Berkeley, Hume concludes that the exercise must fail, and from this draws the moral that abstract general ideas are philosophical fictions.

> An extension, that is neither tangible nor visible, cannot possibly be conceived: and a tangible or visible extension, which is neither hard nor soft, black nor white, is equally beyond the reach of human conception. Let any man try to conceive a triangle in general, which is neither *Isosceles* nor *Scalenum*, nor has any particular length or proportion of sides; and he will soon perceive the absurdity of all the scholastic notions with regard to abstraction and general ideas.[109]

As in the *Treatise*, where he enthusiastically describes Berkeley's refutation of abstract ideas as "... one of the greatest and most valuable discoveries that has been made of late in the republic of letters..."[110], Hume in the first *Enquiry* accepts Berkeley's phenomenological thought experiment as showing that there can be no abstract general ideas. He adopts Berkeley's suggestion that what is called abstract reasoning involves inference about particular ideas delegated by the imagination to represent all members of the same relevant category, from which counterinstances naturally come to mind if reason errs in drawing conclusions for the entire category from properties that are peculiar to any of its specific representatives.[111]

This makes it especially strange to find Hume ranking Berkeley's discovery with the conclusions of skeptics in this

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[107] Ibid., p. 154.

[108] Berkeley, *Three Dialogues*, pp. 192-194; *Principles*, p. 45.

[109] *Enquiry*, pp. 154-155.

[110] *Treatise*, p. 17.

[111] Berkeley, *Principles*, pp. 29-40.

part of the *Enquiry*. The ambiguities are evident in the note to paragraph 122:

> This argument is drawn from Dr. Berkeley; and indeed most of the writings of that very ingenious author form the best lessons of scepticism, which are to be found either among the ancient or modern philosophers, Bayle not excepted. He professes, however, in his title-page (and undoubtedly with great truth) to have composed his book against the sceptics as well as against the atheists and free-thinkers. But that all his arguments, though otherwise intended, are, in reality, merely sceptical, appears from this, *that they admit of no answer and produce no conviction.* Their only effect is to cause that momentary amazement and irresolution and confusion, which is the result of scepticism.[112]

What is surprising is not so much that Hume should classify Berkeley as a kind of skeptic, but that in this context he claims that the arguments of skeptics like Berkeley should *produce no conviction.* This is strange because from the time of the *Treatise* Hume seems to accept Berkeley's arguments against the existence of abstract general ideas.

The passage admits of no easy interpretation. Hume undoubtedly wields Berkeley's pat refutation of abstract general ideas in the second argument against infinite divisibility in the *Enquiry*. This fact seems to be inconsistent with Hume's characterization of Berkeley's argument as 'skeptical'. Yet the 'hint' in Hume's second argument in the note to *Enquiry* 125 trades essentially on Berkeley's renunciation of abstract general ideas, together with the assertion that if mathematicians have no abstract ideas of quantity, but only particular ideas acquired through sensation and imagination, then no ideas are infinitely divisible. Hume states:

> ... when the term Horse is pronounced, we immediately figure to ourselves the idea of a black or a white animal, of a particular size and figure: But as that term is also usually applied to animals of other colours, figures and sizes, these ideas, though

[112] *Enquiry*, p. 155, n. 1. See McConnell, "Berkeley and Skepticism", pp. 43-58.

> not actually present to the imagination, are easily recalled; and our reasoning and conclusion proceed in the same way, as if they were actually present. If this be admitted (as seems reasonable) it follows that all the ideas of quantity, upon which mathematicians reason, are nothing but particular, and such as are suggested by the senses and imagination, and consequently, cannot be infinitely indivisible.[113]

The argument makes reference to the two main features of Berkeley's criticism, that there are no abstract general ideas, and that what is called abstract reasoning involves reasoning from particular ideas that represent other members belonging to the same category. The 'hint' in the argument, which Hume does not sufficiently develop, is formulated as the assumption that no particular idea of extension and no idea of any particular extended thing is infinitely divisible. Hume remarks that: 'all the ideas of quantity, upon which mathematicians reason, are nothing but particular ... and consequently, cannot be infinitely divisible'. The proof can be reconstructed in this way:

1. There are no abstract ideas.
2. What is called abstract reasoning involves reasoning from particular ideas delegated by the imagination to represent the members of the same relevant category.
3. No particular idea of extension or idea of any particular extended thing is infinitely divisible.

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4. No idea of extension is infinitely divisible; there is no adequate idea of extension in space as infinitely divisible. (1,2,3)

The Berkeleyan 'hint' is given here as assumption (3). It states that no particular idea of extension or idea of any particular extended thing is infinitely divisible. To pursue the hint requires further conjecture about the kind of defense Hume might have offered in support of the proposition. The assumption is crucial

[113] *Enquiry*, p. 158.

to the argument, though it is precisely the point Hume leaves to surmise. It is reasonable to suppose that Hume's purpose in the argument is satisfied by the fact that a finite mind making use of finite imagination in a finite amount of (finitely or infinitely divisible) real time cannot infinitely divide any particular idea of extension or particular idea of any extended thing.

The question is not whether the mind could *de re* conceive the idea *of* an infinitely divisible extension or infinitely divisible extended thing, but whether the idea itself could be infinitely divisible. Yet surely dividing a particular idea into infinitely many parts is beyond the limitations of the finite mind's memory, imagination, and reason, operating in finite time. The feat could not be accomplished unless infinite time were available for the mind to undertake the subdivision with infinite imagination, memory, attention, and patience. But these assumptions cannot be introduced without begging the question against Hume's criticism of the very intelligibility of the concept of infinity. A finite mind as a general rule is unable successively to subdivide a single idea of an extended thing for more than a few passes, terminating in the substitution of another particular idea of extension or of another particular extended thing. This new idea might then be further subdivided at greater imagined magnification. Or the process might end in frustration in the failure mentally to keep track of how many subdivisions have been performed, whether each part has been divided or overlooked, after, at best, say, the twentieth division, as in Hume's *Treatise* discussion of the grain of sand thought experiment. At this point, the mind's image is likely to degenerate from a clear distinct idea of an extended thing to a hazy outline of finite extension subdivided by indefinitely many cross-hatch striations.

This defense of Hume's assumption (3) agrees with the phenomenal background of the proof in Berkeley's empiricist criticism of abstract general ideas. The two conceptions go hand in hand. The presumption that the finite mind cannot accomplish the ideational task of infinitely dividing a particular idea of extension or of a particular extended thing powers the main assumption in Hume's argument against infinite divisibility as reconstructed. It complements the Berkeleyan skeptical conclusion that there are no abstract general ideas and hence no adequate abstract general ideas of extension.

Another way to develop Hume's clue to the Berkeleyan refutation of infinite divisibility is to consider that if there are no general abstract ideas, but only particular ones deputized to represent all individuals in the category, then the particular idea of extension, or the particular idea of a particular extended thing, must be something like a mental image of a finite length of a line, surface, or some material entity, of some finite imagined width or thickness. To have an adequate idea of such a particular finite length infinitely divided is impossible on what seem to be the only way of imagining the division of a finite extension into component parts. The best method would be to mark the extension with imaginary short perpendicular line segments, cross-hairs, so to speak, subdividing the extension into discrete distinct parts, or breaking the original extension into distinct proper parts in imagination, and separating them at least some imaginable distance one from another; in effect, marking their subdivision by imaginatively creating spaces between them.

It should be obvious that neither of these devices has any chance of presenting the mind with an adequate idea of an infinitely divisible finite extension. If the way of marking extension by perpendicular subdividers or inserting spaces between its components is supposed to take place successively in progression over time, then no finite mind will have the infinite time needed to score the extension with infinitely many lines or spaces, even if more than one finite number of divisions can clearly be imagined to occur at a time. For the lines perpendicular to and spaces in or on the finite extension, by virtue of being particular ideas, must themselves also have some dimension, a particular width or thickness, no matter how tiny or fine, in an adequate idea or clear and distinct mental image. The subdividing markers of either type must then either be finitely extended or indivisible but constitutive of extension in aggregates of two or more. If the markers are infinite in number, as they need to be in order to score the infinite divisibility of a finite extension, then a finite extension cannot possibly accommodate them. Infinitely many perpendicular line dividers of particular width or thickness, whether finitely extended or indivisible but constitutive of finite extension in aggregates of two or more, cannot be housed within the original finite extension. Collectively by hypothesis they must constitute an infinite extension. Yet infinitely many space dividers of particular width or thickness, whether finitely extended or indivisible, if constitutive of finite extension in aggregates of two or more, must swell and expand the original finite extension to one of infinite length. Hume, consistently with his *Treatise* argument from the addition of infinite parts, emphatically denies that there could ever be an adequate idea of such an infinite extension.

The imagination's limits in dividing extension follow from the assumption that infinitely many dividers of either type are needed to mark the infinite divisibility of a particular idea of finite extension, that two of the (extended or indivisible) dividers placed together or juxtaposed with any particular idea of any part of the finite extension constitutes a finite extension, and that arithmetically dividing an infinite number of either type of mental dividers by two leaves an infinite number remaining. The infinite number of finitely extended juxtapositions of dividers and divided parts of a clear and distinct idea of a finite extension in turn must comprise an infinite extension, as a later formulation of Hume's *reductio* argument from the addition of infinite parts.

It might be objected that an adequate particular idea of infinite divisibility could be manufactured by the imagination putting together first a finite line segment divided in two, and then imagining each one of these halves being brought forward. Such a process is similar to the way in which a camera lens zooms in on a subject in successive stages, to be divided again in two at greater magnification, and so on, in endless repetition. The Humean reply is naturally that in order to provide an idea of infinite divisibility on this model the mind must already have an idea of what it means for a process to be endlessly repeated, in the sense of being infinitely iterated. But to build this supposition into the objection is obviously to reason in a circle with respect to Hume's doubts about whether the mind can have an adequate idea of infinity.

The 'hint' is presented in explicitly ideational terms. Hume states that 'all the *ideas* of quantity ... cannot be infinitely divisible'. But the proof is easily extended to bridge limitations of ideas about the divisibility of space to the divisibility of space itself, again by referring to the ontic requirements of adequate ideas. The result is also confirmed by the *reductio* proof against infinite divisibility reconstructed from Hume's note to paragraph 124. Here it is necessary only to add the assumption:

5. An adequate idea of extension and the divisibility of extension requires that extension is at most finitely divisible into extensionless indivisibles.

The possibility that Hume may allow a transition of this sort from the limitations of adequate ideas of extension to the properties of extension itself is supported in other ways by the shifts he makes from limitations of ideas of space and time to the phenomena of space and time and back again in the *Treatise*.[114]

Hume's Berkeleyan argument in the first *Enquiry* does not explicitly state that the extensionless indivisible constituents of extension in space are sensible or experienceable rather than abstract or ideal. But in the note to *Enquiry* 124 Hume expresses a continued commitment to the existence of sensible extensionless indivisibles as the atomic constitutents of space, which he refers to in this place as 'images' 'present to the fancy or senses' as the 'parts of extension' which 'cannot be divided or lessened'. This may suggest that in the first *Enquiry* Hume does not abandon the sensible extensionless indivisibles of the *Treatise*, even though he does not recount or otherwise appeal

[114] See above, Part One, Chapter 2.

to the inkspot experiment originally offered to confirm their existence.

There is another related Humean argument which Hume does not offer, but that is consistent with his reflections in the note to paragraph 125. It indicates a way in which the proof based on Berkeley's phenomenological refutation of abstract general ideas can be expanded to establish not only that extension is not infinitely divisible and that there must be extensionless indivisibles, but also, in deference to Bayle's trilemma, that extensionless indivisibles are sensible or experienceable. The continuation of the proof is given in terms of an interpolated assumption. The inference is placed in brackets to indicate that the argument is available to Hume, but is not a reconstruction of an argument explicit in Hume's text.

[6. If there are no abstract ideas of extensionless indivisibles, then an adequate idea of extensionless indivisibles requires that extensionless indivisibles are sensible or experienceable rather than abstract.

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7. Extensionless indivisibles are not abstract but sensible or experienceable. (4,6)
8. Extension is constituted by sensible or experienceable extensionless indivisibles; space is at most finitely divisible into sensible extensionless indivisibles. (7)]

In this way, Hume's Berkeleyan argument against infinite divisibility based on the refutation of abstract general ideas in the note to *Enquiry* 125 can also serve as proof in support of the positive doctrine of sensible extensionless indivisibles, and hence as a later counterpart for the inference Hume advances from the phenomenal data of the inkspot experiment in the *Treatise*.

The Berkeleyan empiricist ground of Hume's argument against infinite divisibility in the first *Enquiry* from the nonexistence of abstract or general ideas links it indirectly to the argument against infinite divisibility based on the inkspot experiment in the *Treatise.* Hume was aware of Berkeley's rejection of abstract general ideas when writing the *Treatise*, but seems not to have appreciated its implications for his early critique of infinity and the theory of sensible extensionless indivisibles.[115]

The argument hinted at by Hume in the first *Enquiry* 125, unlike the *reductio* proof of paragraph 124, is a new demonstration against infinite divisibility that is readily expandable to an argument that extensionless indivisibles, in a sense already implied by the opposition between infinite divisibility and extensionless indivisibility, are also experienceable. If Hume's hint is developed in this direction, it provides a complete replacement for the *Treatise* inkspot experiment and argument, and could as such be reasonably interpreted as reflecting his later thinking about the problems of extension and divisibility in the metaphysics of space and time with an argument he does not offer in the early formulation of his finitism.

Hume concludes the note to 125 by recommending the argument to scientists and mathematicians as a way of avoiding skeptical reductions to absurdity of the infinite divisibility thesis. In effect, Hume counsels abandoning the thesis by invoking Berkeley's rejection of abstract general ideas, and thereby in particular the idea of infinite divisibility. Simultaneously, in this way, Hume gives highest approval to the Berkeleyan argument as providing the best criticism of the concept of infinity. "It certainly concerns all lovers of science," he declares, "not to expose themselves to the ridicule and contempt of the ignorant by their conclusions; and this seems the readiest solution of these difficulties."[116]

Infinite Divisibility from the Treatise to the Enquiry

It is sometimes said that Hume lost interest in the problem of infinity, which had been so prominent in the *Treatise*, by

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[115] *Treatise*, p. 17.

[116] *Enquiry*, p. 158, n. 1.

the time he came to write the first *Enquiry*.[117] The internal evidence for such a comparison is that from an extensive forty-page discussion of infinite divisibility in the *Treatise*, ranging from the inkspot experiment, *reductio* arguments and geometry dilemma, Hume reduces his treatment of the question to just two pages in the *Enquiry*, where the topic, moreover, is relegated to footnotes in the text, with no attempt at the same level of rigorous demonstration or sustained assault on the concept of infinite divisibility.[118]

The explication of Hume's two *Enquiry* proofs in the previous chapters testifies on the contrary to the complexity of his ideas about infinite divisibility in the later work, even if the arguments there are admittedly more condensed and schematic. The connection between Hume's early and later approaches to the divisibility of extension, the ideological and methodological continuities and discontinuities from the *Treatise* to the *Enquiry*, nevertheless remains to be addressed. Without working through all possible accounts, it is worthwhile to outline some of the most fundamental alternative explanations and their consequences in a system of categories that may help to locate Hume's attitude toward infinity and infinite divisibility in writings from the two major periods of his philosophical development.

The following distinctions are intended to be suggestive rather than exhaustive. They embody the most obvious combi-

[117] Nidditch, "Introduction" to *David Hume: Enquiries*, p. xiii: "*Space and Time*. It must be admitted that the subject of space and time, as treated in the Treatise, is not very attractive. There is nothing in the Enquiry corresponding to the forty-two pages of the Treatise, in which space and time are treated, except two pages in §xii."

[118] By contrast, compare Fogelin, "Hume and Berkeley on the Proofs of Infinite Divisibility", p. 57: "When he came to write the *Enquiry Concerning Human Understanding*, Hume seems to have changed his mind about the nondemonstrative character of geometrical reasoning, for there he lists Geometry as one of the sciences derived wholly from Relations of Ideas. Yet Hume's fascination with the problem of infinite divisibility is carried over to the *Enquiry*, where the discussion is curious, and, in fact, not altogether forthcoming."

nations of continuity and discontinuity of ideology, methodology, or both. Here are the main possibilities:

(1) The *Enquiry* analysis of infinity represents a continuation of Hume's thesis in the *Treatise*, with no substantial alteration of ideology or methodology.
(2) The *Enquiry* analysis of infinity represents a continuation of Hume's thesis in the *Treatise*, with no substantial alteration of ideology, but with a substantial alteration of methodology.
(3) The *Enquiry* analysis of infinity represents a continuation of Hume's thesis in the *Treatise*, with no substantial alteration of methodology, but with a substantial alteration of ideology.
(4) The *Enquiry* analysis of infinity represents a significant departure from Hume's thesis in the *Treatise*, with a substantial alteration of ideology and methodology.
(5) The *Enquiry* analysis of infinity represents a significant departure from Hume's thesis in the *Treatise*, with a substantial alteration of ideology, but not of methodology.
(6) The *Enquiry* analysis of infinity represents a significant departure from Hume's thesis in the *Treatise*, with a substantial alteration of methodology, but not of ideology.

The distinctions allow for more subtle subdivisions. Further refinements need not be considered until commitment to or elimination of one or more of the main divisions is made and a tentative interpretation is proposed.

The six categories exhaust the two extreme opposing views that Hume's thought in the first *Enquiry* is essentially a continuation of or significant departure from that of the *Treatise*, in ideology, methodology, neither, or both. It remains possible, and in a sense it is perhaps the most attractive possibility, that Hume's *Enquiry* remarks on infinity are in some sense a continuation of and in another compatible sense a significant departure from the more elaborate demonstrations against infinite divisibility in the *Treatise*, in a way that is not readily accommodated by the above scheme. The proposal in any case serves the preliminary purpose of providing a framework within which a more precise classification can be located.

The external evidence for the continuity interpretation as against the significant departure account is equivocal. Hume, in a famous remark in the 'Advertisement' to the second volume of the posthumous 1777 edition of his collected *Essays and Treatises on Several Subjects*, says of himself in the third person, "*Henceforth, the Author desires, that the following Pieces may alone be regarded as containing his philosophical sentiments and principles.*" Some commentators have understood Hume to imply that he thereby rejects the *Treatise* and substitutes the two *Enquiries* in its place as the definitive statement of his philosophy.

Yet Hume admits only that the *Enquiries* 'cast anew' '[m]ost of the principles and reasonings' of the *Treatise*, "... *where some negligences in his former reasoning and more in the expression, are, he hopes, corrected.*"[119] This statement leaves open the question whether Hume also believes that the first *Enquiry* amends earlier metaphysical errors about the problem of infinite divisibility and the idea of spatial extension in the *Treatise*. Hume's prefatory remarks in the 'Advertisement' to the *Essays* might be understood to apply only to his original treatment of moral questions, or to metaphysical issues other than those concerning infinite divisibility in particular, which he does not mention by name. Hume, moreover, does not admit to having made any philosophical mistakes in the *Treatise* critique of infinity or any other topic, but recognizes only '*some negligences*' in reasoning '*and more in the expression*', which might naturally be understood as oversights rather than defects that the later writings are meant to supplement rather than supplant.

Still, Hume distances himself from the *Treatise* by claiming that the '*Author never acknowledged*' '*that juvenile work*', and maintains that the first and second *Enquiry* alone contain his philosophy. Even this can be understood as a largely aesthetic rather than doctrinal or methodological disavowal, based in part perhaps on Hume's much-publicized chagrin at

[119] Hume, 'Advertisement', *Enquiries*, p. 2.

the anticlimactic reception of the *Treatise*, which, as he reports, paraphrasing Alexander Pope, "... fell dead-born from the press without reaching such distinction as even to excite a murmur among the zealots."[120] The question therefore remains whether and to what extent Hume's later writings on any particular subject including infinity, infinite divisibility, and the theory of sensible extensionless indivisibles, is or is not consistent with the *Treatise* arguments. This, it should readily be appreciated, is a problem that can only be decided by a careful reconsideration of the internal evidence in the *Treatise* and *Enquiry*.

In both the *Treatise* and first *Enquiry*, Hume unequivocally rejects infinite divisibility in favor of a strict finitist theory of spatial extension. There may be important differences between Hume's starting place in arguing against infinite divisibility in the two texts. But his more basic agreement indicates a common empiricist thread in Hume's critique of infinity in the early and later periods of his philosophy. This flatly rules out interpretations (3), (4), and (5).

The study of Hume's *reductio* arguments indicates that in the *Treatise* he needs the equivalent of the phenomenal data of the inkspot experiment to justify his claim that extensionless indivisibles are sensible rather than ideal or abstract. It is only in this way that Hume can circumvent the three-pronged attack of Bayle's skeptical trilemma about the ability of reason to furnish a philosophically adequate theory of the divisibility of extension. Further, the linch-pin of Hume's central inkspot argument is his empiricist thesis about the experiential origin of ideas. When he returns to the problem of infinite divisibility in the later work, Hume remains committed to the thesis of the experiential origin of ideas. He reaffirms the doctrine that ideas originate ultimately in immediate impressions of sensation or reflection, of which they then become the faint or faded

[120] Hume, "My Own Life", *David Hume: The Philosophical Works*, Vol. 3, p. 2.

images. The first *Enquiry*, Section II, 'Of the Origin of Ideas', unequivocally states:

> When we entertain ... any suspicion that a philosophical term is employed without any meaning or idea (as is but too frequent), we need but enquire, *from what impression is that supposed idea derived?* And if it be impossible to assign any, this will serve to confirm our suspicion. By bringing ideas into so clear a light we may reasonably hope to remove all dispute, which may arise, concerning their nature and reality.[121]

The fact that the first *Enquiry* reduces the arguments against infinite divisibility of the *Treatise* to a hybrid *reductio* and an undeveloped hint cannot be explained by Hume's rethinking the empiricist theory of the origin of ideas, for in both places he holds essentially the same view. Hume in the *Enquiry*, as in the *Treatise*, distinguishes ideas from sense impressions in virtually the same way, so that no real or substantial difference in the empiricist foundations of Hume's later epistemology or philosophy of mind can reasonably be regarded as obviating the phenomenal data of the inkspot argument in the *Treatise*.[122]

The interpretations that have not already been eliminated each have strengths and weaknesses, and evidence relevant to their acceptance, pro and con. The claim in (1) that the *Enquiry* analysis of infinity is a continuation of Hume's *Treatise*, with no substantial revision of ideology or methodology, is supported in part by the observation that in the *Enquiry* Hume preserves his early empiricist account of the experiential origins of ideas. The fact that in the *Enquiry* Hume's first proof against infinite divisibility, the *reductio* argument of paragraph 124, seems to combine data of the inkspot experiment with the addition of infinite parts *reductio* to produce a hybrid argument reminiscent of the *Treatise reductio* proofs suggests both continuities and discontinuities in his early and later critique of infinity. That the *Enquiry* arguments do not match exactly any of those in the *Treatise*, but add to the six original arguments two different,

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[121] *Enquiry*, p. 22.
[122] Ibid., pp. 17-22.

though compatible, apparently cannibalized arguments, does not exclude the possibility that Hume in the later period might have preferred to reject and replace the *Treatise* arguments with the two new *Enquiry* arguments. Hume in the *Enquiry* does not say that he endorses the *Treatise* arguments or their conclusions, nor does he mention them with approval or try to summarize them. The mere fact that Hume in the *Enquiry* also rejects infinite divisibility and espouses sensible extensionless indivisibles again in itself does not positively rule out the possibility in interpretation (6) that Hume departs significantly from the theory of the *Treatise*, with the substantial alteration of his methodology if not ideology.

The interpretation that appears most in conformity with the facts of Hume's early and later writings is that represented by possibilities (1) and (2). It is unreasonable to suggest that Hume in his later work departs significantly or substantially in his overall conclusions from the earlier position. In the *Enquiry*, as in the *Treatise*, Hume unequivocally rejects infinite divisibility and accepts the existence of sensible 'physical' extensionless indivisibles. Interpretation (6) shares the same fate as (3), (4), and (5), when it is recognized that the methods of the first *Enquiry* are not sufficiently distinct from those of the *Treatise* to warrant the title of being a significant departure from the early work due to any substantial alteration of methodology.

The essential agreement between the *Treatise* and first *Enquiry* on phenomenal considerations in refuting infinite divisibility by *reductio*, and the establishment of the existence of sensible extensionless indivisibles, suggests that the *Enquiry* represents a continuation of rather than significant departure from the *Treatise* critique of infinity. There is perhaps no definitive agreement to be expected here, since what may seem or be made out to seem significant is a matter of judgment and perspective. The hypothesis that Hume has not significantly changed his position about the problem of infinite divisibility from the *Treatise* to the *Enquiry*, but merely offers different, perhaps stronger, compatible arguments in defense of the same conclusions, makes it possible to remain neutral with respect to the question whether Hume intends the new arguments to supplement or replace the original demonstrations, and, if the *Enquiry* arguments are supposed to replace those in the *Treatise*, whether Hume's decision to replace them is of methodological significance or merely stylistic. Hume evidently regards the two arguments in the notes to *Enquiry* 124 and 125 as sufficient to reject infinite divisibility and to uphold the positive thesis of sensible extensionless indivisibles in the metaphysics of extension, to which he remains philosophically committed.

The continuity interpretation of Hume's attitude toward infinite divisibility from the *Treatise* to the *Enquiry* is disputed by J.M.M.H. Thijssen in his essay, "David Hume and John Keill and the Structure of Continua". Thijssen writes:

> In the *Enquiry* Hume upholds the view that an extension is divisible into *physical* points: "Whatever disputes there may be about mathematical points, we must allow that there are physical points" [*Enquiry*, p. 156, n. 1]. Now it could be, of course, that by "physical points" Hume means nothing more than the visible or tangible of the *Treatise* and that his shift from mathematical to physical points is only a matter of terminology, but I do not think that this interpretation holds against the textual evidence; for in this work Hume clearly explains the physical points as "points of extension, which cannot be divided or lessened, either by the eye or the imagination" [ibid.]. They "must be allowed by mathematicians to be infinitely less than any real part of extension" or in other words they are "infinitely small parts of extension" that are, moreover, "absolutely indivisible". In my opinion Hume here expresses the view that a quantity consists of physical *minima* or atoms. The text is not very clear as to whether the number of atoms in a quantity is finite or infinite. The crucial passage reads as follows: "and yet nothing appears more certain to reason, than that an infinite number of them [i.e., physical points] composes an infinite extension. How much more an infinite number of those infinitely small parts of extension which are still supposed infinitely divisible" [ibid.].... Hume here clearly deviates from

> his doctrine in the *Treatise*. The *extensionless* tangible or visible points have turned into physical points *with extension*.[123]

I find Thijssen's interpretation of the *Enquiry* passages unpersuasive. Hume in the *Enquiry* says nothing to contradict the *Treatise* conception of *minima sensibilia*, of sensible indivisibles, as extensionless. The description Thijssen quotes from the *Enquiry* essentially repeats the description of sensible extensionless indivisibles in the *Treatise* inkspot experiment, as incapable of division in vision or imagination.

The *reductio* context in which Hume offers these remarks explains why he speaks of physical points as infinite in number. This is the infinite divisibility hypothesis of the indirect proof which he rejects after deducing the absurdity of an infinitely extended finite extension. Thijssen paraphrases the argument correctly in the first of the two interpretations he considers. Then he suggests that: "... the second interpretation would be that it appears certain to reason that an infinite number of physical points composes an infinite extesnion, but that our abstractive reasoning misleads us. In reality an infinite number of points makes up only a finite extension."[124] The explanation is hard to follow. But Thijssen's conclusion that on either account Hume in the *Enquiry* is committed to extended physical points is clearly unsupported by the text. Hume in the *Treatise* regards extension as constituted by juxtapositions of extensionless indivisible 'points'. If the *Enquiry* version of the argument from the addition of infinite parts holds that there cannot be infinitely many sensible indivisibles or 'physical points' on pain of implying that every finite extension is infinitely extended, then there is no justification for thinking that Hume in the later argument has forsaken the *Treatise* thesis

[123] Thijssen, "David Hume and John Keill and the Structure of Continua", p. 276.
[124] Ibid., p. 277.

of the extensionlessness of the indivisible constituents of finite extension.[125]

It is interesting to compare the difference in emphasis between problems of infinite divisibility and skepticism as they arise in the *Treatise* and *Enquiry*. In the *Treatise*, infinity and questions about space and time are given their own Part II of Book I, and the theme of skepticism is not explored in detail until Part IV, *Of the skeptical and other systems of philosophy*. In the *Enquiry*, by contrast, the focus is more directly on skepticism, and disputes about the infinite divisibility of space and time are introduced merely as examples of how 'Pyrrhonian' skeptics might try in an ultimately self-defeating way to use (natural) reason to refute (metaphysical) reason. Even this apparent difference is softened by the consideration that Hume in the *Treatise* prepares the reader for his discussion of infinite divisibility and the doctrine of extensionless indivisibles by identifying something like the same skeptical attitude found in the first *Enquiry*. Thus, in the *Enquiry*, Hume writes:

> Whatever has the air of a paradox, and is contrary to the first and most unprejudic'd notions of mankind is often greedily embrac'd by philosophers, as shewing the superiority of their science, which cou'd discover opinions so remote from vulgar conception. On the other hand, any thing propos'd to us, which causes surprize and admiration, gives such a satisfaction to the mind, that it indulges itself in those agreeable emotions, and will never be perswaded that its pleasure is entirely without foundation. From these dispositions in philosophers and their disciples arises that mutual complaisance betwixt them; while the former furnish such plenty of strange and unaccountable opinions, and the latter so readily believe them. Of this mutual

[125] This objection undermines the presupposition of Thijssen's observation that: "The text of the *Enquiry* provides no clue as to why Hume may have changed his mind on this point and any attempt at an explanation is complicated by the totally different contexts in which the discussion of continuity occurs in both works. But still I would suggest that Hume may have been motivated to change his account of the composition of continua in the *Enquiry* because of dissatisfaction with his earlier doctrine concerning ideas of space and time."

> complaisance I cannot give a more evident instance than in the doctrine of infinite divisibility, with the examination of which I shall begin this subject of the ideas of space and time.[126]

By transposing Hume's terminology in the *Treatise* to that more directly concerned with skepticism in the first *Enquiry*, it is possible to see in this overture to the early analysis of infinite divisibility the same or remarkably similar skeptical context for critical examination of infinity in the *Treatise* as in the *Enquiry*. In the earlier work, Hume speaks of 'the air of paradox' encountered by the 'vulgar conception' and 'unprejudic'd notions of mankind' in reflecting on the implications of infinite divisibility. Later, in the first *Enquiry*, he refers to the 'extravagant attempt of the skeptics to destroy *reason* by argument and ratiocination', by exhibiting the consequences of 'ideas, which, in common life and to a careless view, are very clear and intelligible', but which nonetheless 'seem full of absurdity and contradiction' when 'they pass through the scrutiny of the profound sciences'. On careful reading, both the *Treatise* and first *Enquiry* debut the question of infinite divisibility as an 'instance' or 'example' of the conflict of natural belief with metaphysical reason, and each offers parallel solutions to the problems engendered by the conflict.

It therefore seems defensible on several fronts to interpret the affinities between Hume's arguments in the *Treatise* and first *Enquiry* as outweighing the differences. There is an essential unity in Hume's rejection of infinite divisibility and assertion of sensible extensionless indivisibles in both works. If he had chosen, and if the arguments had occurred to him at the time, Hume might have seamlessly incorporated the later two *Enquiry* arguments into the original six arguments of the *Treatise*. With no real disruption of the substance if not the style of the *Enquiry*, Hume similarly might have introduced the six *Treatise* arguments into the critique of infinity in the later work. That Hume did not avail himself of the *Enquiry* arguments in the *Treatise*, given the everything-but-the-kitchen-sink approach of

[126] *Treatise*, p. 26.

the first text, is perhaps best explained by the assumption that at that time he had simply not yet thought of them. Of course, Hume was familiar with Berkeley's rejection of abstract general ideas. But he did not seem to appreciate the force of Berkeley's repudiation of abstract general ideas in advancing a powerful and convincing criticism of infinite divisibility with the potential for upholding the doctrine of sensible extensionless indivisibles contained in the 'hint' of *Enquiry* 125. Hume also had the essential elements of the *reductio* argument of the note to *Enquiry* 124 at his fingertips at the time of the *Treatise*, but did not put the pieces together in a single proof until writing the *Enquiry*.

We have seen that the most natural way for Hume to uphold the conceivability of 'mathematical' points against infinitist mathematicians who claim to understand the concept of abstract mathematical points is to invoke Berkeley's refutation of the real phenomenological occurrence of abstract general ideas. Such a maneuver is also available to Hume in principle in the *Treatise*, Book I, Part I, Section VII. Berkeley's rejection of Newton's infinitesimals in *The Analyst*, and the brief windstorm of pamphlets exchanged between Berkeley and the defenders of Newton's infinitary mathematics of fluxions which it spawned, are further indications of Berkeley's continuing spiritual presence in Hume's ongoing dispute with the concept of infinity from the *Treatise* to the first *Enquiry*.[127]

It is now tempting to imagine, at least with respect to the metaphysics of extension and the problem of infinity, that Hume's first *Enquiry* merely rephrases his position about infinite divisibility and sensible extensionless indivisibles in the *Treatise*. He appears in the *Enquiry* to reach the same conclusions in much the same way as in the *Treatise*, without far-reaching ideological or methodological reorientation. He uses partially new or recombined parts of arguments that were equally

[127] Berkeley's refutation of abstract general ideas also implicitly reinforces Hume's empiricist thesis about the experiential origin of ideas in impressions of sensation and reflection. It functions by ruling out the possibility of innate ideas that would otherwise appear to derive *a priori* from nonexperiential origins.

available to him when writing the *Treatise*, that could have been included in the early work without drastically altering its tenor. Hume's philosophical reasons, if he had any, as opposed to stylistic motivations, whatever they might have been, for not appealing to, subsuming, or even mentioning the *Treatise* arguments in the first *Enquiry*, must remain a mystery. Possibly, Hume had an aversion to repeating the arguments of the *Treatise* for aesthetic or literary reasons, or perhaps he assumed that interested readers would seek out his earlier arguments, making it unnecessary to recapitulate them. Again, his purpose in the first *Enquiry* might have been simply to present the persistent conclusions of his early and later philosophy in compact and less technical form, to reach a wider audience, and provide a more easily surveyable study of the topics under a slightly different organization of subject and slightly different emphasis of themes.

Berkeley offers a precedent for this interpretation. He does something remarkably similar in composing his *Three Dialogues Between Hylas and Philonous* after the disheartening reception of *A Treatise Concerning the Principles of Human Knowledge*.[128] In *Three Dialogues*, Berkeley tries to popularize and make his views more accessible in conversational form, less a victim to vulgar misunderstanding, anticipating and answering objections. It is therefore easy to believe, if anything like the same kind of concern was part of Hume's motivation in recasting his critique of infinity among other issues from the *Treatise* to the first *Enquiry*, that he may have wanted to offer only the essentials of the theory with minimal trimmings, to gesture toward conclusions and leave the reader to develop an argument along suggested lines without burdening the second text with unnecessary and unnecessarily controversial considerations. To maintain, as Hume does at the very end of his 'hint' in the note to *Enquiry* 125, that the Berkeleyan refutation properly developed may provide the 'readiest solution' to the difficulties posed by the skeptic's challenge to reason on the

[128] Bracken, *The Early Reception of Berkeley's Immaterialism 1710-1733*.

basis of theoretical commitment to infinity divisibility in the metaphysics of extension indicates that Hume is prepared to let the problem rest on the strength of the Berkeleyan argument that is new to the *Enquiry*, rather than any of the six arguments of the *Treatise*. This, however, does not imply that Hume rejects the earlier arguments. At most we can say only that upon later reflection Hume seems to believe that the object of the *Treatise* critique of infinity can be better achieved by denying the possibility of abstract general ideas of infinite divisibility. The arguments of the first *Enquiry* are clearly different than those of the *Treatise*, and the differences should not be obscured. The two *Enquiry* proofs nevertheless share with their predecessors an underlying unity of strict finitism supported by a characteristically empiricist methodology.

Why, then, with so much riding on Berkeley's argument against abstract general ideas in the first *Enquiry* refutation of infinity, does Hume not expand on the objection, instead of limiting the idea to an undeveloped 'hint'? Why does Hume fail to provide something like a more fully elaborated formulation of the proof in the manner of the *Treatise*? If Hume preferred for aesthetic or polemical reasons to advance an incomplete statement of his arguments as an invitation for the reader to fill in the blanks, perhaps in order to offer a simplified less rigorous presentation of his theory, then he did not simply neglect a problem that had exercised so much of his philosophical energy and ingenuity in the early writing, but instead found a more satisfactory way of expressing the unrepentent conclusions of his original critique.[129]

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[129] On the increased role of skepticism in the first *Enquiry* as compared with the *Treatise*, see Immerwahr, "A Skeptic's Progress: Hume's Preference for *Enquiry I*", pp. 227-238.

CONCLUSION

HUME AGAINST THE MATHEMATICIANS

This may open our eyes a little, and let us see, that no geometrical demonstration for the infinite divisibility of extension can have so much force as what we naturally attribute to every argument, which is supported by such magnificent pretensions.

— Hume, *Treatise*, Book I, Part II, Section IV

CONCLUSION

HUME AGAINST THE MATHEMATICIANS

On the Experiential Origin of Ideas

If the concepts of infinity and infinite divisibility are eliminated, what happens to mathematics, science, and philosophy? Can these disciplines be satisfactorily supported by a Humean strict finitist metaphysics of space and time in which extension is at most finitely divisible into finitely many sensible extensionless indivisibles?

These questions are important only if we already agree that infinitist theories are sufficiently valuable for it to matter whether or not they survive Hume's purge. The refusal to countenance concepts whose empirical lineage cannot be accounted for on the impressions and ideas model may override concerns about the potential loss of sacrificing traditional theories of number, space, and time to Hume's empiricist epistemology and philosophy of mind. If the concepts necessary to a theory cannot be justified, then we are only deluded into thinking we are intellectually indebted to the theory in the first place. A theory that rests on pseudo-concepts is at most a pseudo-theory, if by 'theory' we mean a set of propositions expressing clearly articulated relations of ideas. The pragmatic success of the putative theory must then be explained if possible in some other way, usually by reinterpreting its basic principles without reference to counterfeit concepts. This is what Hume seems ready to do in his jeremiads against the mathematicians.

Hume's eight arguments against infinity in the *Treatise* and first *Enquiry* are directed against those infinitists he usually labels simply 'the mathematicians'. This terminology follows Sextus Empiricus's in opposing mathematics as a discipline with an essentially abstract subject matter beyond the limit of sound empirical methodology, *adversus mathematicos*.[1] That mathematics, ordinarily regarded as setting the standard for exactness in philosophy and the sciences, might not be in command of clear ideas of infinity and infinite divisibility gives Hume's criticism of mathematics its philosophical poignancy.

Hume emphasizes this point in the *Treatise* discussion of the conflict between definitions and demonstrations in mathematics. He explains:

> There have been many objections drawn from the *mathematics* against the indivisibility of the parts of extension; tho' at first sight that science seems rather favourable to the present doctrine; and if it be contrary in its *demonstrations*, 'tis perfectly conformable in its *definitions*. My present business then must be to defend the definitions, and refute the demonstrations.[2]

The distinction between definitions and proofs or demonstrations in mathematics offers the leverage Hume thinks he needs in order to refute traditional infinitistic mathematics, while arguing that basic mathematical definitions of point, line, surface, and extension, are not only compatible with his strict finitism, but are satisfiable in principle only by the doctrine of sensible extensionless indivisibles.

This opens the way for a Humean philosophy of mathematics grounded on the strong empiricist thesis of the experiential origin of adequate ideas of extension. Hume's claim that the definitions and demonstrations of traditional infinitary mathematics are inconsistent or logically at odds with one another underwrites Hume's critique, while leaving open the possibility of reinterpreting mathematics in strict finitist terms that alone

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[1] The phrase '*Adversus Mathematicos*' appears in some titles of Sextus Empiricus' surviving works, also listed as *Adversus Dogmaticos*, and translated into English as *Against the Mathematicians* or *Against the Physicists* by Bury in *Sextus Empiricus*.

[2] *Treatise*, p. 42.

are supposed to do justice to the real meaning of point, line, and so on, in the definitions of traditional geometry. The distinction permits Hume to honor the underlying concepts of mathematics as correct, and to identify the error of infinitary mathematics precisely in its failure to live up to the requirements of its own definitions, going beyond them in unjustifiable ways in its attempted proofs and demonstrations.

The term 'mathematics' might persist as title for the radical overhaul of the theory and calculi of quantity Hume envisions, to bring the formal sciences into line with a correct metaphysics of space and time based on a correct philosophical methodology. Mathematics need not maintain its traditional commitments to infinity, infinite divisibility, and infinitesimals, as shown by modern developments in standard analysis, intuitionistic foundations for mathematics, and finite and discrete mathematics. Hume might also have preferred to break more radically even in terminology from the *mathesis* tradition. Yet Hume nowhere attempts to characterize the mathematics that might emerge if his strict finitism were to be accepted. A strictly finitistic arithmetic, algebra, and geometry, with all practical applications intact, could replace the traditional infinitary systems Hume rejects. There need be no vacuum in what is called abstract thinking in mathematics as a result of Hume's critique of infinity. If Hume were willing to substitute for the concept of infinity a concept of finite but indefinite, indeterminate, or inexhaustible sets and series, then he might not see a special need to propose an entirely new mathematics, which could proceed more or less classically without jeopardizing the sound results of traditional (subtransfinite) pure and applied mathematics.

Hume introduces the conflict between the definitions and demonstrations of geometry in an effort to forestall objections to the concept of sensible extensionless indivisibles from the standpoint of classical infinitary mathematics. He writes:

> A surface is *defin'd* to be length and breadth without depth: A line to be length without breadth or depth: A point to be what has neither length, breadth nor depth. 'Tis evident that all this

> is perfectly unintelligible upon any other supposition than that of the composition of extension by indivisible points or atoms. How else cou'd any thing exist without length, without breadth, or without depth?[3]

The discussion prepares the way for Hume's geometry dilemma, which appears immediately thereafter. Hume takes the view that the definitions of mathematics are unintelligible unless they are given a finitistic interpretation in terms of sensible extensionless indivisibles, and he lets the objection serve as a point of transition to the dilemma against infinite divisibility based on the requirements and origins of the ideas of exact equality and proportion in geometry. Geometry can be resurrected on a correct strict finitist basis if geometrical points are regarded, not as ideal abstract Euclidean (non) entities — the 'nothingnesses of extension' that cannot constitute extension no matter how they are combined — but instead as the sensible extensionless indivisibles discovered by the inkspot experiment. Hume reasons:

> Thus it appears, that the definitions of mathematics destroy the pretended demonstrations; and that if we have the idea of indivisible points, lines and surfaces conformable to the definition, their existence is certainly possible: but if we have no such idea, 'tis impossible we can ever conceive the termination of any figure; without which conception there can be no geometrical demonstration.[4]

> But I go farther, and maintain, that none of these demonstrations can have sufficient weight to establish such a principle, as this of infinite divisibility; and that because with regard to such minute objects, they are not properly demonstrations, being built on ideas, which are not exact, and maxims, which are not precisely true.[5]

There is a dependence relation between adequate mathematical concepts and proper mathematical demonstrations. A

[3] Ibid.

[4] Ibid., p. 44.

[5] Ibid., pp. 44-45.

demonstration is no stronger than the concepts about which it purports to offer conclusions. What may pass for mathematical proof is not really proof at all, if it rests on faulty, obscure, or inexact ideas of mathematical entities that do not adequately apply mathematical definitions. The most elegant syntactically consistent calculus for infinite and higher-order transfinite sets and series would for Hume be no more than an empty symbolism whose terms do not represent adequate ideas, and to which nothing in thought or reality definitely corresponds. The first responsibility in philosophy of mathematics, Hume believes, in striking resonance with Wittgenstein and mainstream contemporary analytic thought, is the clarification of its underlying concepts.[6]

The objection that ideas of infinite divisibility are not clear or precisely true occasions Hume's long detailed discussion of the role of the idea of exact equality and proportion in geometrical demonstrations. Hume prefaces his discussion of the geometry dilemma by showing that only the idea of sensible extensionless indivisibles is able to account for the precision required in the geometrical concepts of quantity, limit, and the like. He argues that the ideas of basic geometrical objects are incompatible with infinite divisibility, and that they demand instead a strict finitist concept of at most finitely divisible spatial extension constituted by finitely many sensible extensionless indivisibles. The objection is that if extension is infinitely divisible, then the concept of a terminus or boundary of lines or surfaces is unintelligible, because any candidate for that role on the traditional assumption will itself be infinitely divisible, and as such cannot constitute a single true endpoint.

> A surface terminates a solid; a line terminates a surface; a point terminates a line; but I assert, that if the *ideas* of a point, line

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[6] In this respect, Hume's attitude is remarkably modern and analytic; it is even tempting to say, Wittgensteinian. See Wright, *Wittgenstein on the Foundations of Mathematics*, esp. pp. 117-141 on Wittgenstein's finitism as a consequence of the surveyability requirement for mathematical proofs in *Remarks on the Foundations of Mathematics*.

> or surface were not indivisible, 'tis impossible we shou'd ever conceive these terminations. For let these ideas be suppos'd infinitely divisible; and then let the fancy endeavour to fix itself on the idea of the last surface, line or point; it immediately finds this idea to break into parts; and upon its seizing the last of these parts, it loses its hold by a new division, and so on *in infinitum*, without any possibility of its arriving at a concluding idea. The number of fractions bring it no nearer the last division, than the first idea it form'd. Every particle eludes the grasp by a new fraction; like quicksilver, when we endeavour to seize it. But as in fact there must be something, which terminates the idea of every finite quantity; and as this terminating idea cannot itself consist of parts or inferior ideas; otherwise it wou'd be the last of its parts, which finish'd the idea, and so on; this is a clear proof, that the ideas of surfaces, lines and points admit not of any division; those of surfaces in depth; of lines in breadth and depth; and of points in any dimension.[7]

The attempt to identify any geometrical object as a true terminus to another is necessarily foiled on the infinitist assumption. Taking hold of one, such as a Euclidean *locus* of geometrical points, regressively implies a horizon of infinitely receding *loci* of points into which every *locus* in turn is infinitely divisible. Thus, any purported limit in the infinitist model frays away indefinitely with no sharp edge or final clean break.

> The *schoolmen* were so sensible of the force of this argument, that some of them maintain'd, that nature has mix'd among those particles of matter, which are divisible *in infinitum*, a number of mathematical points, in order to give a termination to bodies; and others eluded the force of this reasoning by a heap of unintelligible cavils and distinctions. Both these adversaries equally yield the victory. A man who hides himself, confesses as evidently the superiority of his enemy, as another, who fairly delivers his arms.[8]

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[7] *Treatise*, pp. 43-44.

[8] Ibid., p. 44. Bayle in his *Dictionary* article on 'Zeno of Elea' similarly writes: "... it is certain that an infinite number of parts doth not contain any which is first; and yet a body in motion can never touch the second

The problem is avoided, according to Hume, if sensible extensionless indivisibles rather than the infinitely divisible ideal or abstract Euclidean mathematical points are thought to satisfy the traditional definitions of geometry.

> There are few or no mathematicians, who defend the hypothesis of indivisible points; and yet these have the readiest and justest answer to the present question [of explaining the meaning of 'equal to', 'greater or less than' in geometry]. They need only reply, that lines or surfaces are equal, when the numbers of points in each are equal; and that as the proportion of the numbers varies, the proportion of the lines and surfaces is also vary'd.[9]

Hume distinguishes between the solution to the limit or end-point problem afforded by the doctrine of sensible extensionless indivisibles and its practical utility. He believes the solution is correct in theory, but he finds the application 'useless'. The difficulty is that in practice it is virtually impossible to count the exact number of indivisibles in a finite extension. The proposal is therefore of negligible assistance in actually calculating the quantity or equality of geometrical dimensions.[10]

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before the first... And how will [a body] touch [another], since all those parts which you will pretend to be the last, contain an infinity of parts, and infinite number hath no part which can be last? This objection obliged some scholastic Philosophers to suppose, that nature hath intermixed Mathematical points with the parts divisible *in infinitum*, to the end that they may serve to connect, and compose the extremities of bodies. They thought by that means to answer also the objection of the penetrative contact of two surfaces: but this evasion is so absurd, that it doth not deserve to be refuted."

[9] *Treatise*, p. 45.

[10] Berkeley knows that the finitist account of the divisibility of extension implies that not every line segment (in particular those consisting of an odd rather than an even number of *punctiforma*) can be exactly bisected. See Raynor, "'*Minima Sensibilia*' in Berkeley and Hume", p. 199. Hume nowhere discusses the problem, but it might be said that geometry can afford to ignore the possibility in theory and practice, in view of the fact that sensible extensionless indivisibles are so tiny that the difference of one more or less is inconsiderable. There are greater problems encountered

How, exactly, are Euclidean points supposed to figure into the classical infinitary conception of infinitely divisible extension in the first place? It appears, though not in Kant's technical sense, neither analytically nor synthetically; and so, a critic like Hume might conclude, not at all. Not analytically, because the infinite subdivision of any extension never reaches down to individual points, but only to further supposedly infinitely divisible line subsegments that are themselves in a still unexplained sense alleged to consist ultimately of individual Euclidean points. Not synthetically, either, because two Euclidean points can never be put together so as to constitute an extension. This is true not only because of Bayle's objection that they are just so many 'nothingnesses' of extension, but because wherever two Euclidean points appear, according to the infinite divisibility thesis, there is supposed to be at least one other point between them, as though created by spontaneous generation whenever two points are squeezed together in order to define an extended line segment.

There follows next in Hume's geometry dilemma a more detailed account of alternative concepts of quantity and equality that are available to the traditional mathematician committed to the infinite divisibility thesis, which we have already considered. Hume concludes:

> But tho' this answer be *just*, as well as obvious; yet I may affirm, that this standard of equality is entirely *useless*, and that it never is from such a comparison we determine objects to be equal or unequal with respect to each other. For as the points, which enter into the composition of any line or surface, whether perceiv'd by the sight or touch, are so minute and so confounded with each other, that 'tis utterly impossible for the mind to compute their number, such a computation will never afford us a standard, by which we may judge of proportions. No one will ever be able to determine by an exact numeration, that an inch has fewer points than a foot, or a foot fewer than

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here on the infinitist assumption that every finite line segment is infinitely divisible, since then it must be impossible to identify an exact midpoint in any line segment.

> an ell or any greater measure; for which reason we seldom or never consider this as the standard of equality or inequality.[11]

The fact that Hume considers the reference to sensible extensionless indivisibles as providing the proper if practically inapplicable answer is sufficient to indicate that he thinks a finitistic mathematics might be reconceived along traditional lines. Such a program might be implemented by reinterpreting the standard definitions as satisfied, not by ideal abstract Euclidean mathematical points, but by sensible extensionless indivisibles. These in turn would be justified for Hume only by a correctly humanized epistemology and philosophy of mind, an empiricist methodology that does not engender the absurdities and paradoxes of infinitist interpretations of the definitions of geometrical concepts.

The distinction enables Hume to preserve the most useful definitions of traditional mathematics, while rejecting the infinite divisibility thesis, and establishing conceptual foundations for a revisionary finitistic mathematics. The interpretation portrays Hume more accurately as a reformer of definitions rather than assassin of the theorems of traditional mathematics, a neotraditionalist of sorts, who seeks to return mathematics to its original legitimate conceptual basis in the rightful interpretation of its basic concepts, that are correct in spirit but corrupted in meaning by infinitism.

Mathematics and Science Without Infinity

The problem remains whether and to what extent we should accept Hume's empiricist account of the origin of ideas if its repudiation of the concepts of infinity and infinite divisibility contradicts indispensable scientific methods. This is a deep and far-ranging question about the viability of strict finitism. An indication of the potential for finite mathematics, science, and philosophy may suggest that the ideas of infinity and infinite divisibility are not strictly needed for essential theoretical purposes.

[11] *Treatise*, p. 45.

Hume, in the revolutionary fervor of his methodology, will maintain that traditional scientific practices, no matter how useful or well-entrenched, are not worth preserving if they rest on philosophically objectionable foundations. If we have not already made up our minds about the truth of Hume's theory of the experiential origin of ideas, then we will not share his confidence in defying classical infinitism, particularly in the formal and natural sciences. It is important, then, to sketch even if only in outline the prospects of mathematics, science, and philosophy as they might appear bereft of the concept of infinity, to see what if anything of theoretical or practical value Hume must relinquish. We shall briefly consider the impact of Hume's strict finitism from an historical perspective, and from the standpoint of contemporary developments in mathematics and science.

Hume's critique of infinity can be judged either by reference to the state of the art of mathematics and science in his day, by the standards with which he like any philosopher of his time could be expected to be familiar, or by contemporary standards. Both types of comparison are worthwhile. Each addresses different issues about the adequacy of Hume's empiricism in its theoretical ramifications. Hume, it goes without saying, is not responsible for anticipating subsequent innovations in modern mathematics and science, so that from an historical perspective, although it might be reasonable to condemn his finitism if it contradicts Newton's physics and the calculus of infinitesimals, it would arguably be less reasonable to blame Hume's doctrine if it should turn out to conflict with, say, Georg Cantor's transfinite set theory. Yet comparisons of Hume's finitism with modern requirements in mathematics and science are also interesting in retrospective appreciation of his philosophy, for they emphasize some of the deeper conceptual issues with which Hume was concerned, and the implications of his foundational investigations in the metaphysics of scientific methodology.

The distinction between historically contextual and retrospective evaluation is no sooner drawn than its difficulties appear. The attempt to assess Hume's finitism impartially in temporal and cultural terms may unavoidably involve importing philosophical considerations from our own time that were not part of Hume's intellectual milieu. Hume was self-consciously breaking from intellectual traditions that in previous centuries had nourished infinitary concepts in mathematics, science, and philosophy. He projected his critique of infinity with full knowledge of the fact and deliberate intent that it should contradict the Cartesian, Newtonian, and Leibnizian infinitist establishment, confident that truth in the sciences can only be discovered by sound methodology, and that only empiricism provides a sound methodology.

Hume's first-things-first attitude assumes that if a proper foundation is laid for the sciences, then truth will follow by correct application of the method. Whatever cannot be justified by these procedures, despite its usefulness, or the mandarin authority of its scientific sponsors, has no final title to philosophical respectability. Hume must hold that if a proposition is contradicted or cannot adequately be supported by the only proper methodology, then eventually it will come to be seen as dispensable, and scientific disciplines, insofar as they need and deserve to be perpetuated, will find a way to conduct their investigations without commitment to indefensible principles. The faith that future discoveries in mathematics and science will ultimately conform to the requirements of his empiricism makes it appropriate to evaluate Hume's finitism even in historical perspective from the standpoint of latterday advances in the philosophy of mathematics and science. All these factors make the assessment of Hume's project complex but not hopeless.

The first observation to be made in defense of Hume's finitism from both historical and retrospective viewpoints is that infinite quantities are not strictly required in mathematics, science, or philosophy, if finite but indefinite, indeterminate, or inexhaustible quantities will do as well. If infinite divisibility does not obtain, extension, though only finitely divisible, need not be limited to fixed determinate subdivisions or successions of subdivisions, but in principle could continue in Flew's phrase to be divided as (finitely but indefinitely) often as anyone might choose. This may seem to be at odds with Hume's doctrine of sensible extensionless indivisibles, which are meant to be atomic constituents of extension. But the inkspot in Hume's experiment is subjectively indivisible only at a certain distance, while for another subject with greater visual acuity, or for the same subject assisted by an optical aid, or at a lesser distance, the inkspot will no longer be indivisible, but divisible once again into discrete parts. The closer one approaches an object, the more finely divisible it may appear, until distance or the use of high resolution instruments reach their practical limit in perception and imagination. This suggests that the divisibility of extension though finite is practically inexhaustible, and that indefinitely successive divisibility is, in Kant's terminology, a regulative rather than constitutive principle.[12] The search for more powerful magnifying devices might then be inspired by the belief, consistent with Hume's empiricism, that a perceiver using sensation-enhancing equipment need never exhaust the finite divisibility of an extended object into *minima sensibilia relativa*.

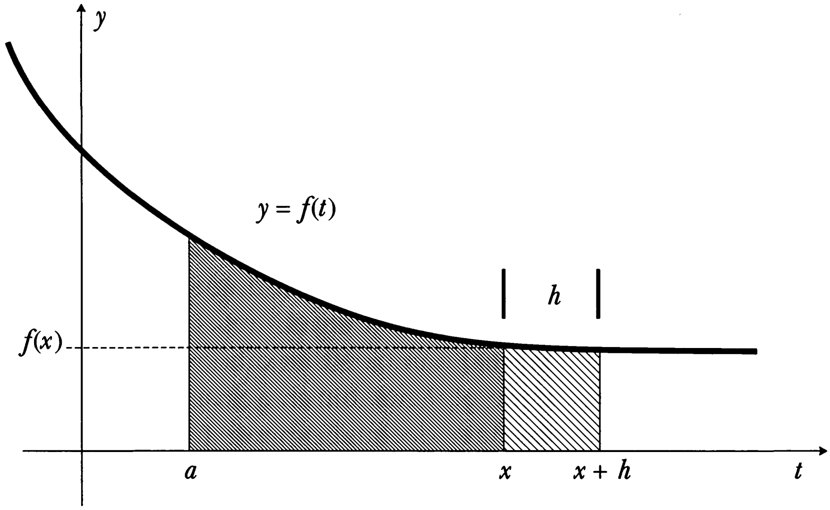
The idea of finite but indefinite, indeterminate, or inexhaustible sets and series of mathematical objects can be defended on two counts against its rival concept of infinite sets and series. Infinity implies inexhaustibility, but not conversely. To be inexhaustible means only that no predetermined limit or end to a set or series will ever actually be attained; not, as in the concept of infinity, that none can ever be ascribed. Finite indefiniteness, indeterminacy, or inexhaustibility appears an adequate replacement for the problematic concept of infinity for the needs of mathematics and science. It is plausibly, moreover, the idea that infinitist mathematicians, scientists, and philosophers actually employ when they claim to be thinking of infinity and infinite divisibility. We may usefully regard finite indefinite, indeterminate, or inexhaustible sets and series

[12] Kant, *Critique of Pure Reason*, A180/B223-A182/B225.

as sequences produced by imaginary machines that generate ordered finite strings of integers, or digits in the decimal expansion of an irrational real number, or other mathematical entities, that begin at and continue through no fixed point in time or operate at no definite speed, and therefore manufacture sets and series of no fixed limit. The concept has the potential of preserving most of the essential mathematics and science of Hume's and the present time, without commitment to the idea of infinity. Among other things, the strict finitist model can be used to define irrational real numbers that otherwise require an infinite decimal expansion of digits, but may instead be defined by finite though inexhaustible decimal expansions. Similarly for more traditional infinite Euclidean point-set series in space and time, Newton's fluxions, Leibniz's infinitesimals, infinitary limits in the modern integral and derivative calculus of Cauchy and Weierstrass, and the set-theoretically defined infinitesimals of Robinson's nonstandard analysis.[13]

All such systems involve calculations performed in finite time using partial finite representations of what are supposed to be infinite sets or series, but that without loss of power can be reinterpreted as finite but indefinite, indeterminate, or inexhaustible sets or series. Thus, when we write the supposedly infinite decimal expansion of an irrational real number like π as 3.14159..., the '...' or 'dot-dot-dot' need not represent an infinite expansion, but an indefinite, indeterminate, or inexhaustible rule-governed progression of digits. In the fundamental theorem of the calculus, the area beneath a curve is determined for any values of $f(t)$ as a continuous function, in which t and y values are supposed in principle to be extendible to ∞ (infinity). But identical values obtain for any chosen finite interval $[a, x + h]$ on t, even if the parameter limit is understood as indefinite, indeterminate, or inexhaustible in extent,

[13] Cauchy, *Leçons de calcul différentiel et del calcul intégral.* By contrast with Weierstrass's eliminative approach, see also Robinson's attempt to revive actual infinitesimals. Robinson, *Non-Standard Analysis*; "The Metaphysics of the Calculus", pp. 28-40. Luxemburg and Robinson, *Contributions to Non-Standard Analysis.*



Fundamental Theorem of the Calculus

If $f(t)$ is continuous on the interval $[a, b]$, and if, for each x in $[a, b]$, area $A(x) = \int_a^x f(t)dt$, then $A(x)$ is differentiable, and $dA/dx = d/dx \int_a^x f(t)dt = f(x)$. $A(x)$ is thereby an antiderivative of $f(x)$.

rather than extending (whatever this is supposed to mean) to infinity.[14]

The finitist reinterpretation in fact has no effect whatsoever on the calculations, theoretical conclusions, or practical applications of classical mathematics. To say that a set or series is unlimited, unbounded, or unending, continues as long as anyone might please, has no end, never stops, etc., is after all what infinitists typically say anyway when they try to explain or stim-

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[14] I will take this opportunity also to criticize the standard lazy-eight symbol for infinity, '∞'. Like the '. . .' or 'dot-dot-dot', there is nothing particularly suggestive of infinity about this convention. The '. . .' may indicate that a set or series continues indefinitely, indeterminately, or inexhaustibly, but to understand that a progression continues infinitely requires a prior understanding of and commitment to the possibility of infinity. Similarly for '∞', which looks like a sign for infinity only if we imagine the loops tracing their pattern back and forth infinitely or over endless infinite time; otherwise, the symbol appears conspicuously finitistic. Why suppose that the movement goes around more than once?

ulate the imagination with a picture of infinity. This makes it tempting to suppose that the concept they are actually thinking of and actually putting to work in pure and applied mathematics in the sciences, despite picturesque references to *the infinite*, is in really no more than a concept of the finite but practically or potentially indefinite, indeterminate, or inexhaustible. It is to this concept rather than the concept of infinity that such predicates more appropriately apply. The attempt to explain the putative concept of infinity in terms that ultimately describe something that is at most finite, and the fact that what passes for pure and applied classical infinitary mathematics can be equally if not better understood as involving finite but indefinite, indeterminate, or inexhaustible sets and series, suggests that it is this idea, rather than the concept of infinity, that mathematicians really understand, and that it is all they really need for the theorems and methods of classical mathematics. A rationale of this type for rethinking the indispensability of the concept of infinity in mathematics reinforces Berkeley's and Hume's dismissal of ideas lacking adequate experiential origins.

Many (not all) intuitionists in the philosophy of mathematics, from a different though related set of ideological scruples, have similarly tried to reform the foundations of logic by eliminating actual infinity or infinite divisibility. Intuitionism, while removing questionable parts of classical mathematics even during Hume's time, nevertheless preserves and reconstitutes what many working mathematicians and scientists believe are sufficient algebraic formalisms to maintain the essential functions of a truncated pure and applied mathematics.[15] The formal success of Cauchy's epsilon and delta methods further attests to the possibility of doing calculus without commitment to in-

[15] Heyting, *Intuitionism: An Introduction*, pp. 32, 39-40. The Fan Theorem for Spread and Choice Sequences in intuitionism allows for denumerable and even nondenumerable infinity. But these are only potential extrapolations of an unending mental construction. See Van Stigt, *Brouwer's Intuitionism*, p. 370. Dummett, with the assistance of Minio, *Elements of Intuitionism*, p. 54: "In intuitionistic mathematics, all infinity is potential infinity: there is no completed infinite." Also, pp. 54-64.

finity, even if Cauchy's once 'standard' analysis is now 'nonstandardly' interpreted as quantifying over actually infinite sets of reals. Such intuitionist projects are often proposed from the standpoint of a humanized concept of truth and proof that is compatible in essentials with Hume's empiricist critique of infinity.

Aristotle, as we have seen, offers a related solution. The distinction between actual and potential infinities serves a similar purpose in avoiding Zeno's paradoxes.[16] Aristotle's concept of potential infinity is in some ways like but not quite the same as the concept of finite but indefinite, indeterminate, or inexhaustible sets and series of mathematical objects. The difference is that for Hume there can be no adequate *idea* even of potential *infinity* in the true sense of the word, as Aristotle's proposal and most intuitionist foundations require. Presumably, however, there can be an adequate idea of *finite but practically or potentially indefinite, indeterminate, or inexhaustible* sets and series of mathematical objects, including ideas of the finite but indefinite, indeterminate, or inexhaustible divisibility of extension, originating in impressions of sensation or reflection. Imagination can present the mind with an adequate idea or clear conception of finite but indefinite, indeterminate, or inexhaustible divisibility of spatial extension from sense impressions of extended bodies in space, to be combined with the adequate idea of indefinite or practically inexhaustible physical phenomena. The latter might be suggested, for example, by the indeterminate number of different visual perspectives a subject can assume with respect to a seen object. This is not the same thing as and is in some ways a weaker, ontically less robust, but for that reason also less empirically objectionable concept, than Aristotle's idea of potential infinity. It is likely, therefore, that Hume would renounce Aristotle's distinction as conceding

[16] Aristotle, *Physics* 263^b3-9: "To the question whether it is possible to pass through an infinite number of units [i.e. intervals] either of time or of distance we must reply that in a sense it is and in a sense it is not. If the units [intervals] are actual it is not possible, if they are potential, it is possible."

too much to infinitism. Hume rejects the conceivability of infinity in any guise, actual or potential. But Hume would not need to dispute the possibility of a reason- or imagination-mediated complex idea of finite but indefinite, indeterminate, or inexhaustible divisibility.

The need for infinitary mathematics in science, particularly in the applied mathematics of mathematical physics, is largely parasitic on the status of infinity in pure mathematics. Applied mathematics makes use of the instruments provided by pure mathematics, though requests for the development of special mathematical methods may often originate in the field of practical application to overcome the limitations of currently available formal tools. Yet it is possible to reform the theory of mathematics so radically or irresponsibly that the scientist is left with insufficient computational machinery. If this happens infrequently, it is only because of a built-in conservatism in the foundations of mathematics that is as jealous of applications as of philosophical considerations. This is seen in the 'battle of books' launched by Berkeley's publication of his finitist tract *The Analyst*, which stirred up such an imbroglio that, as his editors recount, at least one of his mathematician critics "... did not, in so many words, tell the cobbler to stick to his last, [though] it is easy to read his 'hands off mathematics' between the lines of his tart reply."[17]

The history of science in the aftermath of Hume's critique of infinity, on the other hand, has shown at least some sympathy for his disavowal of infinity, at least as it applies to mathematical physics and cosmology. Robert Burton, a contemporary of Descartes, proclaims in *The Anatomy of Melancholy*:

> We may likewise insert with Campanella and Brunus that which Pythagoras, Aristarchus Samius, Heraclitus, Epicurus, Melissus, Democritus, Leucippus maintained in their ages: there be infinite worlds, and infinite earths or systems, *in infinito aethere* [in the infinite ether], which Eusebius collects out of

[17] Luce, "Editor's Introduction", Berkeley, *A Defense of Free-Thinking in Mathematics*, *Works*, IV, p. 105.

> their tenets, because infinite stars and planets like unto this of ours, which some stick not still to maintain and publicly defend, *sperabundus exspecto innumerabilium mundorum in aeternitate perambulationem* [I confidently count upon the eternal movement of innumerable worlds], etc. (Nic. Hill, Londoninensis, *Philos. Epicur.*). For if the firmament be of such an incomparable bigness as these Copernical giants will have it, *infinitum, aut infinito proximum* [infinite, or very nearly infinite], so vast and full of innumerable stars, as being infinite in extent...[18]

How many contemporary scientists, astronomers and cosmologists would join the philosophers Burton lists in accepting the proposition that the universe is infinite in extent? The ruling hypothesis of relativity theory contradicts the idea, and most scientists now seem satisfied with a model of space-time as a kind of Klein bottle. Space on this view is a four-(or more)-dimensional Möbius strip that folds back through, into, and around itself in a single finite, continuous and self-contained topological surface. There is no inside or outside on the surface of space-time, and it is possible only to travel in finite but indefinitely, indeterminately, or inexhaustibly diverse journeys that never reach an endpoint or boundary, but which is in no other sense literally infinite. Truth in science is not decided by popular opinion, even among the best informed, most capable influential scientists and philosophers. The immediate question is not about ultimate truth, however, but only about whether Hume's finitism, by depriving mathematics of infinity, thereby also robs science, and especially applied mathematical physics, of concepts and calculi essential to contemporary cosmology.

The answer must be resoundingly in the negative. Scientists on the whole no longer believe that the universe or physical space is infinite in extent. Infinitists in the physical sciences today, if there are any, place themselves in the position of those ancient Greeks for whom large numbers were so unthinkable, partly because of the limitations of their arithmetical notations,

[18] Burton, *The Anatomy of Melancholy*, Second Partition, Section 2, Member 3, p. 54.

that they elevated any large cardinality to the category of the infinite. Some believed that the grains of sand were so numerous that the world must contain a literally infinite quantity. Archimedes addressed his treatise *The Psammites* or "Sand-Reckoner" to just these 'infinitists'.[19] Following Aristarchus of Samos, Archimedes estimates the size of the universe, from Earth at the center to the outermost limits of the crystalline sphere, where the stars were believed to shine through openings like pinholes in a windowshade. Within this vast but finite space, Archimedes calculates that, far from being infinite, the number of grains of sand needed to fill the entire known universe, let alone the beaches and soil on Earth, is a mere (in modern scientific notation) 10^{63}. If anyone still persists in believing that the universe or physical space is truly infinite in space-time, contains an infinite number of physical particles, or infinite quantity of matter, there are modern day variations of Archimedes' "Sand-Reckoner" in P.J.E. Peebles's *Physical Cosmology* and Stephen W. Hawking and G.F.R. Ellis's *The Large Scale Structure of Space-Time.*[20]

Nor is infinite divisibility necessary in physics. Infinitesimals are optional in standard analysis for the applied mathematics of motion, space, and time. Such an account preserves all the machinery of the Leibnizian-Newtonian calculus, but substitutes the concept of convergence on limits for ontically more suspect infinitesimals or fluxions. Standard analysis recognizes infinity as a limit for functions, integrals, and derivatives. But this too can be understood along strict finitist lines as involving indefinite, indeterminate, or inexhaustible serial progressions. The very idea of infinity as a limit, as something approachable or attainable in the manner of finite limits, can easily be made to seem unintelligible, oxymoronic. What can it mean,

[19] Archimedes, "Sand-Reckoner", *Works*, pp. 221-232.

[20] Peebles, *Physical Cosmology*. Hawking and Ellis, *The Large Scale Structure of Space-Time.* Gamow, *One, Two, Three... Infinity*, esp. pp. 3-23.

except that a series continues indefinitely, indeterminately, or inexhaustibly?[21]

The opinion of at least some well-informed scientists is that space is best understood in terms reminiscent of Hume's theory of sensible extensionless indivisibles as consisting of discrete quanta rather than as classical continua.[22] On this view, there is a kind of graininess to space-time, such that quantum units are ultimately the smallest things in the universe; in effect, true atoms or indivisibles of extension. Quanta can even be construed as sensible extensionless indivisibles in the manner of Hume's inkspot experiment, if they are perceivable by sensation-enhancing devices like particle accelerators and cloud chambers or detector screens. It appears that Hume's repudiation of infinity and infinite divisibility, and his positive theory of sensible extensionless indivisibles, does not contradict, but is at least compatible with if not vindicated by developments in contemporary science.[23]

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[21] Here I expect to be told that I do not understand the concept of limit. But I do as a matter of fact have a very clear idea of the concept, when, for example, 0 is a limit, or 1, or 2, and so on. Then it might be said that I do not have a complete understanding of limits unless I grasp the way in which series can approach and converge on infinity as a limit. My claim, however — and I am merely expressing agreement with Hume's point — is that no one understands infinity as a limit, regardless of their adeptness at manipulating the tokens of a symbol system that purports to take infinity as a limit.

[22] Ludwig, *An Axiomatic Basis for Quantum Mechanics*, Vol. 2, *Quantum Mechanics and Macrosystems*, pp. 174-178. Penrose and Isham, *Quantum Concepts in Space and Time*. Gustafson and Reinhardt, *Quantum Mechanics in Mathematics, Chemistry, and Physics*. Forrest, *Quantum Metaphysics*. But see Franklin, "Achievements and Fallacies in Hume's Account of Infinite Divisibility", p. 88: "On present evidence from physics, space and time are most likely infinitely divisible. This could change at any moment, as the submicroscopic world becomes better known; a few physicists are still actively engaged in investigating the possibility of atomic, or, as they say, 'quantized' space." Franklin refers to Duff and Isham, eds., *Quantum Structure of Space and Time*, and Bacry, *Localizability and Space in Quantum Physics*.

[23] Hilbert, "On the Infinite", pp. 185-186: "In addition to matter and electricity, there is one other entity in physics for which the law of

In philosophy, infinity plays a less significant role than in classical mathematics and contemporary science. The term is not hard to find in philosophical writings. It is especially prevalent in those confections of fanciful metaphysics that so appalled Hume that even *le bon David* wanted them burned. The appeal to infinity and infinite divisibility in scientific philosophy is largely derivative from the widespread historical influence of these concepts in mathematics and science. As with its role in applied mathematics, theoretical physics and engineering, the term 'infinite' might entirely disappear from philosophical (though not historical) commentary on these disciplines, and from philosophical applications of mathematics and science, if it were to be seen as dispensable in the disciplines themselves.

A final philosophical application of the concept of infinity of special interest to Hume concerns the metaphysics of religious belief about the infinite wisdom, power, and goodness of God. We have already noted the connection between these concepts in Hume's critique of infinity and religious skepticism, and similar observations will be made in the Afterword on Hume's aesthetic psychology of the sublime. The idea of infinity is sometimes supposed to be conveyed by trying to imagine God's infinite view of numbers, of infinitely divisible extension, or of infinitesimals. Here I shall do no more than propose a puzzle for finitism and the concept of God. If infinity is rejected, can there be an adequate substitute in the idea of God as a being of finite but indefinite, indeterminate, or inexhaustible wisdom, power, and goodness? Could this more modest idea provide an adequate alternative conception of God to replace what Hume must regard as a faulty attribution to the Deity

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conservation holds, viz., energy. But it has been established that even energy does not unconditionally admit of infinite divisibility. Planck has discovered *quanta of energy*. Hence, a homogeneous continuum which admits of the sort of divisibility needed to realize the infinitely small is nowhere to be found in reality. The infinite divisibility of a continuum is an operation which exists only in thought. It is merely an idea which is in fact impugned by the results of our observations of nature and of our physical and chemical experiments."

of infinite power, wisdom, and goodness? Would a finite but indefinitely, indeterminately, or inexhaustibly knowing, able, and good divine spirit, in Hume's or Philo's concept in the *Dialogues Concerning Natural Religion*, still be, in Philo's phrase, 'worthy of worship'?[24] Or is the *infinite* magnitude of these properties somehow essential to the concept of God?[25]

Hume's Finitism and Cantor's Transfinite Cardinals

An outstanding question is whether Hume's critique of infinity suffers because of its incompatibility with Cantor's transfinite set theory. It is interesting to note that Cantor himself in his 1883 *Grundlagen einer allgemeinen Mannigfaltigkeitlehre* (*Foundations for a General Theory of Multiplicity*) held that only finite numbers are real, and treated all infinities as fictions.[26] In the history of mathematics after Cantor, however, the existence of transfinite cardinals, proven by Cantor's diagonalization method, has been taken much more seriously.[27]

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[24] Hume, *Dialogues Concerning Natural Religion*, pp. 212-213. See Yandell, *Hume's "Inexplicable Mystery": His Views on Religion*, pp. 238-240. Jacquette, "Analogical Inference in Hume's Philosophy of Religion", pp. 287-292.

[25] *Dialogues Concerning Natural Religion*, p. 188: "By this argument, too, we may prove the INFINITY of the divine attributes, which, I am afraid, can never be ascertained with certainty from any other topic. For how can an effect, which either is finite, or, for aught we know, may be so; how can such an effect, I say, prove an infinite cause?" Also, pp. 198-201.

[26] Cantor, *Grundlagen einer allgemeinen Mannigfaltigkeitslehre*, rpt. in Cantor, *Gesammelte Abhandlungen*, pp. 181-182. See Wang, "The Concept of Set", pp. 181-223.

[27] Tiles, *The Philosophy of Set Theory: An Historical Introduction to Cantor's Paradise*, p. 6: "Thus, if one were to proclaim them [infinite and transfinite numbers and transfinite set theory] to be inventions, figments of mathematical imagination, one would not be casting aside centuries of tradition. Indeed, the weight of tradition is firmly opposed to giving credence to talk of any such things. The infinite only gained acceptance and a degree of mathematical respectability because traditional ways of thinking were being cast aside." And p. 95: "It was Cantor's work which gave sense to the question 'How many points are there in a line?', a question which previously lacked any precise sense... Before Cantor developed his theory of transfinite numbers, the natural, and the only available answer, to the question

The attitude is epitomized by the battle cry of David Hilbert's 1925 lecture "On the Infinite", that "No one shall drive us out of the paradise Cantor has created for us."[28] Hilbert, remarkably, like Cantor, also regards infinities and infinitesimals in the calculus as fictions, which he calls 'ideal' constructions, comparing them to imaginary numbers, like the square roots of negative integers.[29] Wittgenstein, in his posthumously edited *Bemerkungen über die Grundlagen der Mathematik* (*Remarks on the Foundations of Mathematics*), reacted to Hilbert's pronouncement with the famous riposte: "I would say, 'I wouldn't dream of trying to drive anyone from this paradise.' I would do something quite different: I would try to show you that it is not a paradise — so that you'll leave of your own accord. I would say, 'You're welcome to this; just look about you.' "[30]

The biographical facts about the caution of the founders of transfinite set theory toward higher-order infinities have not discouraged later mathematicians and philosophers of mathematics from accepting transfinite cardinals as ontically real, entirely on a par with the positive integers. What is now most often meant by classical mathematics includes Cantor's cathedral of transfinite cardinals, and the question of whether or not a mathematical theory implies or is compatible with Cantor's transfinite spiral of transfinite cardinals has become a test for classical adequacy. This is clear, for example, in Bertrand Russell's 'Introduction' to Wittgenstein's *Tractatus Logico-Philosophicus*, when Russell remarks, in blatant disregard

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was 'Infinitely many', and this was a way of saying that there is no number of points in a line, they are without number."

[28] Hilbert, "On the Infinite", p. 191.

[29] Ibid., pp. 195-198.

[30] Wittgenstein, *Wittgenstein's Lectures on the Foundations of Mathematics, Cambridge 1939*, p. 103. See also Wittgenstein, *Remarks on the Foundations of Mathematics*, p. 264: "Imagine set theory's having been invented by a satirist as a kind of parody on mathematics. — Later a reasonable meaning was seen in it and it was incorporated into mathematics. (For if one person can see it as a paradise of mathematicians, why should not another see it as a joke?)"

for Wittgenstein's philosophy of mathematics: "There are some respects, in which, as it seems to me, Mr. Wittgenstein's theory stands in need of greater technical development. This applies in particular to his theory of number (6.02ff.) which, as it stands, is only capable of dealing with finite numbers. No logic can be considered adequate until it has been shown to be capable of dealing with transfinite numbers."[31]

Hume's finitism is obviously incompatible with Cantor's transfinite theory. Cantor's diagonalization requires infinite extensions in two directions, from the decimal expansions of real numbers in the list to which diagonalization is applied, and in the list itself, a hypothetically denumerably infinite roster of all real numbers relative to which diagonally-defined irrational real numbers cannot occur. If the reals are given in a finite specification only, then even if the list is merely indefinite, indeterminate, or inexhaustible in length, it cannot validly be inferred that the irrational real number constructed by diagonalization from the list has no location in the list. It could, for all that the proof would then show, have an address somewhere in the list where the diagonalization, also merely indefinite, indeterminate, or inexhaustible in extent, does not reach. Consider the following list of reals. Cantor shows that by diagonalization we can identify a real number that cannot belong to the denumerably infinite basis list of reals between 0 and 1 from which it is constructed. If the numbers are written in denumerably infinite binary expansions, the list might look like this:

[31] Russell, 'Introduction' to Wittgenstein, *Tractatus Logico-Philosophicus*, p. 21. The irony of Russell's failure to appreciate the deliberate strict finitism of the *Tractatus* should not go unremarked. Russell is wrong both in assuming that Wittgenstein would see it as an improvement in the *Tractatus* logic to extend its foundations of number theory to transfinite ordinals, and that such an enhancement could be supported by Wittgenstein's nonhierarchical picture theory of syntax.

$$\begin{array}{l}
.\underline{0}0100101\ldots \\
.1\underline{1}010110\ldots \\
.11\underline{0}10101\ldots \\
.010\underline{0}1100\ldots \\
.0011\underline{1}010\ldots \\
.10111\underline{1}01\ldots \\
.011011\underline{1}0\ldots \\
.1000111\underline{0}\ldots \\
\vdots \qquad\quad \ddots
\end{array}$$

The diagonal function defines a pathway extending through the matrix of digits, and defines another real number different than any that occurs in the list by changing the digit that appears in the *n*th expansion place of the *n*th row of the list from 0 to 1 or 1 to 0. In the above, the diagonal number relative to the list is .10110001 . . . . This real cannot occur anywhere in the list on pain of contradiction, since for any whole number *n*, if the number *per impossibile* were at row *n*, then by construction it would have both digit *d* (0 or 1) and the complement of *d* (1 or 0) in its *n*th expansion place.[32]

That such numbers are constructible is taken by Cantor and later transfinitists to prove that there are nondenumerably many reals, and in particular that there are nondenumerably many irrationals. It is also supposed to follow that there are more irrationals and functions on integers than positive integers, that irrational numbers and the set of all functions on integers cannot be completely listed, that there is an ascending hierarchy of transfinite cardinals or higher orders of infinity, and that the power set theorem that the cardinality c of any set is less than the cardinality of its power set 2^c ($2^c > c$) is generalizable to the infinite case. The argument is naturally construed as a *reductio* of the assumption that there are as many irrationals as positive integers, or that the two

[32] The diagonalization technique and its interpretation are also found in Cantor, *Contributions to the Founding of the Theory of Transfinite Numbers*. See Dauben, *Georg Cantor: His Mathematics and Philosophy of the Infinite*.

sets can be arranged in one-one correspondence.[33] Since by hypothesis there are denumerably infinitely many reals in the diagonalization basis of what is supposed (for purposes of indirect proof) to be a complete list of reals, the conclusion seems inescapable that the set of all reals is nondenumerably infinite in cardinality, or that there are more irrational real numbers or functions on integers than rational numbers or positive integers.[34]

Cantor, in the Continuum and Generalized Continuum Hypotheses, further conjectures, where $\aleph_0$ denotes the cardinality of positive integers, that the cardinality of the continuum, the number of real numbers or points in a line or line segment, or indeed in all of three-dimensional space, $2^{\aleph_0}$ (adopting the expression as a consequence of the power set theorem), is equal to the first infinite number greater than $\aleph_0$, $\aleph_1$, or that $2^{\aleph_0} = \aleph_1$, and that therefore there are no cardinal numbers between $\aleph_0$ and $2^{\aleph_0}$, $2^{\aleph_0}$ and $2^{2^{\aleph_0}}$, etc. Cantor's Continuum Hypotheses have since been shown to be consistent with but unprovable from and therefore independent of standard set theoretical axioms, and as such the question of their truth or falsehood remain unresolved problems of mathematical logic.[35]

Cantor's diagonalization can be formalized in this way.[36]

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[33] See Simmons, "The Diagonal Argument and the Liar", p. 281.

[34] Other informal expositions of Cantor's results are given by Benardete, *Infinity*, pp. 91-94, and Moore, *The Infinite*, pp. 118-122.

[35] Gödel, *The Consistency of the Axiom of Choice and of the Generalized Continuum Hypothesis with the Axioms of Set Theory*; Gödel, "What is Cantor's Continuum Problem?"

[36] Alternative formalizations are obviously possible. An algebraic demonstration is presented by Davis and Hersh in a compact demonstration in *The Mathematical Experience*, p. 109: "*Cantor's Diagonal Process.* Here is a simple version of it. Consider all the functions f which are defined on the integers $1, 2, 3, \ldots$ *Theorem*: It is not possible to arrange all these functions in a list. *Proof*: Assume that it is possible. Then there would be a first function in the list. Call it f_1. There would be a second function f_2, etc. Now, for each number n, where n takes on the values $1, 2, 3, \ldots$, consider the numbers $f_n(n) + 1$. This sequence of numbers itself constitutes a function defined on the integers and so, by our assumption, it must occur in the list. Call it

Let the formula

$$D(m, n) = (\bar{d}, n, .000\ldots) = \underbrace{.000\ldots\bar{d}\,000\ldots}_{n}$$

express the application of diagonal function D to digit $d(0, 1)$ in expansion place n in row n of the matrix m of binary digits constituted by a denumerably infinite list of real numbers in denumerably infinite binary expansion. The value of the function in $\bar{d}$ is to insert the complement of digit d (1 if $d = 0$; 0 if $d = 1$) in expansion place n in a denumerably infinite expansion of 0's. Then the diagonal number relative to matrix m is the sum:

$$\mathcal{N}\big(D(m)\big) = \sum_{n=1}^{\infty} \big(D(m, n)\big)$$

Thus, if m is the matrix above, then:

$$\begin{aligned}
D\big(m, (0, 1, 1)\big) &= (1, 1, .000\ldots) = .10000000\ldots \\
D\big(m, (1, 2, 2)\big) &= (0, 2, .000\ldots) = .00000000\ldots \\
D\big(m, (0, 3, 3)\big) &= (1, 3, .000\ldots) = .00100000\ldots \\
D\big(m, (0, 4, 4)\big) &= (1, 4, .000\ldots) = .00010000\ldots \\
D\big(m, (1, 5, 5)\big) &= (0, 5, .000\ldots) = .00000000\ldots \\
D\big(m, (1, 6, 6)\big) &= (0, 6, .000\ldots) = .00000000\ldots \\
D\big(m, (1, 7, 7)\big) &= (0, 7, .000\ldots) = .00000000\ldots \\
D\big(m, (0, 8, 8)\big) &= (1, 8, .000\ldots) = .00000001\ldots \\
&\vdots \qquad\qquad \vdots \qquad\qquad\quad \vdots \\
&\overline{}
\end{aligned}$$

$$\mathcal{N}\big(D(m)\big) = \sum_{n=1}^{\infty} \big(D(m, n)\big) = .10110001\ldots$$

The diagonalization proves nothing unless the list of reals that serves as its basis is assumed from the outset to be infinite, or to

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f_k. By definition, $f_k(n) = f_n(n) + 1$, and this is valid for $n = 1, 2, 3, \ldots$. In particular, it is valid for $n = k$, and this yields $f_k(k) = f_k(k) + 1$. Thus, $0 = 1$, a contradiction."

be in one-one correspondence with an infinite set, such as the whole numbers or positive integers.[37] If with Hume it is said that the basis list cannot be infinite in the first place, if there can be no adequate experientially originating idea of infinity, then the fact that a number can be constructed that has no location in a finite but indefinite, indeterminate, or inexhaustible list is hardly surprising, nor does it support what would otherwise be the interesting result that there are higher orders of infinity or a transfinitely ascending spiral of transfinite cardinals.

Hume's strict finitism plainly contradicts the realist interpretation of Cantor's set theory. Yet this fact alone need not reflect negatively on the acceptability of Hume's critique. There are philosophical grounds for suspicion about the meaning of Cantor's diagonalization and the theory of larger infinities. It is worthwhile to consider one such argument based on the empiricist principle of theoretical economy or parsimony for competing scientific or philosophical theories known as Ockham's Razor — the rule that entities are not to be multiplied beyond explanatory necessity, and that theories are to be rejected if they entail the existence of entities for which there is no explanatory requirement.

According to Cantor's transfinite set theory, there is a transfinite hierarchy of infinities. Each order, beginning with the first order of infinity that numbers the set of whole numbers or positive integers, is of successively greater cardinality than that preceding it. For every set S of cardinality n, the cardinality

[37] The power set of a set is the set of all subsets the set contains, including the set itself and the null set. If set $S = \{1, 2, 3\}$, then the power set of S, $\mathcal{P}(S) = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}, \emptyset\}$. The basic formula for determining the cardinality of a power set of a set with cardinality n is 2^n. In the example above, where the cardinality of set S is 3, the cardinality of its power set $\mathcal{P}(S)$ is 2^3 or 8. The generalized power set theorem states that this cardinality relation holds even when the cardinality of a set is (some order of) infinity, or that for any set S, the cardinality of its power set is always greater than the cardinality of the set itself, $C(\mathcal{P}(S)) > C(S)$. The condition is obviously satisfied when the cardinality of S is any finite number n. Cantor's diagonalization proof was needed to establish the theorem in the general case where n is also an infinite cardinal.

of the power set of S is 2^n. In Cantor's notation, the first order of infinity is $\aleph_0$, the second order of infinity is the power set of $\aleph_0$, $2^{\aleph_0}$, the third order of infinity $2^{2^{\aleph_0}}$, and so on. The conflict of Cantor's theory with Ockham's Razor is similar to that arising by analogy in the Third Man objection to Plato's theory of Forms. Although mathematicians claim to have found sets of mathematical objects with cardinality $\aleph_0$, such as the set of whole numbers or positive integers, sets with cardinality $2^{\aleph_0}$, such as the set of all irrational real numbers and the continuum, and even sets with the third order of infinity $2^{2^{\aleph_0}}$, such as the set of all curves, there are no known sets of mathematical objects with the higher cardinality of the fourth, fifth, etc., order of infinity, other than the power sets of transfinite sets themselves. Cantor's cardinals continue transfinitely. By Ockham's Razor, if Cantorian mathematics is ontically committed to the existence of these higher-order cardinalities for which there is no theoretical purpose or explanatory need, then there is an argument for eliminating them from mathematical theory. This economy can only be achieved (as in shaving Plato's beard by Ockham's Razor at the root source of the proliferation of Forms), by rejecting even the first order of infinity at the basis for Cantor's diagonalization.[38]

To this it will be objected that there may eventually turn out to be theoretical uses for cardinalities higher than the third order of infinity. Such a reply might be dismissed as wishful thinking by those who prefer an economical to an inflated ontology of mathematical entities. It might also be objected that if there is a use for the first three orders of infinity, then it is irresponsible to eliminate all higher orders of infinity merely because modelings of cardinals greater than $2^{2^{\aleph_0}}$ have not yet been identified. This criticism overlooks the fact that higher cardinals are thought to express the cardinalities of particular sets of mathematical objects only on the assumption that diagonalization proves their existence. If that assumption is

[38] Plato, *Parmenides* 132-133. Aristotle, *Metaphysics* 990^{b}1-991^{b}9, 1038^{b}31-1039^{a}14.

called into question by Hume's empiricism or Ockham's Razor, then the utilitarian justification for even the first three orders of infinity collapses. The main reason for thinking that the second order of infinity numbers the set of irrational reals is the belief that Cantor's diagonalization proves that the irrationals are greater in infinite cardinality than the set of positive integers. If the diagonalization is cast in doubt, then the presumption that there is a second or higher order of infinity and that the set of reals has an infinite let alone transfinite cardinality is groundless. By now it should be clear that the whole transfinite house of cards tumbles if Cantor's diagonalization is rejected, and that Cantor's diagonalization must be rejected if there is no adequate concept of infinity.

The belief that second-order infinity is the cardinality of the irrational real numbers is unproved in standard set theory. It is at best an educated guess. Because the complement of the irrationals in the set of all rational real numbers is proven to have a lower cardinality than the set of all reals the cardinality of the irrationals is also believed to be the cardinality of the continuum. The conclusion is reinforced by the Pythagorean theorem and its proof of the existence of irrational lengths in geometry. The hunch that the cardinality of the irrationals or of the continuum is the second order of infinity is enshrined in Cantor's Continuum Hypothesis. The related general claim that there are no cardinal numbers between any two neighboring pairs of cardinals in the transfinite hierarchy, between the cardinal of any arbitrary set and the cardinal of its power set, is the Generalized Continuum Hypothesis.

Cantor in later writings repeatedly promises to give a rigorous proof of the Continuum Hypothesis. But he was never able to do so. In the development of modern mathematical logic, Kurt Gödel proved the logical consistency of the Continuum Hypothesis with the axioms of standard set theory,[39] from

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[39] An exception to Gödel incompleteness is proved for infinitary arithmetical systems of integer addition without multiplication by Presburger, "Über ber die Vollständigkeit eines gewissen Systems der Arithmetik ganzer

which Paul Cohen proved its logical independence.[40] The upshot is that the Continuum Hypothesis can neither be proved nor disproved within the resources of standard set theory. It is unnecessary to comment on the philosophical and mathematical significance of this limitation in order to see that from the vantage point of Hume's finitism there would be one less unsolved problem in the foundations of set theory and mathematical logic if the onus of justifying the Continuum Hypothesis were to be eliminated. This simplification and economization of theory entailed by Hume's strict finitism is another dividend of Ockham's Razor.[41] There are no concrete scientific

Zahlen, in welchem die Addition also einzige Operation hervortritt"; Presburger, "On the Completeness of a Certain System of Arithmetic of Whole Numbers in Which Addition Occurs as the Only Operation", translation and commentary by Jacquette. This suggests that the classical limiting metatheorems of Gödel, Church, and Rosser are avoidable in finitist logics with arithmetic in which multiplication can be reduced to addition in the finite but not the infinite case lacking strong (infinitary) induction.

[40] Cohen, *Set Theory and the Continuum Hypothesis.*

[41] Wittgenstein's criticism of Cantor's diagonalization in the *Remarks on the Foundations of Mathematics* is that any legitimate addition to the family of mathematical language games must be justified instrumentally as mathematically interesting, that we must be able to say what mathematical use a proposed formula or method of calculation has, and that Cantor's technique of diagonalization (like Gödel's incompleteness proof) does not meet this requirement. Cantor's method is therefore an 'inflated or puffed-up proof' [*prahlehrischen Beweis*], that purports to prove more than its modest means permit. The only conceivable use Wittgenstein considers as a candidate for Cantor's proof is in attempting to rechannel the mathematician's energies in foregoing attempts to put the irrational reals in a series, or the like, which the proof shows to be as futile as trying to trisect an angle with compass and straight-edge. The philosophical grammar of a 'mathematically interesting' instrumental justification for an addition to the mathematical language game appears to imply for Wittgenstein that if a justification is mathematically interesting, then its opposite must be too. But Wittgenstein suggests that the instrumental value of rechanneling energies away from wasted efforts in mathematics cannot provide a mathematically interesting instrumental justification of limiting conclusions in mathematics, because the effect can always be reversed by what are obviously mathematically uninteresting countermeasures, by

applications of Cantor's transfinite cardinals in mathematical physics or engineering, so that the contradiction of Hume's strict finitism with the theory is confined to pure mathematics and metaphysics with no recognized implications for natural science. Hume, in the spirit of his *reductio* disproofs, would undoubtedly interpret this fact as further confirmation of the absurdity of infinitism.[42]

What, then, would mathematics and science be like without infinity, if Hume's critique were accepted? For the most part these disciplines would remain unchanged. We can preserve while reinterpreting in strict finitist terms all the useful results of number theory, the calculus, and (subtransfinite) set theory, computational mathematics and limits done in the style of Cauchy. Where equations in formal theory and application are standardly understood as implying infinite sets and series, and progressions to infinity as a limit, the same formulas can be proved and used in the Humean recension, but with reference instead to indefinite, indeterminate, or inexhaustible sets, series, and progressions. Not even the notation of traditional infinitary mathematics need be altered, only reinterpreted. Again, it is the ideas and in particular the origins of ideas in terms of which theories in the sciences are supposed to be understood that really matter to Hume. When the interesting concepts are clarified and we have delimited the pretensions of science from the standpoint of a properly humanized epistemology and theory of mind, Hume, in ways that anticipate Wittgenstein's

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'another picture', or, presumably, though Wittgenstein does not mention these additional possibilities, by hypnosis, the philosopher's pill, coercion or reward. Wittgenstein, *Remarks on the Foundations of Mathematics*, pp. 125-142.

[42] Whitehead and Russell, *Principia Mathematica*, Vol. 2, p. 183: "Infin ax. $=: \alpha \in$ NC induct. $\supset . \exists! \, \alpha$ Df. This assumption, like the multiplicative axiom, will be adduced as a hypothesis whenever it is relevant. It seems plain that there is nothing in logic to necessitate its truth or falsehood, and that it can only be legitimately believed or disbelieved on empirical grounds." The fact that standard set theory requires an axiom of infinity strongly suggests that mathematics otherwise has no need for the concept of infinity. See Quine, pp. 280-285. Quine, p. 283, introduces and compares Zermelo's axiom of infinity with Whitehead and Russell's.

attitude, allows philosophy to leave everything pretty much as it is, permitting us to return to billiards and backgammon. From the historical perspective, in terms of the state of knowledge in Hume's day, it appears that Hume's rejection of infinity need not infringe in any way on the appearance or utility of mathematics or natural science. Important work in these areas can continue without disruption, but with a new and better understanding of the true limits of genuine ideas that inform formal theory construction and application. It might even be said that the Humean finitist analysis of mathematical and scientific ideas is revisionary only in the sense of offering a more accurate account of the ideas mathematicians and scientists actually think of and put to use in their equations and derivations all along.

There is one exception from the retrospective view. Hume's strict finitism, as we have seen, is patently at odds with Cantor's transfinite set theory. The diagonalization argument does not go through except on the assumption that a complete list of reals is infinite in length, and that each real is infinite in decimal expansion. Cantor's paradise of transfinite cardinals, at least in its currently popular realist interpretation, is the one obvious casualty of Hume's critique of infinity in contemporary mathematics.

Resilience of Hume's Critique

To summarize Hume's case against infinite divisibility in support of his own theory of sensible extensionless indivisibles, it seems accurate to say that of Hume's eight arguments, none is obviously defective. The proofs are reconstructible as logically circumspect; no invalidities have been uncovered in Hume's inkspot argument, the Berkeleyan refutation of abstract general ideas of infinity, nor in the *reductio* arguments of the *Treatise* and *Enquiry*.

I have not tried to conceal my sympathy for Hume's conclusions. If I do not accept his arguments as entirely decisive, it seems to me that Hume has at the very least accomplished the prodigious task of presenting a novel, coherent alternative to mainstream infinitary mathematics. As such, Hume may justly claim to have 'opened our eyes a little' to the pretensions of infinitism, and guided our imaginations toward another way of thinking about the mathematics and metaphysics of space and time.

The convergence of so many different related kinds and styles of argument in support of the same set of conclusions about the inconceivability of infinity and the existence of sensible extensionless indivisibles in itself argues for the conclusion that Hume's opposition to infinity cannot easily be dismissed. In my survey of recent commentary on Hume, no decisive objections to his proofs against infinity have yet been raised to challenge his critique. I am tentatively inclined to believe that if we accept Hume's empiricist assumption about the experiential origin of ideas, or his adaptation of the Berkeleyan empiricist refutation of abstract general ideas, then there is no choice but to agree with Hume that infinite divisibility is a rationalist philosophical fiction, and that an adequate theory of extension in space and time implies at most the finite divisibility of extension into finitely many sensible extensionless indivisibles. It might even be said that mathematics, science, and philosophy might be improved rather than impoverished by Hume's critique. There is no practical disadvantage in rejecting the concepts of infinity and infinite divisibility, but a definite gain in clarity, simplicity, and economy. The only potential, purely theoretical 'loss' to contemporary mathematical logic and set theory is Cantor's hierarchy of transfinite numbers. It is arguable whether this represents a setback or advance for mathematics and the philosophy of mathematics. Many mathematicians, logicians, and philosophers, have an almost religious sentimental attachment to the beautiful arithmetics that purport to describe the structures of Cantor's transfinite numbers as a mathematical sacred cow. Yet if Hume's critique is sound, then the theory of infinity and higher orders of infinity is an empty formalism devoid of meaning.[43]

The question is whether or not to take the first step down this road with Hume. To concede that ideas must originate in impressions of sensation or reflection, and that there can be no abstract general ideas, appears to me, given the strength of Hume's arguments, to lead us inexorably toward some type of finitism. Moreover, I know of no open-and-shut way either to confirm or discredit Hume's empiricist starting place. The antagonism between rationalism and empiricism is a perennial problem of philosophical methodology, concerning which philosophers can often do no more than confess their intuitive allegiances or suspicions. If no final crushing defeat of Hume's experiential humanizing of epistemology and philosophy of mind is forthcoming, then there is no ground for anticipating an immediate overthrow of Hume's critique of infinity and theory of sensible extensionless indivisibles.

Hume's six *reductio* proofs, five from the *Treatise*, if we include the geometry dilemma, and one from the first *Enquiry* 124, would also need to be refuted in order to counteract his refutation of infinite divisibility. Without a sound defense of the concept of infinity against Hume's multiple challenges of its coherence or intelligibility, there is no prospect of reaffirming infinite divisibility against Hume's thesis of sensible extensionless indivisibles. To undermine Hume's *reductio* arguments is also no easy task, because these proofs if anything are on firmer ground than the inkspot argument and Berkeleyan refutation of abstract general ideas. As negative criticisms, the *reductio* disproofs are freighted with a minimal burden of assumptions to bear, exposing fewer points of vulnerability to countercriticism. There are no apparent weaknesses in any of these assaults on infinity and infinite divisibility in Hume's critique, which reinforces his

[43] What often substitutes for a clear grasp of the idea behind Cantor's hierarchy of transinfinities is a comfortable facility with the elegant notations of transfinite arithmetics. For an attempt to overcome some of the conceptual difficulties encountered in ordinary first-order infinity, see Bolzano, *Paradoxes of the Infinite*.

negative conclusion that mathematics and the metaphysics of extension must disavow the concept of infinity.

We should not be misled by attempts to discredit Hume's critique by facile appeal to contemporary mathematical considerations in which the concept of infinity is simply presupposed. An objection of this sort appears in James Franklin's essay, "Achievements and Fallacies in Hume's Account of Infinite Divisibility". Franklin introduces the criticism with this testimonial statement of faith in modern mathematics:

> To understand what is right about Hume, it will be necessary to review briefly what is now known to be the correct answer on the question of infinite divisibility. Anachronism threatens, of course, but at least we will avoid the error of ignorantly dismissing as impossible what experts with the benefit of all history presume true. In any case, we are dealing with mathematics, where knowledge *is* cumulative.[44]

The following counterargument is then adduced as a knock-down rebuttal of Hume's complaints against the concept of infinite divisibility:

> The infinite divisibility of space and time is possible. (This is because there exists a consistent model which incorporates infinite divisibility, namely the set of infinite decimals.) It follows that all supposed proofs of the impossibility of infinite divisibility, whether mathematical or philosophical, are invalid. There is a small cost to this, in that one must accept that an infinite number of points with zero length can add up to something with a positive length. This is odd, but no more than that; it just means that length is not constituted by counting.[45]

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[44] Franklin, "Achievements and Fallacies in Hume's Account of Infinite Divisibility", pp. 86-87.

[45] Ibid., p. 87. Franklin is criticized by Waxman, "The Psychologistic Foundations of Hume's Critique of Mathematical Philosophy". Franklin's attribution to Hume of a 'sour grapes' attitude in his refutation of infinite divisibility commits Hume to the evidently fallacious inference that we are entitled to conclude that whatever is not experienced does not or cannot exist. This, of course, is not Hume's argument. Hume considers what is supposed to be the content of putative ideas, and rejects them only when

I find this criticism indecisive. Hume questions the possibility of *any* concept of infinity. He would certainly regard it as circular to defend infinite divisibility by invoking one-one modeling correspondences between a supposedly infinite set of decimals and the subdivisions of extension. Hume need only reiterate his concerns about the origin of the idea of an infinite set or series and the paradox of finite minds comprehending infinite quantity, magnitude, and relation, to remind us that his critique is directed against any and every idea of infinity.

Of course, if we could get Hume to agree that the decimals are infinite, then Franklin is right to say that Hume might as well allow the possibility of the infinite divisibility of extension. The concept of one-one correspondence is not such an innovation, even for an eighteenth-century thinker. But Hume disputes the adequacy of the idea of infinity across the board. It is hard to see how Franklin can describe the infinite decimals as providing a consistent model of infinite divisibility in this context, when Hume raises four *reductio* objections to dispute the logical consistency of the concept of infinite divisibility. Franklin's objection also ignores the tradition of strict finitism in mathematics and philosophy of mathematics, which does not begin or end with Hume.

Franklin paints a distorted picture of contemporary mathematics as though it were monolithic in its endorsement of the concepts of infinity and infinite divisibility. Why not say that the decimals (or integers, etc.) are finite but indefinite, indeterminate, or inexhaustible in cardinality? To conclude as Franklin does that no disproof of infinite divisibility can possibly succeed because mathematics today is committed to the existence of infinite sets in which infinite divisibility can be modeled reflects

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he discovers that the ideas cannot possibly have an experiential origin. This conclusion in the case of reputed ideas of infinite divisibility is defended by the grain of sand thought experiment, the inkspot experiment in support of sensible extensionless indivisibles, and the *reductio* disproofs and geometry dilemma, that are supposed to reveal logical inconsistencies showing that any concept of infinity is not merely unexperienced but unintelligible, and hence unexperienceable.

a disregard for the depth of Hume's critique of the concept of infinity generally, and for the details of his arguments as valid *reductio* denunciations of infinite divisibility from the infinitist's own assumptions. An objection of Franklin's sort to Hume's critique is no better than insisting in more explicitly question-begging fashion that infinite divisibility is endless divisibility or divisibility that can always be continued, perhaps in the mind of God, where by 'endless' and 'always' we mean occurring over infinite time, and by the mind of God a consciousness of infinite power, duration, or comprehension.

Finally, Franklin tries to put a good face on what for Hume is an outrageous implication of the infinite divisibility thesis, that infinitely many mathematical points of no length, the 'nothingnesses of extension' as Bayle calls them, are somehow supposed to be able to constitute a positive extended length, and that length is not constituted by counting. The latter conclusion takes no account of Hume's geometry dilemma as a criticism of the inadequacy of ideas of exact geometrical equality and proportion on the infinite divisibility assumption.

Where Hume has seen through infinitist propoganda, he rightly doubts whether we can meaningfully be asked to accept a theory whose concepts are beyond the reach of human understanding. Moreover, Hume proposes in place of the unattainable idea of infinite divisibility an alternative doctrine of the strict finitist divisibility of extension into sensible extensionless indivisibles, the direct experience of which is empirically grounded in sense impressions as highlighted in the inkspot experiment. The concept of infinite divisibility transcends the finite mind's grasp of concepts, and according to Hume is embroiled in multiple contradictions that expose its logical incoherence, as the several *reductio* disproofs of infinite divisibility are intended to show. Even the theoretical usefulness of the concept of infinity in pure and applied mathematics is disputed by Hume, when he argues that infinite divisibility is incompatible with the requirements of adequate ideas of exact equality and proportion in the geometry dilemma.

Now, do we really have a clear idea of how extension is supposed to be constituted by sensible extensionless indivisibles? To repeat Aristotle's worry about the possibility of contact between (ideal abstract Euclidean mathematical) indivisibles, how can Hume's indivisibles touch, with no *interstitia*, yet without having parts? I confess my own inclination to think of what Hume means by the constitution of extension out of indivisibles by imagining indivisibles being magnified to the point where I can see them touch with no daylight showing between the cracks. This unfortunately is well beyond the point where the imagined objects are no longer extensionless, but thick enough to be divisible into halves touching others like themselves on left and right sides. Hume's reply here is to send us back to the inkspot experiment, where the idea of a sensible extensionless indivisible is best experienced. We must hold an indivisible fixed at just the right threshold distance, and then imagine more of the same accumulated in a row or cluster so as to constitute extension. For this, Hume might add, we need only consider that when several indivisibles conglomerate, we can move back ever so slightly and experience their juxtaposition as a new indivisible or *minima sensibilia relativa*.[46] It is naturally a mistake to try imagining the

[46] Franklin offers optical considerations for determining the exact number of Hume's sensible extensionless indivisibles needed to constitute a linear inch. Ibid., pp. 96-97: "A final note: how big is a Humean indivisible? Strictly, this question has no answer, since indivisibles are extensionless. But there remains an almost identical question which must have an answer: how many indivisibles are there to the inch? 'A finite number', according to Hume... Of course, one can see things that occupy an indefinitely small angle, if they are bright enough; this can be confirmed by going outside on a clear night and looking up at any star. But it does make sense to determine the smallest black dot that can be seen against a white background. Such measurements were carried out in Hume's lifetime by Tobias Mayer, who found that under good conditions, the dot is visible if it occupies about 34 seconds of arc. [Franklin refers to Grüsser, "Quantitative Visual Psychophysics During the Period of the European Enlightenment".] This corresponds to a mere 500 indivisibles to the inch. But it turns out that much better results can be got by testing very thin black lines against

constitution of extension out of indivisibles at a closer distance or level of magnification where they are divisible rather than indivisible. As always in Hume, we must distinguish the idea and the problem of its experiential origin from the reality we are psychologically compelled to believe as corresponding to the idea. Indivisibles, we are literally forced to admit, regardless of the vagaries of the phenomenal experiments we may perform to garner an idea of them, are truly and genuinely indivisible.[47]

Hume's strong empiricist thesis about the origin of all ideas in sensation might be weakened to admit the possibility of at least some ideas that do not originate in experience of the world, but that in a sense are preprogrammed or hardwired into the brain of cognitive subjects. Even then it seems farfetched to suppose that the finite brain with its finite data storage and information processing capabilities could possibly be preequipped with an adequate conception of infinity or the infinite divisibility of extension. To attribute such an idea to neural mechanisms in terms of modern cognitive psychology at the very least requires abandoning computational models of the mind. For we know that the formal description of information processing machines can be limited to the operations of finite mathematics. The only other possibility is to suppose that the abstract ideas of infinity and infinite divisibility are bestowed on the mind by an infinite being, God or a god, who by virtue of his infinity may have the requisite complexity to constitute a sufficient source of these ideas. Yet even this desperate proposal

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a white background; in perfect conditions one can see a line so thin it occupies half a second of arc at the eye... Given that the best visibility is obtained about one foot in front of the eye, it can be calculated how many half-seconds of arc are subtended by one inch at the distance of 1 foot. This is the number of indivisibles in an inch. ('As the ultimate standard of these figures is deriv'd from nothing but the senses and imagination, 'tis absurd to talk of any perfection beyond what these faculties can judge of' [T I 2 iv, 51].) The answer is about 35,000."

[47] See Grünbaum, "A Consistent Conception of the Continuum as an Aggregate of Unextended Elements". But compare Cummins, "Bayle, Leibniz, Hume, and Reid on Extension, Composites, and Simples".

begs the question against Hume's critique by assuming that there could be an infinite being.

It begins to appear more and more attractive, more in step with modern science, to join Hume's critique of infinity, and substitute in place of commitments to infinity and the infinite divisibility thesis a suitable version of strict finitism. At one time, scientists and philosophers believed that space and time were infinite in extent, and even that the stars in the sky and sands on the shore were infinite in number. These conceptions have slowly given place to closed curved space-time models, and by an understanding of the enormously large yet finite numbers of material particles, units of distance across deep space, and increments of time, that make up the physical universe. Recent developments in finite mathematics, computational theories of psychology based on the limitations of real time information processing for finite state machines, and other new scientific discoveries and applications, reflect a modern tendency to limit science to finite concrete reality, rather than indulging in ideal abstraction and extrapolation beyond what logically can be counted, and, as Hume would insist, beyond what can be known.

If strict finitism begins to dominate mathematics as it already has the natural and information sciences, if the ideas of infinity and infinite divisibility are abandoned, relegated to a museum of scientific and philosophical curiosities like vortices, phlogiston, the aether, and the reduction of mathematics to logic, then Hume's empiricist critique of infinity will at last be seen as far ahead of its time. Hume will then be recognized as having advanced the elimination of the concept of infinity and infinite divisibility demanded by an 'experimental' theory of knowledge and mind in a brilliant ensemble of arguments that have yet to be countermanded, and yet to be widely understood and appreciated.

AFTERWORD

HUME'S AESTHETIC PSYCHOLOGY OF DISTANCE, GREATNESS, AND THE SUBLIME

... it is the nature of the finite mind to be incapable of comprehending infinity.

— Antoine Arnauld, *Port Royal Logic*

AFTERWORD

HUME'S AESTHETIC PSYCHOLOGY OF DISTANCE, GREATNESS, AND THE SUBLIME

Concepts of the Sublime

The beautiful and the sublime as categories of aesthetic experience attained unique significance in eighteenth-century thought. Hume, as a philosopher of his time, could no more have ignored the Enlightenment's awakening to the pre-Romantic aesthetics of the sublime than he could have ignored the rise of mechanical science in the natural philosophy of Isaac Newton. There is moreover an important connection between the two, because Newtonian mechanics and the sublime as understood by Hume's contemporaries alike are committed to the intelligibility of the concept of infinity, which Hume emphatically rejects.

The original meaning of the sublime is often lost sight of today, when references to subliminal advertising and related artistic phenomena suggest subtlety and concealment or delicacy of structure that is the very opposite of what the sublime first denoted. The sublime is the elevated, lofty, deep or complex, and sometimes terrifying in nature and art, that which implies height and power. Examples of the sublime in philosophical-aesthetic commentary in Hume's day include impressive mountaintop scenes of the Swiss and Italian Alps, river torrents and ocean storms, God's creation of light and the division of night and day in *Genesis* 1.3, and John Milton's poetic evocation of Satan in *Paradise Lost.*

Jean-Jacques Rousseau in *The Confessions* epitomizes the attitude toward the sublime in nature that emerged into artistic and philosophical consciousness in late Neoclassicism at the threshold of its transition into Romanticism, when he writes:

> Never does a plain, however beautiful it may be, seem so in my eyes. I need torrents, rocks, firs, dark woods, mountains, steep roads to climb or descend, abysses beside me to make me afraid. I had these pleasures, and I relished them to the full, as I came near to Chambéry. At a place called Chailles, not far from a precipitous mountain wall called the Pas de l'Échelle, there runs boiling through hideous gulfs below the high road — which is cut into the rock — a little river which would appear to have spent thousands of centuries excavating its bed. The road has been edged with a parapet to prevent accidents, and so I was able to gaze into the depths and make myself as giddy as I pleased... Supporting myself firmly on the parapet, I craned forward and stayed there for hours on end, glancing every now and then at the foam and the blue water, whose roaring came to me amidst the screams of the ravens and birds of prey which flew from rock to rock and from bush to bush, a hundred fathoms below me.[1]

Interest in the aesthetics of the sublime in this period coincides with the translation originally into French and later into English, and subsequent widespread discussion, of Longinus' manual of rhetorical style, *Peri Hypsos* (*On the Sublime*). The concept of the sublime, exemplified for Longinus by the silence of Ajax in Book XI of Homer's *Odyssey* as an archetype of lofty or elevated literary form, became popular enough for Alexander Pope to parody in his satire of Longinus' treatise, *Peri Bathos, or the Art of Sinking in Poetry* (1728).[2] As Samuel H. Monk has documented in *The Sublime: A Study of Critical Theories in XVIII-Century England*, the term 'sublime' derives from the 1674

[1] Rousseau, *The Confessions*, Book Four (1732), pp. 167-168.

[2] Monk, *The Sublime: A Study of Critical Theories in XVIII-Century England*, pp. 10-83. Monk's bibliography includes references to translations of Longinus in French and English editions. A recent translation is given by Grube, Longinus, *On Great Writing* (*On the Sublime*).

French translation of Longinus by Nicolas Boileau-Despréaux, who adapted the Latin *sublimi* for Longinus' Greek *hypsos*.[3] Boileau's *L'Art Poétique* and *Traité du Sublime ou du Merveilleux dans le Discours Traduit du Grec de Longin* excited and shaped British and Continental thinking about the sublime as an aesthetic category, different and in some ways opposite from, yet affording equal pleasure with and therefore complementing the category of the beautiful.[4]

Among philosophical treatments of the sublime in nature and art must be mentioned Edmund Burke's (1757, revised 1759) *A Philosophical Enquiry into the Origin of the Ideas of the Sublime and Beautiful*, and Immanuel Kant's (1764) tract, *Beobachtungen über das Gefühl des Schönen und Erhabenen* (*Observations on the Feeling of the Beautiful and Sublime*), together with the later *Critique of Judgment*, especially the Second Book or 'Analytic of the Sublime', which appeared in 1790.[5] Prior to these, Francis Hutcheson in his *An Inquiry into the Original of our Ideas of Beauty and Virtue* (1725), and Hume, in the *Treatise*, offered philosophical investigations of aesthetic problems about the sublime from the standpoint, in Hutcheson's case, of a theory of a sixth, internal, aesthetic sense, and in Hume's, by his account of the passions based on Locke's associationist psychology.[6] Hume, according to a letter to Adam Smith of 12 April 1759, knew of Burke's *Enquiry*, despite the latter's anonymous authorship, while Burke's introductory 'Essay on Taste', appended to the second edition of the *Enquiry*, in the

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[3] Johnson, *A Dictionary of the English Language*: "The *sublime* is a Gallicism, but now naturalized."

[4] Boileau-Despréaux, *L'Art Poétique* and *Traité du Sublime ou du Merveilleux dans le Discours Traduit du Grec de Longin*, in the *Oeuvres completes de Boileau.*

[5] Kant, *Observations on the Feeling of the Beautiful and the Sublime.* Kant, *Critique of Judgment*, esp. pp. 90-203.

[6] *Treatise*, pp. 427-438. See Hutcheson, *An Inquiry into the Original of our Ideas of Beauty and Virtue in Two Treatises*, second edition, corrected and enlarged, Vol. I, *Concerning Beauty, Order, Harmony, Design.* Locke, *Essay*, pp. 394-401.

opinion of James T. Boulton, is intended as a reply to Hume's "The Standard of Taste".[7]

Hume's aesthetic philosophy is not so thoroughly developed as that of Hutcheson, whose ideas on the beautiful and pathetic influenced Hume, nor as systematically presented as that of Burke or Kant, whose theories Hume may have at least indirectly influenced. Rather, it occurs in what is probably its only proper place in his metaphysics, as a special problem within a more expansive treatment of the psychology of the passions. The comparatively neglected question of Hume's aesthetics of greatness and the sublime is equally important as a counterpart to his more frequently discussed arguments for the standard of taste and pronouncements about rhetoric and tragedy in the posthumously collected *Essays Moral, Political, and Literary*.[8] Hume's aesthetics of the sublime is not neatly packaged in the *Treatise*, but must be extracted from his solutions to three problems about the experience of distance in space and time, and the explanation of the pleasure that these sensations afford, from the persepctive of his early empiricist psychology of the passions.

Infinity, Greatness, and the Sublime

Monk observes that Hume does not contrast beauty and the sublime as exclusive categories, and in the *Treatise* does not even mention the sublime as such or by that name. Monk maintains:

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[7] Hume, Letter to Adam Smith, 12 April 1759, *New Letters of David Hume*, ed. by Mossner and Klibansky, refers to: "... Burke an Irish gentleman, who wrote lately a very pretty Treatise on the Sublime." Burke, *A Philosophical Enquiry into the Origin of our Ideas of the Sublime and Beautiful*, pp. 11-27. See Boulton, Editor's Introduction, pp. xxvii-xxxi.

[8] Hume, "Of Eloquence", "Of Tragedy", and "Of the Standard of Taste", *Essays Moral, Political and Literary*, pp. 97-110, 216-225, 226-249. See Wieand, "Hume's Two Standards of Taste". Carroll, "Hume's Standard of Taste". MacMillan, "Hume, Points of View and Aesthetic Judgments". Baxter, "Hume on Virtue, Beauty, Composites and Secondary Qualities".

> In the *Treatise* beauty and sublimity are not opposed; indeed Hume does not use the word *sublime* at all, but his *greatness* is obviously the same thing. Greatness, he tells us, 'whether successive or extended, enlarges the soul, and gives it a sensible delight and pleasure.' Since beauty gives pleasure also, it would seem that Hume has not taken Addison's hint as to the difference between the two, but has preferred to conclude that the great is simply a larger beauty. It is beauty accompanied 'with a suitable greatness'.[9]

Boulton also writes: "Like Addison, Hume uses the word 'greatness' rather than 'sublimity'."[10] But it is false that Hume *never* uses the word 'sublime' in the *Treatise*, for he does so explicitly in several places in Book II, 'Of the Passions', especially in Section VIII, *The same subject continu'd*, which concludes the argument of the previous section, *Of contiguity, and distance in space and time.* There Hume observes:

> These principles have an effect on the imagination as well as on the passions. To be convinc'd of this we need only consider the influence of *heights* and *depths* on that faculty. Any great elevation of place communicates a kind of pride or sublimity of imagination, and gives a fancy'd superiority over those that lie below; and *vice versa*, a sublime and strong imagination conveys the idea of ascent and elevation.[11]

The *Essays*, moreover, which Monk does not consider, contain at least a dozen occurrences of the words 'sublime' or 'sublimity', though not, with the exception of the essay "Of Tragedy", in the context of examining the concept of the sublime. Yet it must be admitted that Hume does not use these terms as liberally as many of his contemporaries, but prefers in most instances to speak of 'greatness' in distance, height, and depth. The *Enquiries Concerning Human Understanding and Concerning the Principles of Morals* eliminate the subject entirely

[9] Monk, *The Sublime*, p. 64.

[10] Boulton, Editor's Introduction to Burke, p. l. Boulton refers to the *Treatise in David Hume: The Philosophical Works*, Vol. II, 209ff.

[11] *Treatise*, p. 434.

under any terminology, and no continuation of the problems about understanding the experience of the sublime which had exercised Hume in the *Treatise* appear in these later writings. It is to the *Treatise* (and essay "Of Tragedy") that we must turn for Hume's aesthetic psychology of distance, greatness, and the sublime.

An explanation of why Hume usually refers to greatness rather than the sublime may be that for most eighteenth-century commentators on the sublime the concept is supposed to imply or suggest to the mind what for Hume are the philosophically problematic ideas of infinity and God's power. This is evident in aesthetic thought both before and after the time of Hume's composition of the *Treatise*. Joseph Addison in the *Spectator* 413, regards 'greatness' as "... divinely ordained to lead to contemplation of the nature and power of God." John Baillie in his (1747) *An Essay on the Sublime*, states: "Where an Object is *vast*, and at the same Time *uniform*, there is to the Imagination no Limits of its Vastness, and the Mind runs out into *Infinity*, continually *creating* as it were from the *Pattern*."[12]

Burke in the *Enquiry* distinguishes between 'vastness' and 'infinity', and introduces the new category of the 'artificial infinite', which the mind imaginatively constructs from the repetition of figure or the sensation of what appears to be unlimited.[13] But he allows the genuinely infinite as a less frequent source of the sublime, both in infinite extent and divisibility. He admits that the experience of the actually infinite may be rare, but does not exclude the possibility, attributing most of what is ordinarily called infinity to the mind's manufacture of artificial infinity. Burke maintains that:

> Another source of the sublime, is *infinity*; if it does not rather belong to the last [vastness]. Infinity has a tendency to fill the mind with the sort of delightful horror, which is the most genuine effect, the truest test of the sublime. There are scarce

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[12] Baillie, *An Essay on the Sublime*, p. 9.

[13] Burke, *Enquiry*, pp. 73-76, 139-143, Part Four, Section XI, 'The Artificial Infinite'.

> any things which can become the objects of our senses that are really, and in their own nature infinite. But the eye not being able to perceive the bounds of many things, they seem to be infinite, and they produce the same effects as if they were really so. We are deceived in the like manner, if the parts of some large object are so continued to any indefinite number, that the imagination meets no check which may hinder its extending them at pleasure.[14]

> ... as the great extreme of dimension is sublime, so the last extreme of littleness is in some measure sublime likewise; when we attend to the infinite divisibility of matter, when we pursue animal life into these excessively small, and yet organized beings, that escape the nicest inquisition of the sense, when we push our discoveries yet downward, and consider those creatures so many degrees yet smaller, and the still diminishing scale of existence, in tracing which the imagination is lost as well as the sense, we become amazed and confounded at the wonders of minuteness; nor can we distinguish in its effect this extreme of littleness from the vast itself. For division must be infinite as well as addition; because the idea of a perfect unity can no more be arrived at, than that of a compleat whole to which nothing may be added.[15]

The commitment to a Leibnizian-Newtonian concept of infinity and infinite divisibility in mathematics and the physical sciences is pronounced in Kant's related concept of the mathematically sublime, in which he concludes that the estimation of infinity can only be the result of aesthetic judgment rather than sense perception or imagination. The *Critique of Judgment* links infinity explicitly with the sublime, when Kant argues:

> Nature, therefore, is sublime in such of its phenomena as in their intuition convey the idea of their infinity. But this can only occur through the inadequacy of even the greatest effort of our imagination in the estimation of the magnitude of an object.[16]

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[14] Ibid., p. 73.

[15] Ibid., pp. 72-73.

[16] Kant, *Critique of Judgment*, p. 103.

Hume is as opposed to the idea of artificial infinity as an idea *of infinity* as he is to the idea of actual infinity. Hume's arguments in the *Treatise* and first *Enquiry* contradict Burke's concept of the artificial infinite as an adequate idea of infinity constructible by the mind from the experience of repetition or apparent unlimitedness. Hume anticipates the equivalent of Burke's proposal at length in the *Treatise*, only to eliminate the possibility that the agency of reason, memory, or the imagination could synthesize an adequate mind-mediated complex idea of infinite divisibility from the impression or idea of extension.[17] Hume must regard Burke's artificially infinite as equally invalid and misleading in describing the imagination's decidedly less-than-infinite telescopic or microscopic projection of repetitive pattern beyond the limits of experience. There can be no such idea, according to Hume, which makes it objectionable to refer to any phenomenon in Burke's or Kant's terms as or as exhibiting either natural or artificial infinity. For this reason, because of its longstanding associations with the concept of infinity, Hume may have preferred to downplay references to the sublime, and substitute instead the philosophically more responsible and accurate though less dramatic ideas of greatness and vast distance in height and depth.

Hume's religious skepticism and disparagement of appeals to God as an explanatory principle in philosophical contexts have already been remarked. It may therefore be unnecessary to document the claim that for Hume the experience of the sublime as an invitation for the mind to reflect on God's divinity, and especially on God's infinite knowledge, power, and goodness, or God as an infinite being, would be as much anathema to his empiricist aesthetic psychology as to his skeptical fideist philosophy of religion.[18]

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[17] *Treatise*, p. 34. Also pp. 239-240. Compare Locke, *Essay*, pp. 266-267.

[18] Hume, *Dialogues Concerning Natural Religion*, pp. 198-213. See p. 203: "I scruple not to allow, said Cleanthes, that I have been apt to suspect the frequent repetition of the word, *infinite*, which we meet with in all theological writers, to savour more of panegyric than of philosophy, and that any

Hume's Philosophical Psychology and the Aesthetics of Greatness and the Sublime

There is prevailing agreement among many eighteenth-century philosophers about the problem of the sublime, and in general outline about its solution. Hume's commentary on the aesthetics of greatness can be explained in terms of his participation in this ongoing critical examination of the sublime and his differences with received interpretations.

The formula for most Enlightenment theories of the sublime is to recognize its importance in comparison with the beautiful, to propose a definition or set of characteristics to distinguish the two, and then to attempt an account of the unique feelings of pleasure in the experience of the sublime in nature and art. The standard theories in Hume's day all seek to explain aesthetic pleasure in the sublime by way of the sense of interest or threat aroused in the perceiver by the sublime, especially in scenes of natural violence, great distances, heights, and depths, and to a lesser but still significant degree in artistic depictions. These first cause the perceiver to focus attention on the phenomenon, and then, in the safety of that remove, which seems to be an essential ingredient of the experience of the sublime, to enter a transformed state of mind.

Such an experience may combine, in varying degrees, depending on the details of the particular theory, the subsequent accompanying sensations of relief, satisfaction, an 'enlarging' of perspective or outlook, or a sense of pride or increase in the mind's ability to grasp the dimensions with which it is confronted. It is the sensation of mentally overcoming the dangers implicit in the perception, and emerging from the experience victoriously transformed. The sublime, according to such authors as Addison and Kant, acquires its real value by further

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purposes of reasoning, and even of religion, would be better served, were we to rest contented with more accurate and more moderate expressions." Surprisingly, in light of this rejection of the concept of infinity in theological contexts, Hume in the *Treatise*, p. 432, includes eternity in his list of ideas that can inspire awe or experience of the sublime.

leading the mind beyond this point to an appreciation of infinity and the infinite power of God.

Hume's approach to the sublime more or less fits this pattern, though his metaphysics of strict finitism and skeptical religious outlook preclude him from the most highflown speculations about the final significance of the sublime. Hume concentrates on three problems about the concept, concerning: (1) the aesthetic psychology of objects experienced at vast distances; (2) the comparatively greater admiration for objects separated from the perceiver in time than space; (3) the comparatively greater admiration for objects separated in past than future time. Hume assumes that the aesthetic phenomena presupposed by these problems are as he describes, offering a few examples without argument or analysis as though the point in each case was uncontroversial. The problems have a special poignancy in the development of his empiricist epistemology, because as he notes they are in a sense the very opposite of complementary phenomena involving the relations of perceiver and object separated by vast distances in space and time with respect to their effect on conception and passion as opposed to aesthetic admiration. He concludes:

> Thus we have accounted for three phenomena, which seem pretty remarkable. Why distance weakens the conception and passion: Why distance in time has a greater effect than that in space: And why distance in past time has still a greater effect than that in future. We must now consider three phenomena, which seem to be, in a manner, the reverse of these: Why a very great distance encreases our esteem and admiration for an object: Why such a distance in time encreases it more than that in space: And a distance in past time more than that in future. The curiousness of the subject will, I hope, excuse my dwelling on it for some time.[19]

What is additionally at stake in the discussion for Hume's philosophy, beyond its 'curiousness', is its apparent incoherence with his theory of conception and the passions. If the three

[19] *Treatise*, p. 432.

aesthetic phenomena are really as he says, then, as the opposite of what is to be expected from his empiricist psychology of the passions, they indicate conflicting responses to the same objects. Hume has by no means simply contradicted himself, but he needs to reconcile the psychology of aesthetic experience with that of sensation, and in so doing expand upon his previous characterization of the subject matter of aesthetics and the psychology of conception and the passions.

Aesthetics of Great Distance in Space and Time

To resolve the apparent conflict, Hume offers a Hutcheson-style application of Locke's associationist psychology. The mind experiences the specific pleasures of greatness in contrast with the beautiful, according to Hume, by virtue of the effort it must make in assimilating the distances, offering satisfaction in overcoming the difficulties involved. Hume speaks of this as something that 'enlarges the soul' with a 'sensible delight and pleasure'. The mind is active in the aesthetic process of sublime experience as Hume describes it, 'reflecting on the interpos'd distance', admiring the distance itself, and then by Lockean-Hutchesonian association transferring the passion and admiration excited by the contemplation of the distance to the distant object. He explains:

> To begin with the first phenomenon, why a great distance encreases our esteem and admiration for an object; 'tis evident that the mere view and contemplation of any greatness, whether successive or extended, enlarges the soul, and [gives] it a sensible delight and pleasure. A wide plain, the ocean, eternity, a succession of several ages; all these are entertaining objects, and excel every thing, however beautiful, which accompanies not its beauty with a suitable greatness.[20]

There is no doubt that Hume in this passage makes reference to the 'greatness' of natural events in a way that any of his contemporaries would immediately recognize as denoting aesthetic experience of the sublime. If Hume's remarks are

[20] Ibid.

taken literally, they allow that greatness or the sublime can in principle be joined in company with beauty, since Hume holds that objects partaking in great distances are aesthetically more valuable than beautiful things that do not also display greatness. Unlike some aesthetic theorists, Hume appears not to draw a sharp distinction between beauty and greatness or the sublime as mutually exclusive categories.

The psychological problem of explaining the sublime is addressed in this part of Hume's discussion by the perceiver's esteem and admiration 'enlarging' the soul. Hume declares that great distance in space or time adds positive aesthetic value to any object. He combines reflection or contemplation of great distance in the conceptual mode with its effect on the aesthetic experience of the sublime.

> Now when any very distant object is presented to the imagination, we naturally reflect on the interpos'd distance, and by that means, conceiving something great and magnificent, receive the usual satisfaction.[21]

The challenge for Hume is to explain from the resources of his theory of the passions why great distance in space or time should confer this kind of positive aesthetic value on objects so separated from the perceiver. The difficulty is especially acute for Hume, because his pronouncements about distance as weakening the conception while exciting the passions raise an apparent contradiction in the aesthetics of the sublime. Hume's solution, as might be expected, is both ingenious and commonsensical, relying ultimately on Locke's associationist psychology, which Hutcheson had previously applied to aesthetic theory. Hume states:

> But as the fancy passes easily from one idea to another related to it, and transports to the second all the passions excited by the first, the admiration, which is directed to the distance, naturally diffuses itself over the distant object.[22]

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[21] Ibid., p. 433.

[22] Ibid.

The exact mechanism by which experience of greatness or the sublime produces pleasure is later explained by Hume's observations of the mind's experience in overcoming obstacles, the psychological effect of meeting with success after stirring its energies in the effort to prevail over adversity. Like other commentators on the sublime, Hume emphasizes the importance of the difficulties not being overwhelming, as in the case of the terrible in the sublime, the danger not being immediate, which, in the urgency of the moment, would prevent the experience of aesthetic pleasure that can only occur at some safe distance away from the real scene of action (as Rousseau says of his vantage point far above the torrents, 'The road has been edged with a parapet to prevent accidents, and so I was able to gaze into the depths and make myself as giddy as I pleased... Supporting myself firmly on the parapet...'). Hume continues:

> 'Tis a quality very observable in human nature, that any opposition, which does not entirely discourage and intimidate us, has rather a contrary effect, and inspires us with a more than ordinary grandeur and magnanimity. In collecting our force to overcome the opposition, we invigorate the soul, and give it an elevation with which otherwise it wou'd never have been acquainted. Compliance, by rendering our strength useless, makes us insensible of it; but opposition awakens and employs it. This is also true in the inverse. Opposition not only enlarges the soul; but the soul, when full of courage and magnanimity, in a manner seeks opposition.[23]

As a further implication of the associationist solution, Hume connects the aesthetics of greatness in nature to that in artistic production. Thus, a traveler whom we know to have visited distant parts of the world excites admiration, even though he is not actually removed at any considerable distance, but appears together with us in the same room. Artworks and antiques can produce the same effect, because of their association with distance in time, despite being copresent with their admirers.

[23] Ibid., pp. 433-434.

> Accordingly we find, that 'tis not necessary the object shou'd be actually distant from us, in order to cause our admiration; but that 'tis sufficient, if, by the natural association of ideas, it conveys our view to any considerable distance. A great traveller, 'tho in the same chamber, will pass for a very extraordinary person; as a *Greek* medal, even in our cabinet, is always esteem'd a valuable curiosity. Here the object, by a natural transition, conveys our view to the distance; and the admiration, which arises from that distance, by another natural transition, returns back to the object.[24]

Hume repeats the conclusion, which he holds in almost identical terms, in his essay "Of Tragedy". Again, the mechanism of aesthetic pleasure is the mind's excitation by implicit danger:

> Objects of the greatest terror and distress please in painting, and please more than the most beautiful objects that appear calm and indifferent. The affection, rousing the mind, excites a large stock of spirit and vehemence; which is all transformed into pleasure by the force of the prevailing movement.[25]

> Difficulties increase passions of every kind; and by rouzing our attention, and exciting our active powers, they produce an emotion which nourishes the prevailing affection.[26]

As further psychological evidence for the effect of the sublime on the passions through the mind's pleasure in overcoming difficulty, Hume in this place adds:

> Parents commonly love that child most whose sickly infirm frame of body has occasioned them the greatest pains, trouble, and anxiety, in rearing him. The agreeable sentiment of affection here acquires force from sentiments of uneasiness.[27]

Returning to the *Treatise* Book II, Section VIII, Hume delivers his most complete formulation of the associationist psychologi-

[24] Ibid., p. 433.

[25] "Of Tragedy", pp. 220-221.

[26] Ibid., p. 221. I discuss the problem of difficulty and the sublime in aesthetic appreciation in Jacquette, "Bosanquet's Concept of Difficult Beauty".

[27] "Of Tragedy", p. 221.

cal explanation of the aesthetic pleasure resulting from experience of the sublime or greatness in distance, height and depth:

> Since the imagination, therefore, in running from low to high, finds an opposition in its internal qualities and principles, and since the soul, when elevated with joy and courage, in a manner seeks opposition, and throws itself with alacrity into any scene of thought or action, where its courage meets with matter to nourish and employ it; it follows, that every thing, which invigorates and inlivens the soul, whether by touching the passions or imagination, naturally conveys to the fancy this inclination for ascent, and determines it to run against the natural stream of its thoughts and conceptions. This aspiring progress of the imagination suits the present disposition of the mind; and the difficulty, instead of extinguishing its vigour and alacrity, has the contrary effect, of sustaining and encreasing it.[28]

The mind finds conceptual conquest of the potentially overwhelming experience of greatness in distance so satisfying, that by the psychological agency of association it links many other human values to the metaphor of height.

> Virtue, genius, power, and riches are for this reason associated with height and sublimity; as poverty, slavery, and folly are conjoin'd with descent and lowness. Were the case the same with us as *Milton* represents it to be with the angels, to whom *descent is adverse*, and who *cannot sink without labour and compulsion*, this order of things wou'd be entirely inverted; as appears hence, that the very nature of ascent and descent is deriv'd from the difficulty and propensity, and consequently every one of their effects proceeds from that origin.[29]

Hume ventures some of his most revealing metaphors in explaining sublime experience. He depicts the mind's imaginative flight across distances in space and time from the perceiver's viewpoint to the object and back again, and the smooth flowing of ideas like a liquid poured out in mentally traversing space,

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[28] *Treatise*, p. 435.
[29] Ibid., pp. 435-436.

as compared with the mind's efforts in compassing objects separated by great distances in time. In yet another passage of the *Treatise* where Hume refers explicitly to the sublime, he states:

> The mind, elevated by the vastness of its object, is still farther elevated by the difficulty of the conception; and being oblig'd every moment to renew its efforts in the transition from one part of time to another, feels a more vigorous and sublime disposition, than in the transition thro' the parts of space, where the ideas flow along with easiness and facility.[30]

The mind's difficult flight across great distances separating it from sublime objects of aesthetic contemplation is introduced in Hume's attempt to show that distance ordinarily weakens conception and passion by placing objects at removes beyond the subject's immediate practical interests and concerns. He argues:

> When we reflect ... on any object distant from ourselves, we are oblig'd not only to reach it at first by passing thro' all the intermediate space betwixt ourselves and the object, but also to renew our progress every moment; being every moment recall'd to the consideration of ourselves and our present situation. 'Tis easily conceiv'd, that this interruption must weaken the idea by breaking the action of the mind, and hindering the conception from being so intense and continu'd, as when we reflect on a nearer object.[31]

Whether Hume intends these references to the mind's movement over distances, and its comparative smooth or rough transition through the parts of space and time, literally or figuratively, may be impossible to determine. The problem remains in any case of understanding what Hume may mean to convey by these rhetorical analogies. Evidently he means to describe a mental activity involved even in split-second reflection on distances from subject to object in the experience of greatness and the sublime, and a difference between the two in the ease or difficulty the mind encounters in its efforts to assimilate the

[30] Ibid., p. 436.
[31] Ibid., p. 428.

information implicit in the presence of a distant object such as a mountain peak or gorge.

The same may be true of a different kind of difficulty which the mind experiences in coming to terms with the terrifying, which Burke in the *Enquiry* makes the hallmark of the sublime, as contrasted with the beautiful. This is manifest in Rousseau's statement of his experience of the sublime. Rousseau's conception does not simply involve the great distance between his safe observation point on the parapet and the riverbed below at the Pas de l'Échelle, but equally if not more importantly the rushing torrents of water with their obvious implications of tremendous power and danger. It is not just the deep canyon that impresses, it is the *abyss*.

The second problem (2) Hume promises to resolve is dealt with in similar fashion, concerning the more enhanced aesthetic impression of distance in time than space. Hume again seems to overstate the sufficiency of distance to provoke sublime aesthetic experience:

> But tho' every great distance produces an admiration for the distant object, a distance in time has a more considerable effect than that in space. Antient busts and inscriptions are more valu'd than *Japan* tables: And not to mention the *Greeks* and *Romans*, 'tis certain we regard with more veneration the old *Chaldeans* and *Egyptians*, than the modern *Chinese* and *Persians*, and bestow more fruitless pains to clear up the history and chronology of the former, than it wou'd cost us to make a voyage, and be certainly inform'd of the character, learning and government of the latter.[32]

Similarly, with respect to problem (3), concerning the asymmetry of aesthetic admiration and esteem for distant objects in or from past as opposed to future time, Hume holds:

> 'Tis not every removal in time, which has the effect of producing veneration and esteem. We are not apt to imagine our posterity will excel us, or equal our ancestors. This phaenomenon is the more remarkable, because any distance in

[32] Ibid., p. 433.

> futurity weakens not our ideas so much as an equal removal in the past. Tho' a removal in the past, when very great, encreases our passions beyond a like removal in the future, yet a small removal has a greater influence in diminishing them.[33]

The answer Hume offers in all three cases, to which the phenomena of all three problems in different ways testify, is that aesthetic delight is produced by the mind's overcoming the difficulties involved in assimilating the information and potentially threatening implications presented by the mind's apprehension of objects experienced at great distances in space and time. Hume's response to problem (3) further supports this interpretation, as he promises, "The third phaenomenon I have remark'd will be a full confirmation of this."[34] Here Hume contends:

> In our common way of thinking we are plac'd in a kind of middle station betwixt the past and future; and as our imagination finds a kind of difficulty in running along the former, and a facility in following the course of the latter, the difficulty conveys the notion of ascent, and the facility of the contrary. Hence we imagine our ancestors to be, in a manner, mounted above us, and our posterity to lie below us. Our fancy arrives not at the one without effort, but easily reaches the other: Which effort weakens the conception, where the distance is small; but enlarges and elevates the imagination, when attended with a suitable object. As on the other hand, the facility assists the fancy in a small removal, but takes off from its force when it contemplates any considerable distance.[35]

The difficulties are greater for conception as for perception. The aesthetic pleasure potentially derivable is also proportionally greater for the experience of distant as opposed to near objects, for objects in time as opposed to space, and for objects in the past as opposed to the future. The difficulties first arouse the mind's interest and attention, as Hume would have

[33] Ibid., pp. 436-437.
[34] Ibid., p. 436.
[35] Ibid., p. 437.

it, and then, as they are surmounted, produce the delight that arises naturally as in other human endeavors through pride of accomplishment in mastering an imposing cognitive challenge.

Greatness, Difficulty, and Hume's Aesthetics of the Sublime

The question should now be addressed whether or to what extent Hume's account of the aesthetic psychology of the sublime is successful. More interestingly, perhaps, it should also be determined whether his theory resolves the apparent tension in his conceptual-emotional treatment of distance, distance in space as opposed to time, and past as opposed to future time, as weakening rather than exciting the mind's interest in perceived objects.

Assessing his contributions to the philosophical-psychological study of greatness and the sublime, Monk credits Hume with extending Hutcheson's subjective approach to the experience of the sublime by a more thoroughly elaborated psychology, and paving the way in this direction for subsequent aesthetic-psychological interpretations in the work of Baillie and Burke. Monk argues that:

> In comparison with many essays on sublimity in the eighteenth century, Hume's remarks are incomplete enough, but they are none the less new departures, for they are concerned in the main not with the object *qua* object, but with the experiences of the mind that perceives the object. Hutcheson had carried the subject into the sphere of the subjective by establishing the sixth sense, but he had been powerless to analyse the experience because he lacked a psychology. Hume began the application of psychology to the discussion of the sublime which Baillie was to carry on in the next decade and which Burke was to take to exhaustive lengths.[36]

It is an exaggeration for Monk to say that Hutcheson entirely lacked a psychology for his analysis of aesthetic experience, for, as Monk himself says, Hutcheson also applies a version of Locke's associationism. But Hume undeniably develops psy-

[36] Monk, *The Sublime*, p. 65.

chological principles for the subjective exploration of greatness and the sublime more thoroughly than Hutcheson or any of his more literary and less philosophical predecessors.

One of the main difficulties with Hume's aesthetic psychology of distance is that it appears phenomenologically implausible. This is evident particularly in consideration of instances in which an experience of the sublime is so immediate that there does not seem to be adequate time for the imagination's flight to and from a distant object, and the transference of admiration and esteem from the distance to the object. The effect of some sublime objects on the mind occurs so quickly that the only way to harmonize Hume's associationist interpretation of the experience with the raw phenomenological data of the encounter may be to say that the subject is able at once and without further preparation to experience it as great or sublime. In other words, some objects are experienced as sublime without sufficient time for the mind to mentally fly to the far endpoint of space occupied by an object from the viewer, and then to transfer admiration for the distance to the object itself. Hume can only say either that the imagination accomplishes this so quickly in at least some cases that the process is phenomenologically inscrutable, or that the mind acquires the category of the great and sublime at some point in its history by imaginatively compassing a great distance in space or time and transferring its admiration for the distance to the distant object. This provides a Humean explanation of the origin of the idea of greatness and the sublime, so that thereafter it is unnecessary for the perceiver to repeat the procedure every time a great or sublime object is experienced. The mind can rely on its previous painstaking efforts to classify the great and sublime by association with a well-established paradigm, without having to repeat the process again on every new exposure.

If I have experienced greatness in other circumstances prior to some particular encounter, then I may be able to recognize the phenomenon almost immediately as such by a flash of association, without going through a step-by-step process of first grasping the distance by flight of imagination and transferring the admiration from distance to distant object. If as a child I had never before enjoyed anything of the kind to which my mind might make comparison by association, then in the first presence of the sublime I may need to take whatever time is necessary to digest the facts about the object's distance and transfer my admiration from distance to the object. But as things are now I can take psychological shortcuts, in which aesthetic pleasure in greatness and the sublime obtains immediately by association, without actually struggling to overcome the difficulties of experiencing vast distance on every occasion of contact with the sublime. Children as a matter of fact do seem either to spend more time admiring the great and sublime, or else are so overwhelmed that they turn away with no appreciation of the phenomena.

Yet there are also experiences of the sublime that seem to have nothing to do with admiration and esteem for distance, such as the sublime of the minutely divisible and the horror and power projected in Milton's portrait of Satan. With respect to minute divisibility, it would be inadequate for Hume to reply that distances or divisions imaginably experienced by tiny beings may account for the sublimity of worlds within worlds which they might encounter as vast distances from their diminutive perspective, for in that case nothing fails to qualify as sublime. But while this seems a reasonable reply, it raises another important aspect of the aesthetic psychology of greatness and the sublime that Hume's analysis overlooks.

The most objectionable gap in Hume's aesthetics of the sublime is that it does not explain why the same distant objects are sometimes experienced as great or sublime and sometimes not. The stars are not always seen with awe, even when their distance from the viewer is conceptualized. Distance, despite Hume's assertions, does not seem to be a sufficient condition for sublime experience; it is only on special occasions that a distant object is regarded as great or sublime. Whatever motivates this attitude on the part of percipients is a missing ingredient from Hume's psychology that leaves his explanation incomplete. It is a specific kind of attention and attitude directed toward great and distant objects that eludes Hume's analysis. Satan in Milton's story might be thought to be sublime by association with great distance if it is agreed that the horror inspired by his power can be understood as the ability to cause evil over vast reaches of space and time that extend from an imaginary past. For some, Milton's description apparently has this effect, since the example is one of the most common in eighteenth-century discussions of the sublime. Yet distance alone, together with the flexible resources of an associationist psychology, seems to provide neither necessary nor sufficient conditions for sublime aesthetic experience.

An adequate aesthetic psychology of greatness and the sublime must account for the difference between episodes of passively perceiving and conceptualizing great distances and objects at great distances, and episodes of aesthetic appreciation of distances and distant objects *as* great or *as* sublime. This Hume's analysis fails to do. He may have isolated some of the elements of some aesthetic experiences of greatness, but he cannot be said to have identified its nature or essence. And perhaps, in the context in which the subject arises in his larger discussion of the passions, this is not really his purpose. What is remarkable about Hume's theory of greatness and the sublime is its reduction of experience of the sublime to a particular psychological attitude toward distance in space and time, and whatever can plausibly be associated with it. The beauty, or perhaps the sublimity, of such an account is that it tries to explain complex aesthetic psychological phenomena in terms of the conception and likely effect on the passions in experience of the most elemental metaphysical properties of space and time. This makes Hume's project extraordinary for the insights it extends via associationist psychology to some of the deepest problems of aesthetics.[37]

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[37] Ibid.: "Thus by a cumulative process of association [according to Hume] distant objects awaken sublime emotions in the soul. We have here an early and an interesting effort to analyse psychologically the experience then called sublime."

Hume distinguishes the weakness of conception and the passions and strength of esteem and admiration as equally the result of greatness of distance from subject to object in space and time. He is aware of the conflict, but appears uninterested in resolving the problem as a genuine contradiction. Indeed, having established the first three weakness-distance theses for conception and the passions, Hume proceeds to argue for the second set of matching opposite strength-distance theses for esteem and admiration. Hume's adaptation of associationist psychology is the central part of his solution. But association by itself does not entail the distinction by which great distance in space, and in past time as opposed to space or future time, diminishes conception and interest, but increases aesthetic admiration. The distinction on which Hume's aesthetic psychology trades in resolving these three sets of conflicting theses is more fundamental. It does not derive from association, but must obtain as a basis for distinguishing the different roles, functions, and consequences of association in the two types of cases, in order for the associationist resolution to be applied. What constitutes this underlying basis of distinction for Hume?

The answer must be partly speculative, because the text does not precisely spell out Hume's intentions. I want to propose for consideration an hypothesis that may help to explain how association serves to differentiate between the strength or weakness of psychological attitude in the two sets of theses. It appears that Hume's theory requires a distinction, to which he is in any case committed, between reason and passion, and the alignment of conception as passion, about which Hume is explicit, and of aesthetic admiration or esteem as a function of reason, which is at best implicit in Hume's exposition, and conjectural in the interpretation of his aesthetics of the sublime. The suggestion is that association yields stronger conception as a passion and weaker aesthetic admiration as a function of reason in inverse proportion to the great distance of objects in space as opposed to time, and past as opposed to future time. Thus, association, combined with the distinct natures of reason and passion, accounts for the difference in aesthetic admiration versus conception and interest in Hume's psychology of the sublime.

The evidence for this interpretation is indirect but persuasive. (1) Hume explictly links conception and interest with the passions in the passages quoted above in the context of his discussion of the distinction between reason and the passions. (2) Hume poses a conflict of three sets of theses, partly resolved by appeal to the concept of association, but requiring some further distinction as basis. (3) The distinction between reason and passion satisfies this need; the conflict is plausibly resolved by appeal to the distinction between reason and passion as a basis by which association might discriminate between the two sets of cases. (4) The dichotomy in the context of Hume's discussion of the distinction between reason and the passions is naturally completed, like the inference to a missing term in a ratio, linking aesthetic admiration to reason as Hume explicitly relates conception and interest to the passions. (5) Immediately following his presentation of the second set of distance theses for aesthetic admiration and the associationist resolution of their conflict with the first set for conception, possibly to emphasize its relevance to his analysis of the sublime, Hume recapitulates the distinction between reason and the passions.

The distinction, which Hume elsewhere describes as an 'opposition', is a vital theme of the *Treatise*.[38] Hume summarizes the categories in this way:

> It may not be improper, before we leave this subject of the will, to resume, in a few words, all that has been said concerning it, in order to set the whole more distinctly before the eyes of the reader. What we commonly understand by *passion* is a violent and sensible emotion of mind, when any good or evil is presented, or any object, which, by the original formation of our faculties, is fitted to excite an appetite. By *reason* we mean

[38] *Treatise*, pp. 415-416, 457-458.

> affections of the very same kind with the former; but such as operate more calmly, and cause no disorder in the temper...[39]

The location of Hume's remarks about reason and the passions following the three theses and complementary theses concerning the effect of distance on conception and the passions, and the aesthetic appreciation of greatness in space and time, suggests that the distinction between reason and the passions may also have more specific importance to the conflict. It implies that Hume sees the distinction as underlying the associationist resolution of the conflicting effects of distance on conception as passion and on aesthetic admiration, if the proposed hypothesis is correct, as a function of reason.

It may appear that Hume's categories are at cross-purposes here. On the one hand, he distinguishes between reason and passion, which seems to divide cognitive from noncognitive mental events on the basis of whether or not they involve judgments of matters of fact and relations of ideas. Hume, on the other hand, further distinguishes between violent and calm 'affections', which seems to indicate a subdivision of types within the category of the passions. If this were the correct picture of Hume's distinctions, then his account of reason would be inconsistent. For while Hume separates reason from passion, he also describes reason as a 'calm' affection. If calm affections are a subspecies of passion, then reason is both a passion and not a passion. The error of interpretation, which is natural but not necessitated by Hume's text, is in regarding affection as synonymous with or as a special kind of passion. Hume, despite these intuitive associations, seems to use the terms in a more technical sense. Reason and passion are 'affections' of the mind, or ways in which the mind is affected. But reason and passion are distinguished as different kinds of affections of mind by the careful use of a psychological, introspective criterion, in conjunction with evidence about their respective functions. Passion is typically a violent affection, though Hume also acknowledges the existence

[39] Ibid., p. 437.

of calm passions such as resentment and benevolence, while reason is comparatively calm.[40] Hume remarks on the difficulty of distinguishing between reason and certain calm passions 'by all those, who judge of things from the first view and appearance'.[41] The difference, even in the case of these superficially hard to discriminate calm mental states, centers on his further characterization of reason unlike passion as the faculty distinctively concerned with matters of judgment.[42] On this interpretation, the cross-categorical inconsistency in Hume's statements about reason does not arise, since there is something more to distinguish them even in exceptional situations where they are, superficially, phenomenologically similar.

The account also preserves Hume's discussion from an objection that otherwise threatens his distinction between reason and passion as the basis for his associationist solution to the problem of how great distance in space, and in past time as opposed to space or future time, diminishes conception and interest as passive, but increases aesthetic admiration in the judgment of reason. There is no need to limit Hume's conception of aesthetic admiration as involving only calm passions if reason is a calm affection but not a passion or subspecies of passion, calm or otherwise. The point of recognizing the associationist role of reason in aesthetic

[40] Ibid., p. 417: "Now 'tis certain, there are certain calm desires and tendencies, which tho' they be real passions, produce little emotion in the mind, and are more known by their effects than by the immediate feeling or sensation." See pp. 418-419, where Hume further distinguishes between calm and violent and strong and weak passions.

[41] Ibid.

[42] Ibid.: "When any of these passions are calm, and cause no disorder in the soul, they are very readily taken for the determinations of reason, and are suppos'd to proceed from the same faculty, with that, which judges of truth and falshood. Their nature and principles have been suppos'd the same, because their sensations are not evidently different." Elsewhere Hume does not limit the function of reason to truth value judgments, but in drawing distinctions generally, among which distinguishing between truth and falsehood is a particularly important application. See pp. 25, 43, 73.

judgment is not to exclude passionate, even violently passionate, response, as an accompaniment to some if not all aesthetic experience. But by finding a place for reason alongside passion in aesthetic admiration of the sublime, a secure foundation is provided for the use Hume makes of associationist psychology in explaining his assumptions about the difference in conceptual and aesthetic attitudes toward great distance in space and time, and past and future time.

Hume regards perception and conception as falling within the province of the passions. If the proposed interpretation is correct, Hume also believes that in aesthetic judgment in general or aesthetic judgment of greatness and the sublime in particular, the mind's esteem and admiration for sublime aesthetic qualities of nature and art primarily and essentially belong to reason and reflection rather than exclusively to the passions.

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INDEX